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On Math, Matter and Mind

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We discuss the nature of reality in the ontological context of Penrose's math-matter-mind triangle. The triangle suggests the circularity of the widespread view that math arises from the mind, the mind arises out of matter, and that matter can be explained in terms of math. Non-physicists should be wary of any claim that modern physics leads us to any particular resolution of this circularity, since even the sample of three theoretical physicists writing this paper hold three divergent views. Some physicists believe that current physics has already found the basic framework for a complete description of reality, and only has to fill in the details. Others suspect that no single framework, from physics or other sources, will ever capture reality. Yet others guess that reality might be approached arbitrarily closely by some form of future physics, but probably based on completely different frameworks. We will designate these three approaches as the fundamentalist, secular and mystic views of the world, as seen by practicing physicists. We present and contrast each of these views, which arguably form broad categories capturing most if not all interpretations of physics. We argue that this diversity in the physics community is more useful than an ontological monoculture, since it motivates physicists to tackle unsolved problems with a wide variety of approaches.

KEY WORDS: ontology, mathematics, physics, consciousness

1. INTRODUCTION: THE ROLE OF METAPHOR

Although physicists agree on the formalism of their theories and the methodology of their experiments, they often disagree about the question of what it all means. A few hundred years ago, this was only to be expected,

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since at its inception, physics covered very limited aspects of the world. However, now physics can arguably lay proper claim to providing an effective description of all material processes, if not in practice then at least in principle. And with no obstacles visible to a full understanding of the dance of matter and energy in space and time, what aspect of the world would not be amenable to an analysis by physics, again at least in principle?

The obvious objection would be to point out that notions such as meaning or beauty or responsibility have been explicitly filtered out of physics, as part of its methodology. To believe that you could remove some of the most important aspects of human experience, and then hope to fully reconstruct them through the mathematical formalism of physics strikes many as absurd. This view, that physics can only cover limited aspects of our experience, we will call the secular view. The opposite view that holds that a straightforward application and further exploration of the current framework of physics will eventually cover and explain all of reality we will call the fundamentalist view.

There is a reason for choosing religious terms for our metaphors. First of all, they indicate a mind set, a psychological attitude that corresponds quite closely to the ones we will discuss among physicists. Secondly, they are universal, in that the tension between the more fundamentalist and the more secular views of religion occur everywhere, in Christianity, Judaism, Islam, Hinduism, Buddhism, and so on.

There is one other type of viewpoint that is widespread across religions, and that is the mystic view of the world. Many religions have produced mystics who use precise methods and descriptions in order to find deeper forms of truth that will eventually transcend both methods and descriptions, leading to insight that is utterly down-to-earth but that cannot be captured by the net of description.

Among physicists, the mystic view holds that a future form of physics may come arbitrarily close to a genuine understanding of reality, both from a pragmatic and an "insight" point of view. However, to what extent this insight can be captured in any way through presently familiar forms of mathematical physics or any other future form of physics relying on description, is not clear and is left as an open question. This may sound presumptuous, but there are good historical reasons to expect fundamental changes in what is considered legitimate physics. For example, the discovery of quantum mechanics toppled some of the notions held most near and dear by physicists, such as reproducibility of experiments.

In summary, we will discuss three broad categories of world views that physicists may hold, which we label fundamentalist, mystic, and secular. They differ in their view of whether a physics framework will

ultimately give access to all of reality. The three answers are, respectively: yes, the current framework; yes, a future framework; no, no framework present or future. In what follows, the fundamentalist view is advocated by Max Tegmark, the secular view by Mark Alford, and the mystic view by Piet Hut.

A useful starting point for the debate is the Matter-Mind-Math triangle (Figs. 1–4), put forward by Penrose [1, 2].

This figure encapsulates the idea that matter somehow embodies mathematics, the mind arises from matter, and mathematics is a product of the mind. If such a circular production seems unsatisfactory, we can ask which of these three arrows should be removed. In Sec. 2 we see that the fundamentalist, secular and mystic viewpoints each find different parts of the diagram to be the weakest, and we set forth their arguments and counter-arguments. This sets the stage for Sec. 3, where each viewpoint gives its version of how the diagram should be. Their strengths and weaknesses are debated vigorously in Sects 4–6. We close with the three different visions of the future in Sec. 7 and a brief conclusion in Sec. 8.

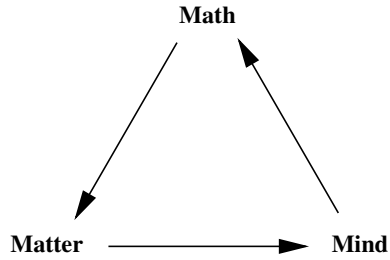

Fig. 1. Our starting diagram, based on a similar one by Penrose [1].

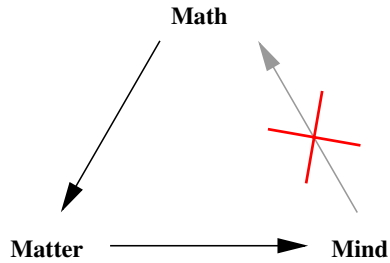

Fig. 2. Fundamentalist critique of the Penrose diagram.

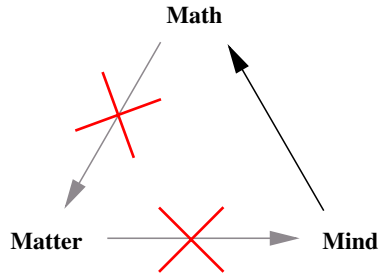

Fig. 3. Secular critique of the Penrose diagram.

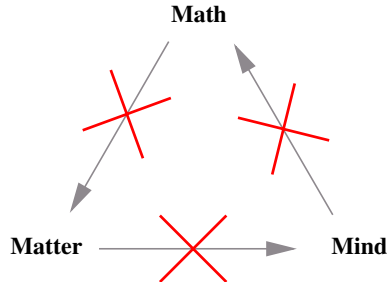

Fig. 4. Mystic critique of the Penrose diagram.

2. THREE CRITIQUES OF THE PENROSE DIAGRAM

2.1. The Fundamentalist Critique

I am a mathematical fundamentalist: I single out math as the underlying structure of the universe, and disagree strongly with the symmetry between math, mind and matter that is expressed in the Penrose diagram. I have no problem with the reduction of the world around us, including our minds, to mathematical laws of physics—rather, I find it elegant and beautiful. I am therefore happy with the Math→Matter and Matter→Mind links, but object to the Mind→Math link.

2.1.1. Against Mind → Math Link

I adopt the formalist definition of mathematics: it is the study of formal systems. Although this pursuit itself is of course secondary to the human mind, I believe that the mathematical structures that this process uncovers are "out there", completely independently of the discoverer.

Consequently, math is not a product of the human mind, and there should be no Mind→Math link.

Math is also unique in its ability to circumvent the classic problem of infinite regress, where every explanation of a statement in human language must be in the form of another unexplained statement. The trick is the emergent concepts idea of Sec. 2.E in [3]. Although whims of human fashion influence the choice of which particular formal systems we explore at any one time, and which aspects thereof, we are continually increasing the amount of charted territory. The street map of mathematical structures is "out there", and any intelligent entity who begins to study any corner of it will inevitably discover at least the main plazas and connecting boulevards, even if many charming back alleys and sprawling suburbs are missed due to cultural prejudice. The key is that the explorer needs no a priori explanation of what concepts like integers, vectors or groups mean, since she herself will introduce notation for them and create her own interpretation of them. Mathematics is thus the same whether it is discovered by us, by computers or by extraterrestrials.

2.1.2. Defense of Math → Matter Link

Although I respect the Secular "shut-up-and-calculate" attitude (Sec. 2.2) I feel that the evidence favors the Math→Matter Link. Physical theories are considered successful if they make predictions that are subsequently verified. The view that the physical world is intrinsically mathematical has scored many successes of exactly this type, which in my opinion increase its credibility. The idea that the universe is in some sense mathematical goes back to the Pythagoreans, and appears again in Galileo's statement that the Universe is a grand book written in the language of mathematics, and in Wigner's discussion of the "unreasonable effectiveness of mathematics in the natural sciences" [4]. After Galileo promulgated the idea, additional mathematical regularities beyond his wildest dreams were uncovered, ranging from the motions of planets to the properties of atoms. After Wigner had written his famous essay, the standard model of particle physics revealed new "unreasonable" mathematical order in the microcosm of elementary particles, and my guess is that history will repeat itself again and again. I know of no other compelling explanation for this trend than that the physical world really is completely mathematical, isomorphic to some mathematical structure.

Let me briefly elaborate on what I mean by this hypothesis that mathematical and physical existence are equivalent. It can be viewed as a form of radical Platonism, asserting that the mathematical structures in Plato's realm of ideas or Rucker's "mindscape" [5] exist in a physical sense. It is

akin to what John Barrow refers to as "π in the sky" [6], what Robert Nozick called the principle of fecundity [7] and what David Lewis called modal realism [8]. However, it is crucial to distinguish between two ways of viewing a physical theory: from the outside view of a physicist studying its mathematical equations, like a bird surveying a landscape from high above it, and from the inside view of an observer living in the world described by the equations, like a frog living in the landscape surveyed by the bird. From the bird perspective, the physical world is a mathematical structure, an abstract, immutable entity existing outside of space and time. If history were a movie, the structure would correspond not to a single frame of it but to the entire videotape.

Consider, for example, a world made up of pointlike particles moving around in three-dimensional space. In four-dimensional spacetime – the bird perspective – these particle trajectories resemble a tangle of spaghetti. If the frog sees a particle moving with constant velocity, the bird sees a straight strand of uncooked spaghetti. If the frog sees a pair of orbiting particles, the bird sees two spaghetti strands intertwined like a double helix. To the frog, the world is described by Newton's laws of motion and gravitation. To the bird, it is described by the geometry of the pasta – a mathematical structure. The frog itself is merely a thick bundle of pasta, whose highly complex intertwining corresponds to a cluster of particles that store and process information. Our universe is of course far more complicated than this example, since we do not yet know to what mathematical structure it corresponds. Part of the challenge here is that reality can appear dramatically different in frog and bird perspectives, and the phenomenology linking them can be more difficult than finding the correct mathematical structure itself. It took the genius of Einstein to realize that frogs living in Minkowski space would perceive time to slow down at high speeds, and that of Everett to realize that a single deterministically evolving quantum wavefunction in Hilbert space contains within it a vast number of frog perspectives where certain events appear to occur randomly.

2.1.3. Defense of Matter → Mind Link

I believe that consciousness is the way information feels when being processed. Since matter can be arranged to process information in numerous ways of vastly varying complexity, this implies a rich variety of levels and types of consciousness. The particular type of consciousness that we subjectively know is then a phenomenon that arises in certain highly complex physical systems that input, process, store and output information. This implies that there is nothing wrong with the Matter → Mind link.

This hypothesis has clearly not been proven. However, this can hardly be held against it, since it can strictly speaking never be proven: I cannot

even prove to my colleagues that I personally am self-aware—they simply have to take my word for it. Moreover, the spectacular successes of computers and neural networks over the past decades have arguably made the hypothesis appear less implausible than before.

Although believing in this hypothesis may make some people feel less happy about what they are, I have so far seen no hard scientific evidence against it. Rather, many objections seem to be based on a combination of human vanity and wishful thinking. Humanity has already had its collective ego deflated so many times (by Copernicus, Darwin, Freud, infinite Universe cosmology, Deep Blue, etc.) that yet another demotion would not bother me at all. I am what I am and will continue to enjoy feeling the way I subjectively feel regardless what the underlying explanation turns out to be.

2.2. The Secular Critique

I am a secular scientist. I enjoy practicing science, and believe that it is of great practical importance and intellectual value. Everyone should take science seriously, while remembering that it is a human creation, not an all-embracing metaphysics. I am therefore happy with the Mind→Math link. I appreciate the scientific interest and usefulness of physiological explanations for human behavior, but I am unenthusiastic about the Matter→Mind link as a metaphysical claim. I disagree strongly with the Math→Matter link.

2.2.1. Against Math → Matter Link

All physical scientists are impressed by the "unreasonable effectiveness of mathematics in the natural sciences" [4]. That effectiveness is indeed spectacular. But the Math→Matter linkage in the Penrose diagram tries to go further, inferring from the effectiveness of math that it is itself the ultimate substance of the universe. Wigner, however, avoiding metaphysical speculation, simply concluded that we should be grateful for the effectiveness of mathematical methods, and get on with the business of exploiting them. I agree with him.

I will give my detailed criticism of the Math→Matter link in Sec. 6.1, where we discuss the Fundamentalist's version of the MMM diagram, in which this link plays the foundational role. For now I just want to point out that although our current theories of physics and our current way of doing mathematics may seem inevitable, we should be wary of assigning them any kind of fundamental status. History shows that both physics and math have changed markedly over time.

The Fundamentalist suggests that math is a particularly secure foundation because it can resist the infinite regress of explanation via "emergent concepts". But an infinite regress of explanations only arises when one is confronted by the ultimate skeptic, who demands proof of everything. Such a person will not believe the "emergent concepts" claim either, since it cannot be proved. For any normal person, explanations will be accepted at some point, the infinite regress will never arise, and math will not be needed as a foundation for knowledge.

2.2.2. Against Matter → Mind Link

Like the Mystic, I am unenthusiastic about the Matter→Mind link, if it is taken as a form of reductionism that states that the mental phenomena are "really just material processes", as if the material were somehow more fundamental than the mental. Of course it is conceivable that everything could be reduced to physics, but it is also conceivable that it could not. Our scientific theories are not definitive accounts of nature's secrets, so there is no point in worrying about what it would be like if they were. And what happens as our theories of the material world become better and better? Is there a gradual reduction of the mental to the physical? Not at all. We can find out as much as we like about the brain, but this does not provide a new and better foundation for our concepts of mental things [9]. For example, science may find brain processes that correlate with mystic experiences, or genes that determine shyness. Such progress is extremely interesting, but it happens not to tell us anything about the *meaning* of "the mystical" or "shyness".

Of course, scientific progress does influence our beliefs and attitudes. Rather than saying that our friend is possessed by demons or went crazy we may now talk about whether he is suffering from paranoid schizophrenia or manic depression. These are not just synonyms for "crazy", they represent a richer picture of the person involved, a more specific set of assumptions about his probable behavior, even a different view of his curability and humanity. Our mental concepts have evolved, but they have not been reduced to physical ones. Even if, as the Mystic fears (Sec. 2.3), we started talking in materialistic terms—"My brain processes are causing me to leave the room"—would that mean we were now right where we had been wrong before?

I therefore find myself sharing the Fundamentalist's professed indifference to scientific discoveries about how the brain works. Would people feel diminished if it became possible to build Turing-test-passing machines? I don't think so. The machines we currently make are stupid. If we made dazzlingly intelligent, kind and beautiful machines we would see nothing

wrong in being like them. We would probably claim they were patterned after our image.

2.2.3. Defense of Mind→ Math Link

The Fundamentalist (Sec. 2.1.1) and the Mystic (Sec. 2.3.1) both express the view that Math cannot be dependent on the human mind because mathematical correctness has a solid "out-there" feeling about it. I do not find this a compelling argument. Many conceptual systems with which we are deeply familiar give us that feeling of inevitable rightness. For example, the grammar of ones native language has exactly this character: one "stumbles over" the incorrectness of ungrammatical sentences. Even morality has a solid core of indisputably right and wrong types of actions to which we have an immediate and visceral response, though we normally do not discuss these because we take them for granted and spend our time arguing over the more ambiguous cases.

The denial of the Mind → Math link corresponds to the Mathematical Platonist doctrine that mathematical objects have an independent existence of their own. It is interesting in the context of the MMM diagram to note that the main argument for Mathematical Platonism is the "Quine-Putnam indispensability argument" [10, 11] which exploits the Math-Matter connection. Quine and Putnam point out that mathematics is an essential component of science, as are other exotic entities such as electric fields and protons. So if we grant that electric fields and protons exist independently of us, then we should give the same status to the mathematical objects (real numbers, Hilbert spaces, etc) that we use in science. Many philosophical objections have been raised against the Quine-Putnam argument [12]. As a physicist I find it unconvincing because there is a wide range of entities that are posited in physical theories, from fairly concrete ones like electric fields to more abstract ones like an object's wave function, or its center of gravity, that we don't (all) think of as "existing" or "being real" in the normal sense of those words. So just because something plays a role in our theorizing doesn't mean it is a real object. And when a physicist argues that something is real he or she invokes a specific instance where the object is present: "of course electric fields are real: you can tell that there's an electric field around a proton because it pushes other positive charges away." One couldn't speak of Hilbert space or the complex plane as giving away its presence in a similar way. This is related to the fact that what physicists do when they check whether their theories give a good description of the real world is very different from what mathematicians do when they check whether their results are true. In fact, by scientific standards (and that is what Quine and Putnam are ultimately appealing

to) mathematics really doesn't have a "check against nature" step in its procedures at all. Checking whether a theorem is true is not like an experiment, it is more like checking whether a sentence is grammatical.

In his critique of the Mind→Math link, the Fundamentalist asserts that any intelligent entity will agree with us about math. For example, aliens from another planet might have incomprehensible ideas about many things, but their math would map on to (or extend) ours. This is a rather vague statement because it would be up to us to decide which parts of the alien *œuvre* we call "math". It could just reduce to the empty statement that if we pick out the parts that look like our math, then they will map onto our math. The Fundamentalist takes it as an article of faith that a nice mapping, covering a reasonable amount of material, would necessarily be found, but this need not happen. We might be faced with "inscrutable aliens", in whose behavior we cannot cleanly identify anything that corresponds to "doing math". For example, their successful feats of engineering might not be based on a coherent mathematical theory of physics but on what looks to us like a set of rules of thumb. Another possibility is "pragmatic aliens" [13], who study mathematical questions, but with a different approach from our own. Like us, they have proven Fermat's last theorem: they simply computed millions of examples, and declared it true. Their mathematicians have developed such numerical methods to a high art, and have numerical "proofs" of many theorems that we had not even conjectured. They seem unimpressed by the rigor of our analytic proofs, arguing that there is little fundamental difference between the small chance that their numerical searches have missed errors in their theorems, and the small chance that we have failed to notice errors in our analytic proofs.

Of course, even if we find we can agree on math with all the aliens that we have met, it still wouldn't cast doubt on the Mind→Math link. It would just mean that in certain respects our minds are alike. One could speculate on evolutionary reasons for that, or simply accept it as a fact. And of course, the next batch of aliens might turn out to be different.

2.3. The Mystic Critique

I take a mystic view of science. Unlike the Secularist, I expect that science will ultimately give us profound insights into the real nature of the world. But unlike the Fundamentalist, I believe these will not emerge in any straightforward way from science as it is currently constituted. Rather, I expect science to metamorphose into something so different that it is literally inconceivable for us. So in that sense I agree with the Secularist

that physics will probably see upheavals even (far) more fundamental than the discovery of quantum mechanics. And I agree with the Fundamentalist that Science will ultimately come arbitrarily close to a full understanding of reality.

The reason I like the word 'mystic' is that the future science I envision will be so different from current science, and the role of elements such as math and experiments will be so different from what they are now, that we have not the foggiest idea of what these will look like. The structures of a future science will remain a mystery, and the only thing we can be pretty sure of is that our current lines of reason will be seen to be naive and superficial, compared with the newer and deeper insights.

So let me be clear: the word 'mystic' for me points to a form of probing into mysteries, as it was meant in Medieval times. Note that mystics were very keen to try to show structure and to enumerate parts of that structure – the term 'mystic' just happened to get a bad rap later on, and is now unjustifiably associated with attempts to confuse and muddle a situation.

As for the Penrose diagram, I have deep doubts about all the links. Making these links now, before a future unification, seems premature. I strongly believe that the process of unification, which has successfully uncovered intrinsic links between, e.g., electricity and magnetism, space and time, matter and energy, will continue. What can be more different than matter and energy? Their unification was totally unexpected. If history is any guide, future unifications will occur that are currently equally unexpected. And one example may well involve our three M aspects, matter, mind, and math. These three can then no longer can be treated as independent notions that have the power to point to each other.

Drawing arrows, in my view, is simply a precursor to the program of unification, in which nature is discovered to be already unified more than we had thought. It was through tracing the arrows between electricity and magnetism – how exactly can an electric charge generate a magnetic field, and a magnet generate an electric field – that electromagnetism was discovered.

Science, like any human activity, is ultimately given in experience, and understood through the lens of conscious experience. Within experience, we can discern subject and object poles. The trend of science, so far, has been to explain/reduce all phenomena to processes that are described purely in terms of objects. The rise of the subject is seen as somehow being a byproduct of sufficiently complex phenomena, taking place in brains, material systems that can be fully described objectively. While not denying the correlations between subjective experience and objective processes in our nervous system, I do not want to buy into an unquestioned prior status of the object pole over the subject pole of experience [14].

The Fundamentalist defends the Matter→Mind link on the basis of advances in neurophysiology. I do not deny that a deep understanding of the material structure of the human brain will shed a lot of light on the way *in which* we experience; but the very fact *that* we experience may completely fall outside such an explanatory framework. My guess would be that such a question requires a shift to a wider horizon of knowledge/meaning/explanation [15, 16].

Let me give an analogy. Imagine a world in which there are no periodic phenomena, and hence no clocks of any type in nature. Someone living in such a world may not easily discover the concept of time, and certainly not a concept of a linearly progressing time, that can be mapped onto a one-dimensional line of real numbers. But it would be wrong to draw the conclusion: no clocks no time, hence clocks produce time. Similarly, it is too simple to say: no brains no experience, hence brains produce experience. Reality may be a lot more subtle. Brains may tune into an aspect of reality that is explicitly filtered out when setting up laboratory experiments and when formulating mathematical regularities summarizing the lab experiments. This is just one analogy, and merely meant to illustrate the fact that there is a lot more room within the "horizon of knowing," than we normally consider.

To try to explain our experiences as somehow arising out of matter is a tempting project. And of course there is a very tight correlation between the thoughts we think and the precise electromagnetic and chemical processes in and between our brain cells. However, the problem with identifying brain states with experience is that we are short-changing ourselves. When people talk about their experience by invoking scientific images, such as "my hormones drive me to do such and such" or "my taste buds enjoyed so and so", they are using technological thinking as a mode of alienation.

When the Fundamentalist says "I am what I am", it strikes me as hopelessly naive. While a sudden new discovery will not instantaneously change the way we experience the world, by the time the new knowledge seeps into the way we view the world, it definitely colors the way the world presents itself to us. The existence of placebos is just one striking example of this phenomenon.

2.3.1. Transcendence of All Three Links

I neither want to attack nor defend the three links in the Penrose diagram, in any absolute way. Rather I want to transcend them, after first defending them in a relative way.

The proper defense of all three links is not by ascribing to them any power in terms of causality, but by pointing to meaningful correlations that exist between the three M-elements. To take the example of a movie: within the movie, all kind of phenomena seem to 'cause' other phenomena, in rather precise ways, but we know that the real cause lies in the projector, and in the process of shooting the film in the first place. The correlations are still important, since without them the movie would be just a heap of flickering lights without meaning. But the importance of the correlations is strictly limited to the framework of the movie, and has no fundamental meaning whatsoever.

So perhaps this is what makes me a mystic, in that I am uttering seemingly contradictory statements. Among the three of us, I am the only one who can accept all three links, without being bothered by the circular nature of the 'vicious triangle' links. Since I only ascribe relative meaning to them, I have no need for any foundation. None of the elements within the story of reality is absolute or basic; all elements emerge simultaneously from a deeper unification, speaking in physics lingo. In Buddhist lingo, for example, this could be called co-dependent arising [17].

The Platonic position of the Fundamentalist, that mathematical truth exists all by itself, is extremely appealing because it seems to correspond closely to our experience. When we struggle with a mathematical problem, and finally find a solution, we sometimes "stumble" upon it in a way that is rather similar to the way we stumble over a chair. The resistance that mathematical objects show to our attempts to prove what later turns out to be false is akin to the resistance that physical objects show when we try to wish them away – both appear to have an existence independent of the presence or absence of individual humans. In this sense, I am sympathetic to Alain Connes' notion of archaic mathematical reality as being as real as physical reality [18].

The Secular view that matter and mind have their own meaningful existence I also find extremely appealing, within our every-day explanatory framework. In practical terms, it makes sense to deal with the world around us in terms of material objects and energetic processes, and it also makes sense to treat our experience as something that has equal pride of place.

The problem arises when we try to isolate elements from this story, and point to some of them as truly fundamental. In my view, a future physics will transcend any story we have woven so far.

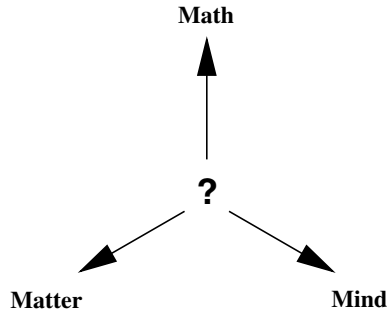

Fig. 5. Mystic's view: other-source diagram.

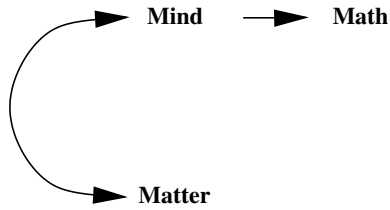

Fig. 6. Secular view: no-source diagram.

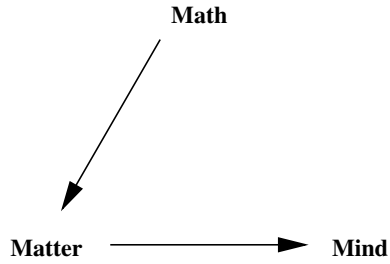

Fig. 7. Fundamentalist view: radical Platonist diagram.

3. BEYOND THE TRIANGLE: THREE VIEWS

3.1. The Mystic View: the Other-Source Diagram

Mystic
I'm against all three arrows in the original picture. In my alternative picture, "?" stands for an origin that cannot be easily described, the way each of the other three can. Our three M's are more like the shadows on the wall of Plato's cave; or in another metaphor, they are the fish that can be

dragged up with the nets of discursive/conceptual thinking. The Source or Origin lies beyond that, and is more real than any particular element of what we conventionally take reality to be.

Note here that in fact, upon finer scrutiny, the separation between "?" and the three M's is only illusory. The real mind cannot be captured in a description, nor can the real matter. I'd say that even the real math cannot, if you include the living intuitive process of discovery.

3.2. The Secular View: the No-Source Diagram

Secularist
In so far as the relationship of matter, mind, and math can be expressed in the form of Penrose-type diagrams, I prefer Fig. 6. It presents mind and matter as two different concepts, but ones that are used together in a single picture of the world. It places math at a lower level than matter and mind: math is a particular mental activity, among many others. There is no reason to give it a privileged position above, say, morality or language use.

Although mind and matter are separate in this diagram, I do not want to imply that they are disjoint (dualism). I see them as two aspects of the world that we happen to distinguish quite sharply.

3.3. The Fundamentalist View: the Radical Platonist Diagram

Fundamentalist
My view is that mathematical structures (the cube, manifolds, operator algebras, etc.) exist quite independently of us humans, so math must be promoted to the fundamental vertex, as in Fig. 7. The human mind then emerges from math, as a self-aware substructure of an extremely complicated mathematical structure [3, 13, 19–21]. Each such substructure subjectively perceives itself as existing in a physically real sense. Given the mathematical equations that describe our Universe, an infinitely intelligent mathematician could in principle deduce the properties of both its material content and the minds of its inhabitants.

4. DEBATING THE MYSTIC'S OTHER-SOURCE DIAGRAM

4.1. Secular Critique of Mystic's Other-Source Diagram

Firstly, I would say that the main difference between the Mystic and the Fundamentalist is in the question of timing. The Fundamental-

ist thinks we already know the ultimate constituents of the world; the Mystic thinks we will come to achieve such enlightenment in the future. This makes it harder to criticize the Mystic's approach, because it does not yet have any specific form, but the basic objection is the same: why will we be entitled to declare the final victory at some particular moment? When is the right time to promote whatever picture we have found workable to metaphysical status?

Secondly, I do not think it is meaningful to invoke a hidden or ineffable source. If it cannot even be described, what role can it play in our quest for knowledge? Playing with the definition of the word "real" seems to me to be a similarly arbitrary exercise.

Thirdly, I do not think that math deserves to be put on the same footing as mind and matter. There are non-mathematicians, but there is nobody who is non-mental or non-material! What determines which human activities we include as separate poles connected directly to the ineffable origin? Why not include language, music, art, and so on?

4.2. Mystic's Response

I see science as converging further and further toward what is true about reality, but I don't expect there to be a particular point at which we can declare victory. The road toward deeper insight may be unending, simply because the degrees of what can be called insight may be inexhaustible. Classical mechanics seemed to give complete insight, in principle, but quantum mechanics offered a type of insight that was qualitatively deeper and other. I have no reason to expect this process of discovery to stop. My main point is not that we will reach a final truth, but rather that we will never reach an absolute boundary beyond which we cannot progress in getting closer to the truth.

As for the ineffability of a source, that is just a relative notion. When all you have are concepts such as electricity and magnetism, then electromagnetism is ineffable, since it doesn't fit into your framework. It has to be invoked as a new type of source that can be projected in your existing framework, very much like the shadows in Plato's cave. But then you enlarge your framework in order to include the new source.

By the way, the requirement that any source should be describable is not something I want to buy into. Just as quantum mechanics has tossed out some of the deepest held convictions of classical mechanics, similarly I expect a future science to throw out some of what we now still see as absolutely essential for the scientific method. Our current notion of a description may be one of them. We already see a harbinger of this

shift in quantum mechanics, where the perfectly effable classical idea of an exhaustive description of all degrees of freedom of a particle has turned out to be impossible.

As for the Secularist's existence proof for non-mathematicians, I agree. However, I had in mind something wider than technical mathematics. If we want to classify what we find around and in us under the three headers of matter, mind, and math, we'd better take each of these terms rather generally. Under matter we take anything physical, such forms of pure energy like radiation, or even the vacuum nowadays. Similarly, under math we should take any conceptual structure that we use in our mind in order to make sense of both mind and (material) world. Language, logic of all sorts (including common sense reasoning), any distinctions that are found to be useful I would group here under "math" – take counting, for example, as a mapping operation to divide things in the world into distinct categories. In this way, I think you will agree that there are no non-conceptual humans, as long as they function normally.

In other words, I don't think we are dealing with mathematics here as just one of out many academic subjects, such as physics or biology or art or literature. The first two subjects apply more to the "matter" corner of our triangle, the latter two more to the "mind" corner of our triangle, but "math" stands out as the surprising fact of the regularity of the world, or if you like, the surprising fact that we can view the world in such unexpectedly regular ways and get away with it, without contradictions, and with the ability to make accurate predictions.

4.3. Fundamentalist Critique of Mystic's Other-Source Diagram

My main objection to both the other source view and the no-source view is an admittedly emotional one: I perceive them as defeatist. Figures 5 and 6 both indicate that the quest for a fundamental theory of everything is ultimately doomed, and that the best we can do is go off and meditate or optimize a ball bearing. Although the Mystic hopes for great conceptual breakthroughs ahead, I am not persuaded by his hypothesis that we will come arbitrary close to a complete understanding while remaining precluded from ever attaining it. I also find the Mystic battle plan disturbingly vague, failing to understand what specific steps we should take to explore the object pole. In contrast, Fig. 7 can be taken as a specific battle cry to forge ahead in search of the equations of a truly fundamental theory. This theory may turn out to be too complex to be comprehensible to us humans, but at least there's hope. We shouldn't give up without trying!

4.4. Mystic's Response:

I applaud the enthusiasm of the Fundamentalist, and I share it. I only differ in the direction in which I think we are likely to find the answers. The Fundamentalist is in effect looking for the lost key under the lamp post. He takes the objectivistic program, in which everything is explained in third-person terms as interacting objects, and hopes that program will carry the day. But I see that as a limiting case of a wider program, in which first-person experience and a study of the subject *qua* subject will augment our already very detailed studies of objects. Within that wider program, I think we have much more of a chance to find a horizon in which we can see how all of our experiences hang together in a meaningful way, without artificially reducing everything to a particular subset of privileged phenomena.

What we really need to do is to go back to the state of science as it was in the days of Galileo. He focused on some very simple experiments, in which he rolled balls from inclined planes and the like. I think we should similarly focus on very simple experiments in which we take the subject equally seriously as the object, and try to see where that leads us. Such an approach may well point the way to deeper forms of unification, including the unification of subject and object [14].

5. DEBATING THE SECULARIST'S NO-SOURCE DIAGRAM

5.1. Fundamentalist Critique of the Secularist's No-Source Diagram

The Secular view seems even more defeatist to me than the Mystic one. In the Mystic's view, there is at least an underlying unity that we may one day be able to understand better. The Secularist doesn't even want to bother searching for an underlying reality, since it simply doesn't exist. We are admonished to stop seeking a deeper understanding since there is nothing deeper to understand. I am convinced that there is a deep explanation for it all, the only question being whether we can understand it from the frog perspective of our limited human minds.

5.2. Secular Response

The Fundamentalist needs to open his eyes to the world that lies outside theoretical physics. Of course there is a thrill in the thought that your research is opening up the blueprints of the universe, but it is the guilty

pleasure of surrendering to vanity and parochialism. To submit to it, you must convince yourself that we are living at a special time, when the basic structure of math has been determined ("formal systems"), and the final theory of physics is at hand. These metaphysical yearnings are easy to empathize with, but the whole approach is shamelessly ahistorical. It has beneficial side-effects (the ability to raise public enthusiasm, for example) but also leads to a devaluation of the normal process of day-to-day science.

The Fundamentalist says we should be searching for an "underlying reality". I agree that the laws of nature are not immediately obvious, but reality itself is not lying under anything. It is available for us to explore by scientific or other means. Scientific methods have been highly successful in giving us a coherent picture of and control over the world we live in. In any reasonable sense of the word "understand", I believe there are many aspects of the world that we either already understand or can hope to understand in the future. However, the Mystic and Fundamentalist both want to go further, to an *unimprovable* understanding. They have some concept of a hidden universal truth. But this can only be defined as that which is yielded by the method they suggest for finding it, and there is no reason to think that this method will converge to a unique result.

5.3. Mystic's Critique of the Secularist's No-Source Diagram

Even though the Secularist claims not to be a dualist, and affirms unity between the aspects of mind and matter, his description of mind and matter as two different "aspects" makes the two so separate as to make it very hard for me to see where the unity lies. At least a material reductionist has a clear form of unity to show for, in that everything is considered to be reducible to (properties of) matter. In the Secular view, the two "aspects" that we "happen to distinguish sharply" form an effectively dualistic picture of the world.

The whole notion of dualism as having any ultimate standing, explicit or implied, goes against all that we have learned in the history of physics. Time and again, we started studying two seemingly separate topics, like space and time, or electricity and magnetism, or matter and energy, only to find out that they were merely poles or aspects of a single more fundamental entity, such as spacetime, electromagnetism, or matter-energy. In other words, physics teaches us that dualistic descriptions are nothing but harbingers of a deeper level of non-dual understanding.

Even apart from my intuition as a physicist, all I ever see in dualistic thinking is some form of either laziness or fear. Yes, it is much simpler to keep the world neatly partitioned, and then to study either side of the fence at leisure. Crossing the fence requires more work and is more risky, in that it may well undermine, at either side of the fence, many prejudices held near and dear for a long time.

Specifically, the strongest argument against dualism is: if mind and matter really have nothing to do with each other, how come they show up in the same world, and how come they do *seem* to interact so tightly?

5.4. Secular Response

Perhaps the Mystic and Fundamentalist are reading too much into my diagram: it is drawn in a pragmatic rather than a metaphysical spirit. By their standards I should refuse to draw any diagram at all. I see my diagram as similar to the common-sense one that divides living beings into plants and animals. These are obvious categories that reflect important distinctions. That is all it is trying to say.

6. DEBATING THE FUNDAMENTALIST'S RADICAL PLATONIC DIAGRAM

6.1. Secular Critique of the Fundamentalist's Radical Platonic Diagram

The Math→Matter link plays a crucial role in the Fundamentalist's diagram. It corresponds to a very strong set of statements:

(F1) Physics is converging to some set of ultimate laws,

(F2) Our study of mathematics is converging towards an "ultimate math".

(F3) The ultimate laws will be expressed in terms of the ultimate math. Thus math has a special relationship to the material world.

(F4) It is plausible (or perhaps just aesthetically appealing) that the ultimate math is not just an external reality, but that our Universe consists of it.

I find all these claims to be either meaningless or dubious.

Against (*F1*)
It is not clear that physics is converging to a set of ultimate laws. Physics has changed greatly even in the last 100 years. Our basic understanding of space and time was revolutionized, and even the core concept of the determinism of physical law was radically revised by quantum mechanics, which introduced elements of randomness that classical physicists would have found unbelievable. There is no reason to think that physics is now immune to such upheavals, or that it ever will be. A related question is: when will we declare that the convergence is complete? The cost of testing our "fundamental" physical theories has escalated dramatically over the last few decades as the relevant energy scale rises. It seems likely that as these theories are successively improved, it will become harder and harder to do the experiments that would test them. The theory that survives uncontradicted may do so mainly because the difficulty of falsifying it becomes insuperable. Another way to say this is that the time between revolutions in physics will become longer, and at some point one will not be able to tell whether it is infinite. When this happens, the human need for novelty will lead science down other, more rewarding, paths than the search for a fundamental theory.
Against (*F2*)
Math is certainly progressing, but there is no reason to think that it is converging to a definite structure, with fixed questions and established ways to address them. Consider the history of math. There was mathematics before Hippasus the Pythagorean showed that $\sqrt{2}$ is irrational; mathematics before Russell's paradox, when it was thought that arithmetic could be reduced to set theory; the famous battle between intuitionists and formalists, before Gödel showed that there were true but undecidable propositions; the computer proof of the 4-color theorem by Appel and Haken. In each case we find that progress did not just fill in some gaps, but modified the idea of what mathematics is or should be. Mathematics has taken radical turns in the past and will do so in the future, stepping out in new directions that confound attempts to systematize it once and for all. I think this makes it impossible to formulate a coherent concept of the "ultimate mathematics".
Against (*F3*)
It is striking that mathematical methods have been outstandingly successful in physics. But what is the content of the assertion that any supposed ultimate laws of physics must be "mathematical"? As I argued above, mathematics has evolved over time. It has also been strongly influenced by physics. Five hundred years from now physicists may be using methods that we would find hard to recognize, and mathematics may have grown to include those methods simply because they work for physicists. Or perhaps the intellectual map will have changed to the point where we can't cleanly identify a domain that we would call "physics". Perhaps the

Fundamentalist would prefer to claim that ultimate physical laws will have the character of what we *now* call mathematics. That is simply a statement of faith, and the Fundamentalist does not offer any reason to believe it.
Against (*F4*)
(a) Plausibility. The Radical Platonist position is just another metaphysical theory like solipsism, and even materialism. In each case one starts with a decent theory (mathematical descriptions of nature, sense-data as the basis of cognition) and then raises it to an object of worship, as having finally captured the essence of reality. Philosophers from Kant to Wittgenstein have criticized such thought for taking loose use of language to an extreme where it fosters intellectual illusions [22]. In the end the metaphysics just demands that we use a different language for saying what we already knew. But what is the difference? Nothing is gained by reformulating "it is dark in here" as "I have few visual sensations" or "the local intensity of electromagnetic radiation is low". Talk of conscious beings as "self-aware substructures" is a similarly empty transcription.
(b) Aesthetic appeal. It is not clear that Math $\rightarrow$ Matter is aesthetically attractive. Even if there were some ultimate mathematics, it might be nothing like what we *now* call math. It could easily seem as ugly and contrived to us as irrational numbers and Russell's "repaired" set theory did to their contemporaries when they were formulated.

Finally, the Fundamentalist describes himself as a formalist and a Platonist, but these are contradictory. A Platonist believes mathematical truths are truths about some world of mathematical objects, while a formalist believes that math is just the sum of all strings that you can get by manipulating symbols according to rules, starting with arbitrary axioms. As Gödel's theorem shows, these are two different things: the methods allowed by formalists cannot prove all the theorems in a sufficiently powerful system. There are systems as powerful as arithmetic that are consistent and complete, and that therefore *cannot be axiomatized* (Ref. [13], p. 126), and so are outside the formalist structure. This spells doom to the Fundamentalist's project. The idea that math is "out there" is incompatible with the idea that it consists of formal systems.

6.1.1. Fundamentalist Reply

I would rephrase my assumptions as a Fundamentalist as follows:

- **Assumption A1:** That the physical world (specifically our Level III multiverse [21]) is a mathematical structure.
- **Assumption A2:** Mathematical multiverse: that all mathematical structures exist "out there" in the same sense.

The Secularist (F1) critique of A1 is focused on the question of whether our understanding of physics will converging towards fundamental laws. I believe that if we fail in this quest, if will be because of the limitations of our human minds rather than because of the nature of reality: I view it as almost tautological that there are some fundamental laws that nature obeys: I assume that there is an external reality that exists independently of us humans, and the laws of physics are how this reality works. Denying this external reality would be flirting with solipsism, and I view it as human vanity taken to the extreme.

Although the (F2 & F3) critique above suggests that "ultimate math" is vague and undefined, there is nothing vague about my two assumptions. The notion of a mathematical structure is rigorously defined in any book on Model Theory. The integers are well-defined even though most of them have never been used in human calculations, and mathematical structures are likewise well-defined even though most of them have yet to be explored by mathematicians. The Secularist argued in Sec. 2.2.3 that alien mathematics might be unrecognizable to us. If so, it would only be because we are uncovering a different part of what is in fact a consistent and unified picture, so math is converging in this sense. The reason why this is far from apparent is that our development of mathematics is in a very early stage, nowhere near a systematic classification of even the most basic structures/formal systems. Our attention is therefore drawn to interesting features in the mathematical landscape such as theorem X or formal system Y, and we may not yet see the forest for all the trees.

The secularist criticizes (F4) for loose use of language. To me, *all* use of human language is necessarily loose and hence insufficient for describing an external reality existing independent of us. This is why nature speaks the language of mathematics and why I am advocating mathematical language to describe reality.

The objection that (F4) may be aesthetically unappealing takes the anthropic principle to a comical extreme, suggesting that the universe must be devised so as to make us like it (note also that by Assumption 2, we are part of merely one particular mathematical structure in the vast Level IV multiverse of all structures, some subjectively more elegant than others). I view the mystic's objection in Sec. 2.3 to technological thinking as a mode of alienation as either invoking similarly wishful thinking ("our universe must be devised so as to not make us feel alienated") or as encouraging thought control.

Aside from Wigner's above-mentioned ubiquity of mathematics in physics, a second argument supporting assumption A1 (and F3) is that abstract mathematics is so general that *any* fundamental "theory of everything" that is definable in purely formal terms (independent of vague

human terminology) is also a mathematical structure. For instance, a TOE involving a set of different types of entities (denoted by words, say) and relations between them (denoted by additional words) is nothing but what mathematicians call a set-theoretical model, and one can generally find a formal system that it is a model of. In other words, if the physical world exists independently of us humans, it is not obvious that it can avoid being a mathematical structure.

Given A1, a second argument for assumption A2 (and F4) is that if two entities are isomorphic, then there is no meaningful sense in which they are not one and the same [23]. This implies assumption A2 when the entities in question are a physical universe and a mathematical structure describing it, respectively. To avoid this conclusion that mathematical and physical existence are equivalent, one would need to argue that our universe is somehow made of stuff perfectly described by a mathematical structure, but which also has other properties that are not described by it. However, this violates assumption A1 and implies either that it is isomorphic to a more complicated mathematical structure or that it is not mathematical at all. The latter would be make Karl Popper turn in his grave, since those additional bells and whistles that make the universe non-mathematical by definition have no observable effects whatsoever.

Finally, I find the objection involving Gödel to be very interesting and subtle. My hypothesis is that only Gödel-complete (fully decidable) mathematical structures have physical existence. This drastically shrinks the Level IV multiverse, essentially placing an upper limit on complexity, and may have the attractive side effect of explaining the relative simplicity of our universe. If you define mathematical structures as (equivalence classes of) models of axiom systems [3], then they are guaranteed to be Gödel-complete (consistent). Please note that although we conventionally use a Gödel-undecidable mathematical structure (including integers with Peano's recursion axiom, etc.) to model the physical world, it is not at all obvious that the actual mathematical structure describing our world is a Gödel-undecidable one—lacking a theory of quantum gravity, we have certainly not found it yet. Even a world corresponding to a Gödel-complete mathematical structure could in principle contain observers capable of thinking about Gödel-incomplete mathematics, just as finite-state digital computers can prove certain theorems about Gödel-incomplete formal systems like Peano arithmetic.

The classification project for consistent mathematical structures may well prove far too difficult for us humans to complete, but even partial success in this endeavor could be useful, since the tiny fraction of mathematics uncovered so far has gone such a long way in understanding the physical world. So let's not give up without trying!

6.2. Mystic Critique of the Fundamentalist's Radical Platonic Diagram

On the convergence of math, I remain agnostic. I agree that, historically, new developments were completely unpredictable, and unexpected. And yes, there is a "resistance" to mathematical objects that makes them in many aspects like physical objects. As I mentioned earlier, you can stumble on properties of triangles, like you can stumble on furniture. In that sense, there are certainly not a matter of fashion or convention (although the framework, within which there are defined, may be; I guess that is Secular's main point). The Fundamentalist's belief that the current framework is more than fashion is, I guess, a belief. I don't see clear arguments in favor of it, although I must say, intuitively, it is rather appealing.

I share the Secularist's doubts about the idea that the world is "made of math". My argument against this is that it seems to me like a category mistake. The category of existence of physical objects is different from the category of existence of mathematical objects. For one thing, this particular chair here exists, and is distinct from a "chair in general'. Triangles always belong to the latter category; I've never come across a particular triangle with an individual existence like that of that of a chair.

My intuition tells me that mathematics can never be exhausted. Whenever we have tried to formalize a system in the past, we wound up formalizing some neatly delineated piece of turf, very interesting by itself, but leaving out unexpected other developments that did not fit the mould. Unless the Fundamentalist comes up with a very good argument for the possibility of finding an overarching "space of all models" together with a clear structure governing that space, I don't see how his project could ever work. In mathematics, asking such questions as "what is the set of all sets" has always run into inconsistencies. I would expect something similar to happen here.

In summary, I love the Fundamentalist project of classifying mathematics to such an extent that the parallels between "what is there" in math and "what is there" in physics become compelling. I don't expect this project to be possible, in the end, but I do expect interesting insight to come out of trying to make it work, so therefore I'm all for it.

6.3. Fundamentalist Response

The resolution to the category issue is that the entire Universe is a single mathematical structure, subsuming within it all particular physical objects.

I share the Mystic's concern that the Radical Platonist program may be extremely difficult for us humans to carry though even if the underlying

assumption is correct. However, it's worth a shot! My description of my vision for the future (below) lists a few areas where it should be possible to make at least limited progress.

7. THREE VISIONS OF THE FUTURE

7.1. The Fundamentalist Vision

If the radical Platonist view is correct, both arrows in Fig. 7 deserve intense study since there is real hope of understanding them better. This means continuing with "business as usual" in both fundamental physics research and brain/consciousness research. However, it also suggests research in some slightly unconventional directions as described below.

If all mathematical structures are equally real, then the one we inhabit is but one in a vast ensemble, and should be expected to be the most generic one compatible with our existence. In the terminology of [21], this ensemble or "Level IV multiverse" is vastly more diverse than the Level I multiverse of spatial horizon volumes, the Level II multiverse of different post-inflationary domains where the same fundamental laws have produced different effective laws from its landscape of possibilities, and the Level III multiverse of unitary quantum mechanics. To test this prediction, it is interesting to work out how the Universe would differ if physical constants or equations were altered, quantifying the degree to which it appears fine-tuned for life.

The ultimate classification problem in mathematics would be to classify all formal systems. Very little progress has been made in this direction because of the great difficulties involved, but any further insights about the structure of mathematics could shed more light on the nature of physics.

As described in Sec. 2.1.2, a mathematical structure can be viewed in two ways: from the bird perspective of a mathematician or from the subjective frog perspective of a self-aware substructure in it, like us. Since the relation between these two perspectives can be extremely subtle, a more systematic study of such phenomenology issues will be important—otherwise our candidate mathematical theories will not make testable predictions, and we may not even recognize the correct equations if we stumble across them.

Finally, since all mathematical structures are equally real in this view, we need to keep a very open mind as to what we are looking for. There is no room whatsoever for subjective nostalgic bias towards structures that resemble cozy classical concepts, or rejection of theories because they are "too crazy".

In conclusion, I feel that the fundamentalist vision is the most specific and falsifiable of the three, subsuming the Secularist's to-do list and adding to it items that are less vague than the Mystic's. There are real calculations to be done here, and my opinion is that controversial philosophical arguments should never be used as excuses not to make calculations.

7.2. The Secular Vision

Can a Secularist express a vision for the future? The whole point of the secular view is to avoid grandiose posturing. I have already given my expectations about the future of fundamental physics (Sec. 2.2): as it becomes harder to obtain experimental data against which to test our most fundamental theories, it will become harder to know whether we should believe them. One cannot predict how fundamental physics will then progress. It might continually find unexpected directions in which to proceed, such as the one hoped for by the Mystic. It might change so drastically that we would no longer recognize it as physics, or even science. Finally, it might simply atrophy away.

If I grapple with any deep mysteries, they are not about the meaning of science, but about the way people see the world. Science is the process of building a picture of the world that is, where possible, quantitatively accurate. It strikes some balance between conservatism (making minimal changes to established pictures) and radicalism (abandoning pictures that do not work). But many people feel that science is incomplete without a deeper claim than mere predictive power. They want to tilt the balance towards conservatism, treating the picture, current or future, as metaphysical truth. The mystic and fundamentalist thinkers show this tendency, and, in a wider context, some of the most successful religions display an extreme form of such conservatism, requiring adherence to a rigid set of beliefs that vary very slowly. It is striking, and mysterious to me, that so many people feel a strong need for such a frozen picture.

What impresses me is the degree to which our current understanding of the universe would be utterly incomprehensible to previous generations. It seems very reasonable to expect that human ideas will continue to develop in unpredictable ways, and that the theories of the future will explore directions that we cannot even imagine today. Ironically, it may be the fundamentalist and mystically inclined thinkers who bring this about. I am inclined to believe that those who think that their theories uncover the deep structure of the world are the ones who are driven most strongly to make essential and revolutionary contributions to the progress of theoretical physics.

7.3. The Mystic Vision

We have painted ourselves in a corner, scientifically, by describing the whole world in objective terms, and finding less and less room for ourselves to stand on. The challenge we now face is not to reduce ourselves also to objects, but to explore ways to let science naturally widen its area of investigation, while staying true to its methodology of peer review, based on an interplay between theory and experiment, with experiment having the last word.

For the last four hundred years, natural science has studied the object pole of experience in ever increasing detail. While this has so far been a sensible approach, we are now reaching the limits of a purely object-oriented treatment. In various areas of science, from quantum mechanics to neuroscience and robotics, the subject pole of experience can no longer be neglected. Most likely, science will change qualitatively with this required extension of its methodology.

This will not happen overnight. I expect this program to be carried out over a period of time comparable to the time it took to get the science of the object up and running, perhaps a century, quite likely a few centuries. But this process cannot and should not be hurried by wishful thinking or by external agendas. Imagine what would have happened if physicists had listened to William Blake, two centuries ago, who fulminated against their clockwork picture of the Universe. If they had tried to start a new "poetic physics" by trying to force Blake's notions into the accepted framework of physics, they would have been led astray. Nothing has come, fortunately, from "communist biology" in the days of Lysenko or the "Aryan physics" of the Nazis.

Real progress in physics can only come from within, from a necessity to introduce wider frameworks of explanation and interpretation to accommodate experimental facts that cannot be satisfactorily dealt with in the existing frameworks. After Blake's complaints, more than a hundred years passed before physicists discovered quantum mechanics, which showed that the material world is indeed really a far cry from a clockwork universe. The original dogmas of repeatability and strict causality, in the classical sense in which nothing happens spontaneously, were shown to be incorrect. These ideas had been extremely helpful to get physics off the ground. But in the end they were forms of dogma that had to be discarded.

The beautiful thing about physics is that it has an orderly way to conduct revolutions, and to discard older ideas; or more accurately, to assign older ideas to their proper domain of limited significance, while moving on to more accurate descriptions of reality. I expect that a thorough

exploration of the presence and ways of functioning of the subject, on a par with that of objects, will revolutionize science in a constructive way, within this century and the next.

8. CONCLUSION

We have discussed the nature of reality in the ontological context of Penrose's math-matter-mind triangle. Physicists have widely differing views of the deeper meaning of physics, and we have found that we three authors reflect this disparity. Despite our similarities in research interests, knowledge and cultural background, we espouse conflicting views that we have termed fundamentalist, secular, and mystic. A key message for non-physicists reading this paper is therefore that they should be deeply suspicious of any self-proclaimed popularizer or other ambassador claiming to speak on these matters on behalf of the consensus of the theoretical physics community.

A second noteworthy conclusion is that despite these disagreements, we advocate rather similar paths forward. This illustrates that physicists can vary strongly in their interpretations of the meaning of physics, while agreeing quite well on how it should be done.

Looking to the immediate future of physics, one thing that we agree on is that the diversity in views in the physics community is healthier than an ontological monoculture. Our best hope of making progress on the open questions of physics is to tackle them with a wide variety of ideas.

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