# TRACE DYNAMICS AND IMPLICATIONS FOR MY WORK OF THE LAST TWO DECADES

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- OVERVIEW OF TRACE DYNAMICS

- CONNECTIONS TO MY RECENT PROJECTS

# OVERVIEW OF TRACE DYNAMICS

"TRACE DYNAMICS" IS A NONCOMMUTATIVE GENERALIZATION OF CLASSICAL LAGRANGIAN AND HAMILTONIAN DYNAMICS

HOW IT WORKS (BOSONIC CASE)

$\{q_r\}$ $\{\dot{q}_r\}$ $\cdot = \partial/\partial t$

↑ NONCOMMUTING COORDINATES - OPERATORS ON UNDERLYING COMPLEX HILBERT SPACE

$$L = L[\{q_r\}, \{\dot{q}_r\}]$$

ORDERING IMPORTANT, SO $\frac{\delta L}{\delta q_r}$ NOT DEFINED

NOW USE CYCLIC INVARIANCE OF TRACE: DEFINE TRACE LAGRANGIAN $\mathbf{L}$

$$\mathbf{L} = \mathrm{Tr}\, L$$

FORM $\delta \underset{\sim}{L}$ AND REORDER CYCLICALLY SO THAT

ALL $\delta q_r$, $\delta \dot{q}_r$ STAND ON THE RIGHT

DEFINITION

$$\delta \underset{\sim}{L} = \mathrm{Tr} \sum_r \left( \frac{\delta \underset{\sim}{L}}{\delta q_r} \delta q_r + \frac{\delta \underset{\sim}{L}}{\delta \dot{q}_r} \delta \dot{q}_r \right)$$

THIS DEFINES $\delta \underset{\sim}{L} / \delta q_r$ $\quad \delta \underset{\sim}{L} / \delta \dot{q}_r$

CAN NOW SHOW

(1) $$0 = \delta \underset{\sim}{S} = \delta \int_{-\infty}^{\infty} dt \, \underset{\sim}{L} \Rightarrow \frac{\delta \underset{\sim}{L}}{\delta q_r} - \frac{d}{dt} \frac{\delta \underset{\sim}{L}}{\delta \dot{q}_r} = 0$$

DIRECTLY GIVES OPERATOR EULER-LAGRANGE EQUATIONS

HAVE A DYNAMICS THAT IS MORE GENERAL

THAN QUANTUM MECHANICS

TO RECOVER QUANTUM MECHANICS, WE CONSIDER THE EQUILIBRIUM STATISTICAL MECHANICS OF THIS DYNAMICS

HAVE THREE GENERIC CONSERVED QUANTITIES:

① TRACE HAMILTONIAN

DEFINE $$p_r = \frac{\delta \mathbf{L}}{\delta \dot{q}_r}$$

$$\mathbf{H} = \mathrm{Tr}\left(\sum_r p_r \dot{q}_r\right) - \mathbf{L}$$

THEN
$$\frac{\delta \mathbf{H}}{\delta q_r} = -\dot{p}_r \qquad \frac{\delta \mathbf{H}}{\delta p_r} = \dot{q}_r$$

$$\frac{d\mathbf{H}}{dt} = \mathrm{Tr} \sum_r \left( \frac{\delta \mathbf{H}}{\delta q_r} \dot{q}_r + \frac{\delta \mathbf{H}}{\delta p_r} \dot{p}_r \right)$$

$$= \mathrm{Tr} \sum_r \left( -\dot{p}_r \dot{q}_r + \dot{q}_r \dot{p}_r \right) = 0$$

② **OPERATOR** $\tilde{C} = \sum_n [q_n, p_n] = -\tilde{C}^\dagger$ (ANDREW MILLARD)

WITH FERMIONS, $\tilde{C} = \sum_{n,B} [q_n, p_n] - \sum_{n,F} \{q_n, p_n\}$

SUPPOSE $\underset{\sim}{H}$ IS GLOBAL UNITARY INVARIANT - THAT IS, IT INVOLVES NO NONCOMMUTATIVE CONSTANTS

$\tilde{C}$ IS CORRESPONDING NOETHER CHARGE OPERATOR $\frac{d\tilde{C}}{dt} = 0$

③ **PHASE SPACE MEASURE**

$$d\mu \equiv \prod_{n,m,n,A} d\langle m|q_n|n\rangle^A \; d\langle m|p_n|n\rangle^A$$

↑ INDEXES REAL COMPONENTS (index $A$)

$d\mu$ IS INVARIANT UNDER CANONICAL TRANSFORMATIONS

$$\delta p_n = -\frac{\delta \underset{\sim}{G}}{\delta q_n} \qquad \delta q_n = \frac{\delta \underset{\sim}{G}}{\delta p_n}$$

⇒ GENERALIZED LIOUVILLE THEOREM

CAN USE STATISTICAL MECHANICS

# CANONICAL ENSEMBLE

$$\rho = Z^{-1} e^{-\tau \underset{\sim}{H} - \mathrm{Tr}(\tilde{\lambda} \tilde{C})}$$

$$\int d\mu \, \rho = 1 \quad \Rightarrow \quad Z = \int d\mu \, e^{-\tau \underset{\sim}{H} - \mathrm{Tr}(\tilde{\lambda} \tilde{C})}$$

$\tau, \tilde{\lambda}$ ARE ENSEMBLE PARAMETERS

$$\langle \tilde{C} \rangle_{AV} \equiv i_{eff} D$$

$$i_{eff}^2 = -1 \qquad i_{eff}^{\dagger} = -i_{eff} \qquad [i_{eff}, D] = 0$$

SIMPLEST CASE IS D = MULTIPLE OF UNIT MATRIX

$$D = t_1 \mathbb{1}$$

$\langle \tilde{C} \rangle_{AV}$ A FUNCTION OF $\tau, \tilde{\lambda} \Rightarrow \tilde{\lambda} = i_{eff} \lambda$

## UNITARY FIXING

$\tilde{\lambda} = i_{eff}\,\lambda \Rightarrow \rho$ INVARIANT UNDER $U_{eff}$ FOR WHICH

$$U_{eff}^{\dagger}\, i_{eff}\, U_{eff} = i_{eff}$$

SINCE

$$\mathrm{Tr}\,\tilde{\lambda}\,\tilde{C} \to \mathrm{Tr}\,\tilde{\lambda}\,U_{eff}^{\dagger}\,\tilde{C}\,U_{eff} = \mathrm{Tr}\,U_{eff}\,\tilde{\lambda}\,U_{eff}^{\dagger}\,\tilde{C} = \mathrm{Tr}\,\tilde{\lambda}\,\tilde{C}$$

THUS THE CANONICAL ENSEMBLE ONLY PARTIALLY BREAKS THE GLOBAL UNITARY INVARIANCE

$\Rightarrow$ WE WILL AVERAGE TOO MUCH IF WE USE $\int d\mu\, \rho$

LET $\quad d\mu' = d\mu\big|_{\text{OVERALL } U_{eff} \text{ FROZEN}}$

USE THIS RESTRICTED MEASURE TO FORM THERMODYNAMIC AVERAGES

**EFFECTIVE COMPLEX PART:** LET $X$ BE ANY $q_n$ OR $p_n$

$X_{eff} \equiv \frac{1}{2}\left(X - i_{eff}\,X\,i_{eff}\right)$ = PART OF $X$ THAT COMMUTES WITH $i_{eff}$

$$i_{eff}\,X_{eff} = \frac{1}{2}\left(i_{eff}\,X + X\,i_{eff}\right) = X_{eff}\,i_{eff}$$

PHYSICAL OBSERVABLES ARE TRACES, WHICH ARE INDEPENDENT OF UNITARY FIXING

$$\mathrm{Tr}\, \mathcal{O} = \mathrm{Tr}\, U^{\dagger}_{eff}\, \mathcal{O}\, U_{eff}$$

THIS SUFFICES, SINCE TRANSITION PROBABILITIES CAN BE WRITTEN AS TRACES

$$|\langle\alpha|\mathcal{O}|\beta\rangle|^2 = \langle\alpha|\mathcal{O}|\beta\rangle\langle\beta|\mathcal{O}^{\dagger}|\alpha\rangle = \mathrm{Tr}\, \rho_\alpha\, \mathcal{O}\, \rho_\beta\, \mathcal{O}^{\dagger}$$

$$\rho_\alpha = \frac{1}{2\pi i}\oint_{\gamma_\alpha} dz\, \frac{1}{z - \mathcal{O}_\alpha}$$

NON-TRACE QUANTITIES ARE UNITARY FIXING DEPENDENT

# HOW QUANTUM FIELD THEORY EMERGES

$$\frac{\int d\mu' \rho\, x_{1\,eff} \cdots x_{n\,eff}}{\int d\mu' \rho} \Leftrightarrow x_{1\,eff} \cdots x_{n\,eff}$$

↑ OPERATORS IN A QUANTUM THEORY WITH ROLE OF $i$ PLAYED BY $i_{eff}$

CAN DERIVE EQUIPARTITION THEOREMS (WARD IDENTITIES) FOR THESE AVERAGES

WHEN (i) $\tau \sim \frac{1}{\text{MASS}}$ VERY SMALL (ii) $\tilde{C} \to \langle \tilde{C} \rangle_{AV} = i_{eff}\, \hbar$

↑ PLANCK MASS?

THESE WARD IDENTITIES HAVE THE STRUCTURE OF QUANTUM MECHANICS:

$$[q_{r\,eff}, p_{s\,eff}] = i\hbar\, \delta_{rs} + O(\tau)$$

$$[q_{r\,eff}, q_{s\,eff}] = 0 + O(\tau)$$

$$\dot{x}_{eff} = \frac{i_{eff}}{\hbar} [H_{eff}, x_{eff}] + O(\tau)$$

INSIDE AVERAGES (LOCALITY IS EMERGENT)

# SKETCH OF GENERAL WARD IDENTITY DERIVATION

$$\frac{\int d\mu' \rho\, \mathcal{O}}{\int d\mu' \rho} \equiv \langle \mathcal{O} \rangle_{AV}$$

TAKE $\mathcal{O} = \{ \tilde{C}, i_{eff} \} W$

- USE $\int d\mu' \, \delta[\rho \mathcal{O}] = 0$ — TRANSLATION INVARIANCE OF MEASURE $d\mu'$
- NEGLECT $\tau$ TERM COMING FROM $\delta\rho$ — DECOUPLING
- REPLACE $\tilde{C} \to \langle \tilde{C} \rangle_{AV}$ IN INTEGRANDS
- MAKE VARIOUS CHOICES FOR $W$

$W \propto$ CANONICAL $q$ OR $p$ — GIVES CANONICAL ALGEBRA INSIDE $\langle\,\rangle_{AV}$

$W \propto H =$ OPERATOR HAMILTONIAN — GIVES HEISENBERG EQUATION OF MOTION INSIDE $\langle\,\rangle_{AV}$

$W \propto G = G^{\dagger}$ GIVES UNITARY CANONICAL TRANSFORMATION INSIDE $\langle\,\rangle_{AV}$

CAN INCLUDE SOURCES IN $\rho$ SO THAT $\langle\,\rangle_{AV}$ HAS SOURCE TERMS THAT CAN BE VARIED

## REMARKS

① $O(\tilde{\lambda})$ TERM IN WARD IDENTITIES VANISHES BECAUSE

$$\{\dot{x}_{eff}, [\tilde{\lambda}, x]\} \propto [\tilde{\lambda}, x_{eff}] = 0$$

② CONDITIONS FOR NEGLECT OF $O(\tau)$ TERMS IN WARD IDENTITIES

- NEED BOSON FERMION BALANCE
- SUFFICES FOR $\dot{x}_{eff}$ AND $\tilde{C}_{eff}$ TO HAVE DISJOINT SUPPORT ON OPERATOR PHASE SPACE

$$\langle [q_s, p_r] \rangle_{AV} = i_{eff}\, \hbar\, \delta_{rs} \qquad \text{ONLY BOSONS}$$

$r = s$ AND SUM $\quad \langle \prod_r [q_r, p_r] \rangle_{AV} = \langle \tilde{C}_{AV} \rangle = i_{eff}\, \hbar = i_{eff}\, \hbar \prod_r 1 = i_{eff}\, \hbar\, N_B$

WITH BOTH BOSONS AND FERMIONS

$$\langle \prod_{rB} [q_r, p_r] - \prod_{rF} \{q_r, p_r\} \rangle = \langle \tilde{C}_{AV} \rangle = i_{eff}\, \hbar$$

$$\Downarrow$$

$i_{eff}\, \hbar\, (N_B - N_F) + \prod O(\tau) \qquad$ NO CONTRADICTION IF $N_B = N_F$

# WHY THERE ARE BROWNIAN MOTION CORRECTIONS

$$\tilde{C} = \langle \tilde{C} \rangle_{AV} + \Delta \tilde{C}$$

$\langle \tilde{C} \rangle_{AV} = i_{eff}\,\hbar$ ; $\Delta \tilde{C}$ ← RAPIDLY FLUCTUATING

$$\tfrac{1}{2}\{\tilde{C}, i_{eff}\} = -\hbar + \underbrace{\tfrac{1}{2}\{\Delta\tilde{C}, i_{eff}\}}_{-\hbar(\mathcal{K}+\mathcal{N})}$$

$\mathcal{K}$ = FLUCTUATING C-NUMBER

$\mathcal{N}$ = FLUCTUATING MATRIX, WITH OPERATOR ANALOG

FROM $\mathcal{N}$ TERM: APPROPRIATE ANSATZ CORRESPONDS TO MASS-PROPORTIONAL FORM OF "CONTINUOUS SPONTANEOUS LOCALIZATION" (CSL) STOCHASTIC SCHRÖDINGER EQUATION

HAVE PLAUSIBILITY ARGUMENTS, BUT NOT A DERIVATION OF CSL

STOCHASTIC SCHRÖDINGER EQUATION FOR POINTER WITH CENTER OF MASS $q$

(THIS IS LEADING SMALL DISPLACEMENT APPROXIMATION TO GRW AND CSL MODELS)

$$d|\psi\rangle = -\frac{i}{\hbar} H|\psi\rangle dt - \frac{\eta}{2}(q-\langle q\rangle)^2|\psi\rangle dt + \sqrt{\eta}\,(q-\langle q\rangle)|\psi\rangle dW_t$$

$\langle q\rangle = \langle\psi|q|\psi\rangle$ EXPECTATION VALUE OF $q$ IN STATE $|\psi\rangle$

$dW_t$ = BROWNIAN NOISE

- BECAUSE $|\psi\rangle$ APPEARS IN $\langle q\rangle$, THIS IS A NONLINEAR STOCHASTIC DIFFERENTIAL EQUATION
- THIS EQUATION CAN BE PROVED TO GIVE STATE VECTOR REDUCTION ON POSITION EIGENSTATES WITH BORN RULE PROBABILITIES

## SUMMARY: TRACE DYNAMICS AS PRE-QUANTUM MECHANICS

- THERMODYNAMICS - VIA EQUIPARTITION THEOREMS (WARD IDENTITIES)

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  UNITARY EVOLUTION OF QUANTUM MECHANICS

  HEISENBERG EQUATIONS OF MOTION, SCHRÖDINGER EQUATION

- BROWNIAN MOTION CORRECTIONS - VIA CSL PHENOMENOLOGY

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  PROBABILITY INTERPRETATION, BORN RULE

  OF REDUCTION POSTULATE OF QUANTUM THEORY

## OVERVIEW

**CLASSICAL MECHANICS** ----- VARIABLES COMMUTATIVE, ALL ∝ UNIT MATRIX

CANONICAL QUANTIZATION ↓ ↑ CLASSICAL LIMIT $\hbar \to 0$

**QUANTUM MECHANICS** ----- VARIABLES SPECIAL (INFINITE) MATRICES

$$[q_l, p_m] = i\hbar\,\delta_{lm}$$

$$[q_l, q_m] = 0 \qquad \text{etc.}$$

STOCHASTIC SCHRÖDINGER EQUATION ← QUANTUM MECHANICS

THERMODYNAMICS, EQUILIBRIUM STATISTICAL MECHANICS ↑

BROWNIAN MOTION CORRECTIONS ↑

**"TRACE DYNAMICS"** ----- VARIABLES GENERAL MATRICES

or

"GENERALIZED QUANTUM DYNAMICS"

WITH **GLOBAL UNITARY INVARIANT DYNAMICS**

NO A PRIORI COMMUTATIVITY PROPERTIES

$$[q_l, q_m] \neq 0 \qquad \text{etc.}$$

# CONNECTIONS TO MY RECENT PROJECTS

(i) STUDIES OF CONTINUOUS SPONTANEOUS LOCALIZATION (CSL)

THE GHIRARDI - RIMINI - WEBER - PEARLE MODEL
OF OBJECTIVE REDUCTION

MANY PAPERS 2000 TO 2021

SOME HIGHLIGHTS:

- WITH L.P. HORWITZ - COMPLETING HUGHSTON'S BORN RULE PROOF
- BOUNDS ON CSL PARAMETERS FROM LATENT IMAGE FORMATION
- WITH A. BASSI - CSL WITH NON-WHITE NOISE
- MANY OTHER PAPERS ON CSL PHENOMENOLOGY

(ii) DARK MATTER AS A DECOUPLED $i \to -i$ SECTOR

$i_{eff}$ IS A 2X2 REAL MATRIX REPRESENTATION OF THE IMAGINARY UNIT, ACTS AS $i$ ON ONE SECTOR AND $-i$ ON THE OTHER

IF SECTORS ARE UNCOUPLED OR WEAKLY COUPLED, $-i$ SECTOR COULD BE ASTROPHYSICAL "DARK MATTER"

- ONE PAPER, A GRAVITATIONAL ESSAY, PROPOSING THIS

(iii) UNIFICATION MODELS FOR STRONG, ELECTROWEAK FORCES WITH BOSON-FERMION BALANCE BUT NO FULL SUPERSYMMETRY

- STUDIED SU(8) MODELS WITH SPIN-3/2 PARTICLES
- PROPERTIES OF GAUGED SPIN-3/2 AND ASSOCIATED ANOMALIES
- SEEMS TO NEED QUARK-LEPTON COMPOSITENESS TO GET STANDARD MODEL FAMILIES
- NO DEFINITIVE CONCLUSION
- NEED EXPERIMENTAL CLUES AS TO WHAT LIES BYOND "STANDARD MODEL" OF PARTICLE PHYSICS

## (iv) INCORPORATING GRAVITY INTO TRACE DYNAMICS

- CANONICAL ENSEMBLE HAS A "WEYL SCALING" $g_{\mu\nu}(x) \to \lambda^2(x)\, g_{\mu\nu}(x)$ INVARIANCE

- SUGGESTS THAT THE NONDERIVATIVE PART OF THE INDUCED GRAVITATIONAL ACTION IS ALSO WEYL SCALING INVARIANT

- ALTERNATIVE "COSMOLOGICAL CONSTANT" ACTION

$$S_{eff} = -\frac{\Lambda}{8\pi G}\int d^4x \left({}^{(4)}g\right)^{1/2} \left(g_{00}\right)^{-2}$$

WEYL SCALING INVARIANT BUT ONLY 3-SPACE GENERAL COORDINATE INVARIANT

- EXTENSIVE STUDIES OF THIS IDEA 2013 - 2021
- HORIZONLESS "BLACK" HOLES $g_{00} > 0$ ALWAYS

(N) HORIZONLESS "DYNAMICAL GRAVASTAR" MODELS

- SOLVE TOLMAN-OPPENHEIMER-VOLKOFF (TOV) EQUATIONS WITH EQUATION OF STATE JUMP

  $\rho = 3p \rightarrow \rho + p = \beta > 0$ ($\beta$ SMALL .1 .01 .001)

- p MUST BE CONTINUOUS & $\rho$ JUMPS TO $\rho < 0$ (ALLOWED IN QUANTUM THEORY)

- MATHEMATICA NOTEBOOKS FOR EXOTIC COMPACT OBJECT CLOSELY APPROXIMATING EXTERIOR SCHWARZSCHILD GEOMETRY BUT NO HORIZON OR TRAPPED SURFACES $f_{00} > 0$

- POSSIBLE ASTROPHYSICAL IMPLICATIONS OF MATTER "LEAKING" OUT