Foundations for Learning in the Age of Big Data

Maria-Florina (Nina) Balcan

Carnegie Mellon University

Two Core Aspects of Machine Learning

Algorithm Design. How to optimize? — Computation

Automatically generate rules that do well on observed data.

- E.g.: Adaboost, SVM, gradient descent, etc.

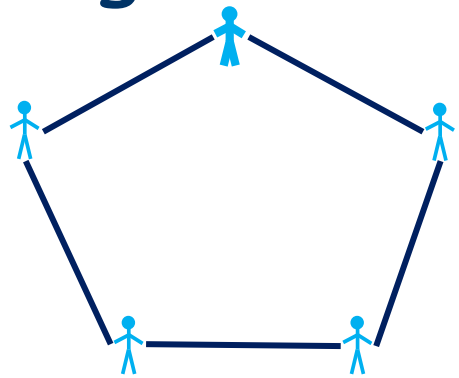
Confidence Bounds, Generalization — (Labeled) Data

Confidence for rule effectiveness on future data.

$$O\left(\frac{1}{\epsilon}\left(\mathrm{VCdim}(C)\log\left(\frac{1}{\epsilon}\right)+\log\left(\frac{1}{\delta}\right)\right)\right)$$

Distributed Machine Learning

Modern ML: massive amounts of data distributed across multiple locations.



E.g., video data, scientific data, medical data

Key new resource:

This talk: models and algorithms for reasoning about communication complexity issues.

- Supervised Learning

  [Balcan-Blum-Fine-Mansour, COLT 2012] Runner UP Best Paper

  [TseChen-Balcan-Chau AISTAT'16]

- Clustering, Unsupervised Learning

  [Balcan-Ehrlich-Liang, NIPS 2013]

  [Balcan-Kanchanapally-Liang-Woodruff, NIPS 2014]

Distributed Learning

Many ML problems today involve massive amounts of data distributed across multiple locations.

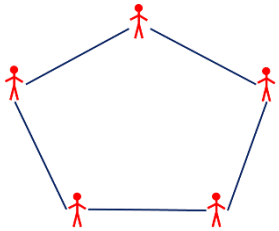
Often would like low error hypothesis wrt the overall distrib.

E.g.:

- different hospitals with different distributions of patients; want to learn a classifier to identify a common misdiagnosis.

- different research groups collected scientific data; wish to perform learning over the union of all these datasets.

Distributed Learning

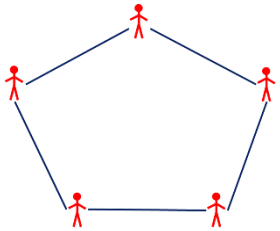


- Data distributed across multiple locations.
- Each has a piece of the overall data pie.
- To learn over the combined D, must communicate.

Important question: how much communication?

Plus, privacy & incentives.

Distributed PAC learning [Balcan-Blum-Fine-Mansour,COLT 2012]



- X – instance space. s players.
- Player i can sample from D_i, samples labeled by c*.
- Goal: find h that approximates c* w.r.t. D=1/s (D_1 + … + D_s)
- Fix C of VCdim d. Assume s << d. [realizable: c* ∈ C, agnostic:c*∉ C]

Goal: learn good h over D, as little communication as possible

- Total communication (bits, examples, hypotheses)
- Rounds of communication.

Efficient algos for problems when centralized algos exist.

Overview of Our Results

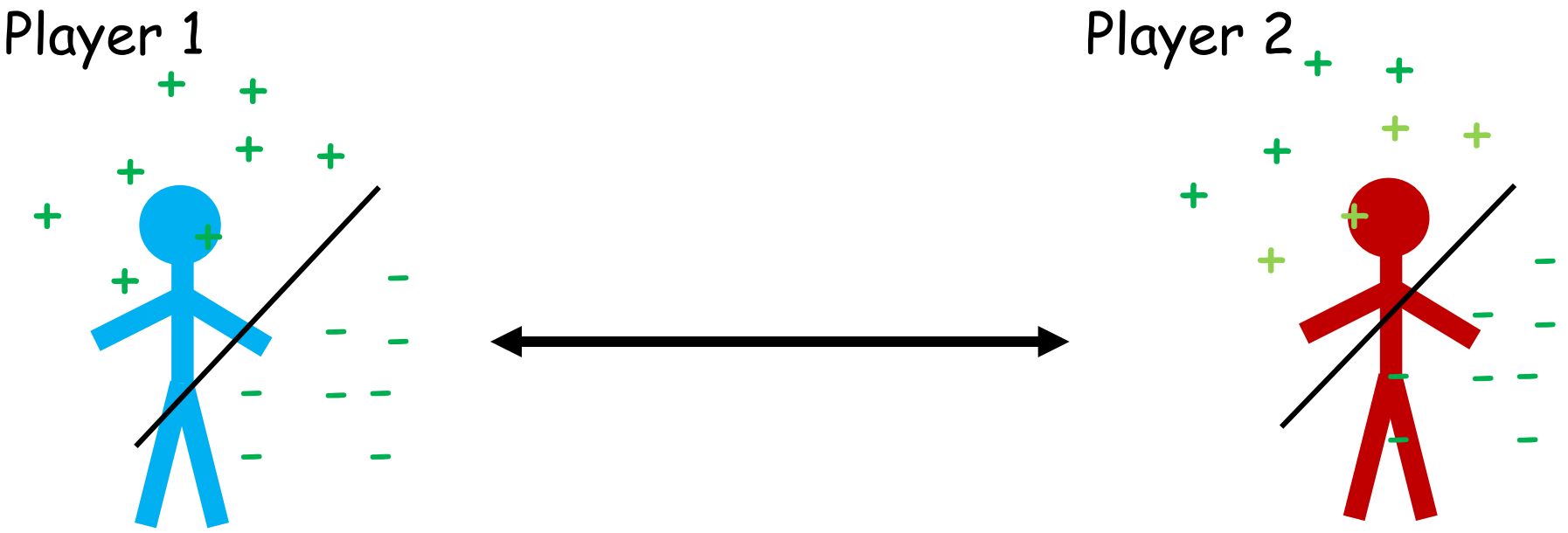
Introduce and analyze Distributed PAC learning.

- Generic bounds on communication.
- Broadly applicable communication efficient distributed boosting.
- Tight results for interesting cases (e.g., conjunctions, parity fns).

Interesting special case to think about

s=2. One has the positives and one has the negatives.

- How much communication, e.g., for linear separators?



A simple communication baseline.

Baseline

$d/\epsilon \log(1/\epsilon)$ examples, 1 round of communication

- Each player sends $d/(\epsilon s) \log(1/\epsilon)$ examples to player 1.
- Player 1 finds consistent $h \in C$, whp error $\leq \epsilon$ wrt D

Improving the Dependence on $1/\epsilon$

Baseline provides linear dependence in d and $1/\epsilon$

Can get better $O(d \log 1/\epsilon)$ examples of communication!

Recap of Adaboost

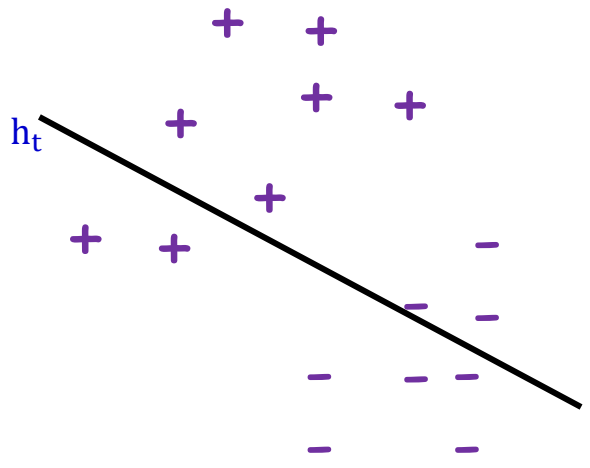
- Boosting: algorithmic technique for turning a weak learning algorithm into a strong (PAC) learning one.

Recap of Adaboost

- Boosting: turns a weak algo into a strong (PAC) learner.

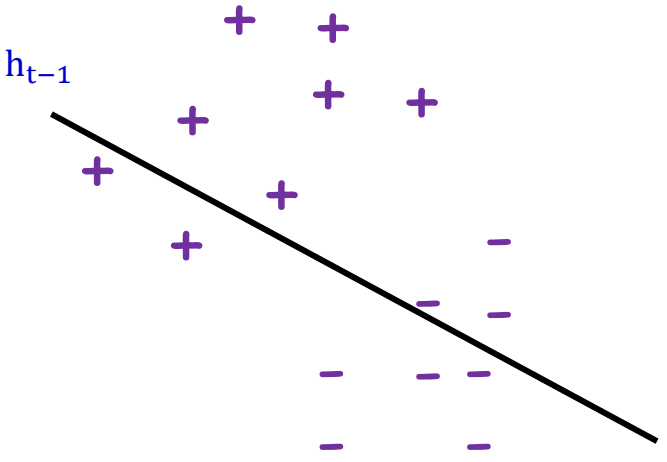
Input: $S=\{(x_1, y_1), \ldots, (x_m, y_m)\}$; weak learner A

- Weak learning algorithm A.
- For t=1,2, … ,T
  - Construct D_t on $\{x_1, \ldots, x_m\}$
  - Run A on D_t producing h_t
- Output H_final=sgn($\sum \alpha_t \ h_t$)



Recap of Adaboost

- Weak learning algorithm A.
- For t=1,2, … ,T
  - **Construct D_t on $\{x_1, \ldots, x_m\}$**
  - Run *A* on D_t producing h_t



- D_1 uniform on $\{x_1, \ldots, x_m\}$
- D_{t+1} increases weight on x_i if h_t incorrect on x_i ; decreases it on x_i if h_t correct.

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} e^{\{-\alpha_t\}} \text{ if } y_i = h_t(x_i)$$

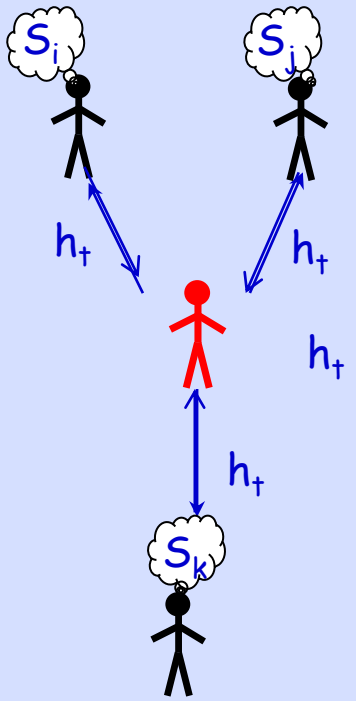
$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} e^{\{\alpha_t\}} \text{ if } y_i \neq h_t(x_i)$$

Key points:

- $D_{t+1}(x_i)$ depends on $h_1(x_i), \ldots, h_t(x_i)$ and normalization factor that can be communicated efficiently.
- To achieve weak learning it suffices to use O(d) examples.

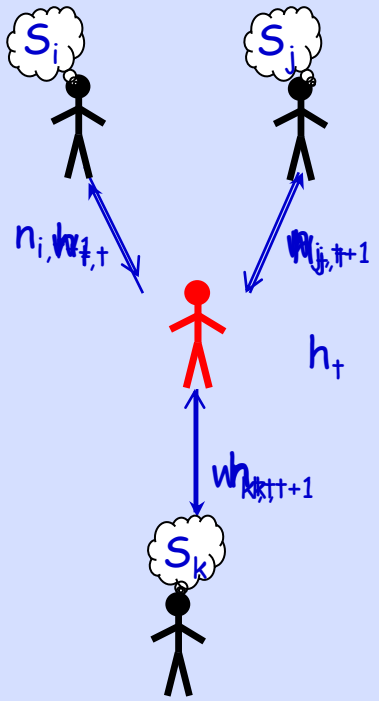
Distributed Adaboost

- Each player i has a sample S_i from D_i.
- For t=1,2, … ,T
  - Each player sends player 1, enough data to produce weak hyp h_t.
    [For t=1, O(d/s) examples each.]
  - Player 1 broadcasts h_t to other players.



Distributed Adaboost

- Each player i has a sample S_i from D_i.
- For $t=1,2,\ldots,T$
  - Each player sends player 1, enough data to produce weak hyp h_t.
    [For t=1, O(d/s) examples each.]
  - Player 1 broadcasts h_t to other players.
  - Each player i reweights its own distribution on S_i using h_t and sends the sum of its weights $w_{i,t}$ to player 1.
  - Player 1 determines the #of samples to request from each i [samples $O(d)$ times from the multinomial given by $w_{i,t}/W_t$].



Distributed Adaboost

Can learn any class C with $O(\log(1/\epsilon))$ rounds using $O(d)$ examples + $O(s \log d)$ bits per round.

[efficient if can efficiently weak-learn from $O(d)$ examples]

Proof:

- As in Adaboost, $O(\log 1/\epsilon)$ rounds to achieve error ϵ.
- Per round: $O(d)$ examples, $O(s \log d)$ extra bits for weights, 1 hypothesis.

Dependence on $1/\epsilon$, Agnostic learning

Distributed implementation of Robust halving [Balcan-Hanneke'12].

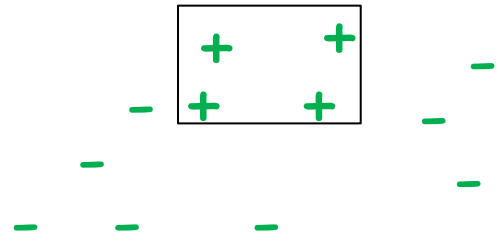
- error $O(OPT)+\epsilon$ using only $O(s \log|C| \log(1/\epsilon))$ examples.

Not computationally efficient in general.

Distributed implementation of Smooth Boosting (access to agnostic weak learner). [TseChen-Balcan-Chau'16]

Better results for special cases

Intersection-closed when fns can be described compactly .



C is intersection-closed, then *C* can be learned in one round and *s* hypotheses of total communication.

Algorithm:

- Each i draws S_i of size $O(d/\epsilon \log(1/\epsilon))$, finds smallest h_i in *C* consistent with S_i and sends h_i to player 1.
- Player 1 computes smallest h s.t. $h_i \subseteq h$ for all i.

Key point:

h_i, h never make mistakes on negatives, and on positives h could only be better than h_i ($\mathrm{err}_{D_i}(h) \leq \mathrm{err}_{D_i}(h_i) \leq \epsilon$)

Better results for special cases

E.g., conjunctions over $\{0,1\}^d$ [$f(x) = x_2x_5x_9x_{15}$]

- Only $O(s)$ examples sent, $O(sd)$ bits.
  - Each entity intersects its positives.
  - Sends to player 1.
  - Player 1 intersects & broadcasts.

1101111011010111
1111110111001110
1100110011001111
1100110011000110

[Generic methods $O(d)$ examples, or $O(d^2)$ bits total.]

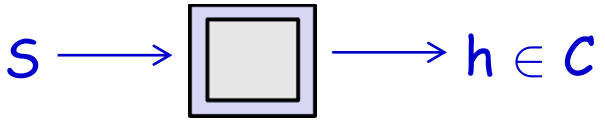
Interesting class: parity functions

- $s = 2, X = \{0,1\}^d$, C = parity fns, $f(x) = x_{i_1} \text{XOR } x_{i_2} \ldots \text{XOR } x_{i_l}$
- Generic methods: O(d) examples, $O(d^2)$ bits.
- Classic CC lower bound: $\Omega(d^2)$ bits LB for proper learning.

Improperly learn C with O(d) bits of communication!

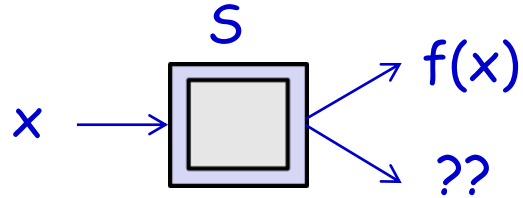
Key points:

- Can properly PAC-learn C.



[Given dataset S of size $O(d/\epsilon)$, just solve the linear system]

- Can non-properly learn C in reliable-useful manner [RS'88]



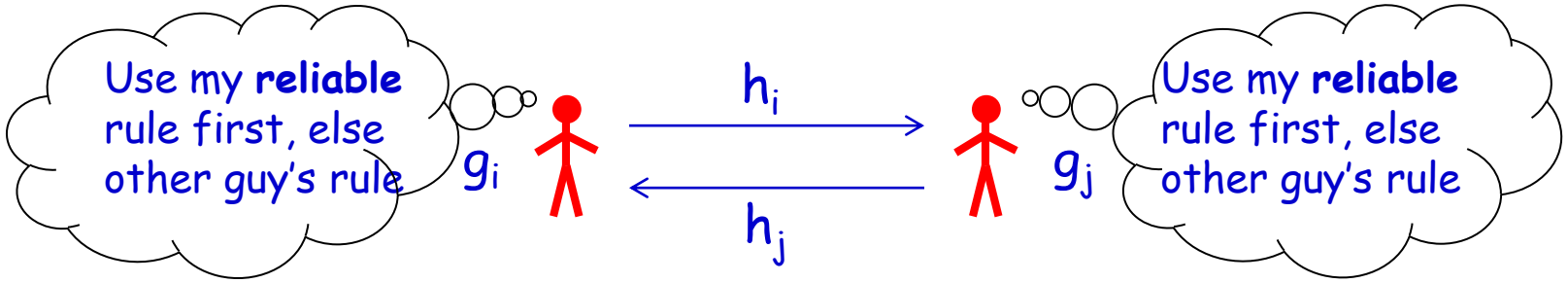
[if x in subspace spanned by S, predict accordingly, else say "?"]

Interesting class: parity functions

Improperly learn C with $O(d)$ bits of communication!

Algorithm:

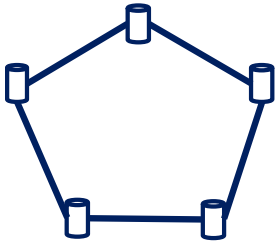
- Player i properly PAC-learns over D_i to get parity h_i. Also improperly R-U learns to get rule g_i. Sends h_i to player j.
- Player i uses rule R_i: "if g_i predicts, use it; else use h_j"



Key point: low error under D_j because h_j has low error under D_j and since g_i never makes a mistake putting it in front does not hurt.

Distributed PAC learning: Summary

- First time consider communication as a fundamental resource.

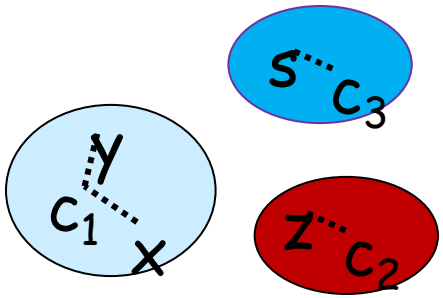


- General bounds on communication, communication-efficient distributed boosting.
- Improved bounds for special classes (intersection-closed, parity fns, and linear separators over nice distributions).

Distributed Clustering

[Balcan-Ehrlich-Liang, NIPS 2013]

[Balcan-Kanchanapally-Liang-Woodruff, NIPS 2014]



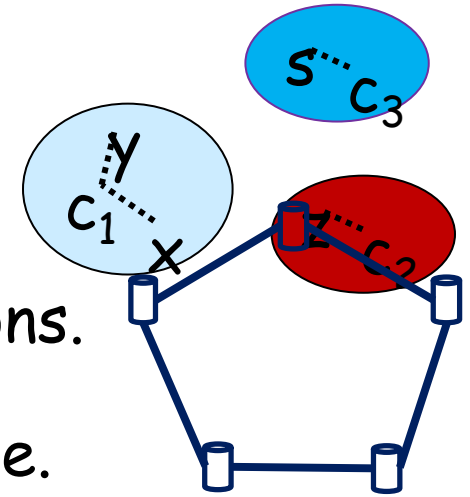
Distributed Clustering [Balcan-Ehrlich-Liang, NIPS 2013]

k-median: find center pts $c_1, c_2, \ldots, c_k$ to minimize $\sum_x \min_i d(x,c_i)$

k-means: find center pts $c_1, c_2, \ldots, c_k$ to minimize $\sum_x \min_i d^2(x,c_i)$

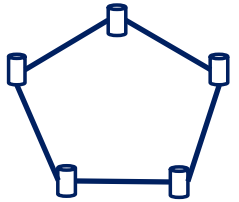
Distributed Clustering

- Dataset S distributed across s locations.
- Each has a piece of the overall data pie.



Goal: cluster the data, as little communication as possible

Distributed Clustering [Balcan-Ehrlich-Liang, NIPS 2013]



- Data distributed across s locations.
- Each has a piece of the overall data pie.

Goal: cluster the data, as little communication as possible

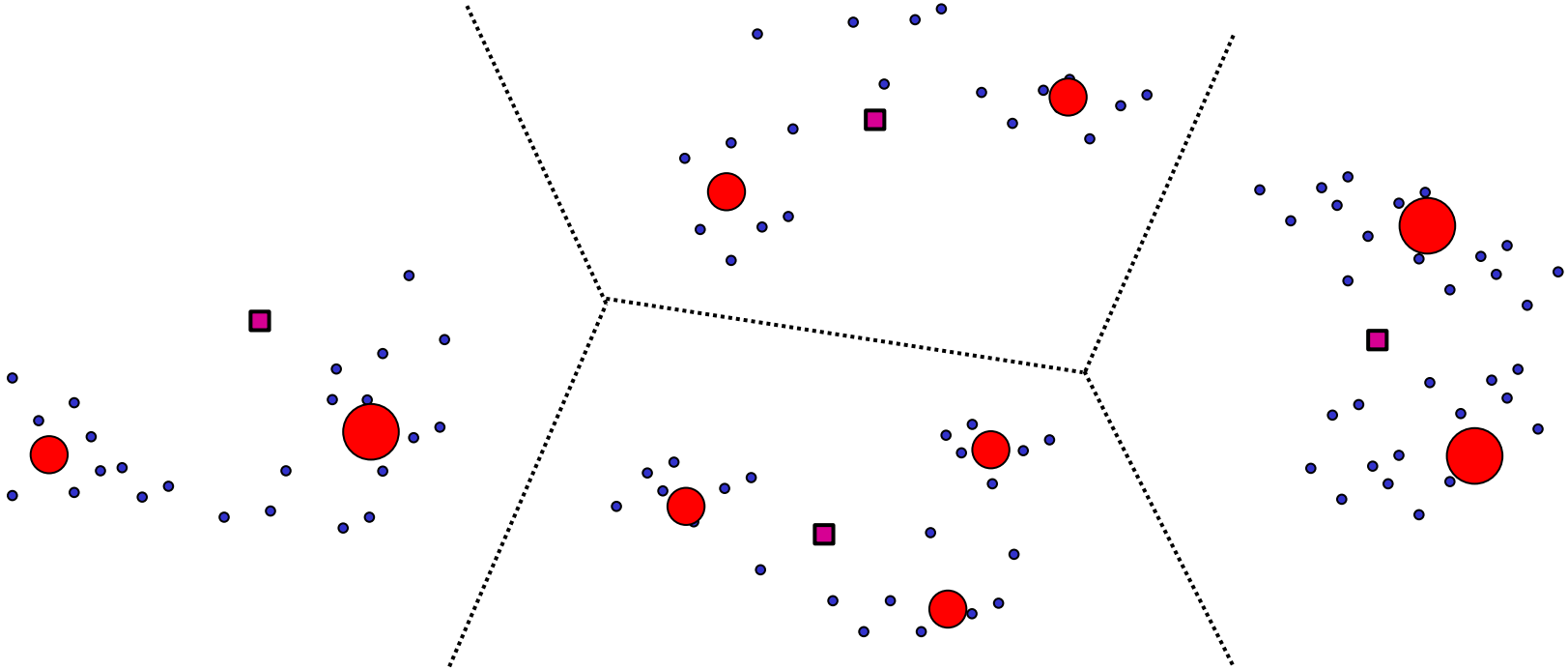
Key idea: use coresets, short summaries capturing relevant info w.r.t. all clusterings.

- By combining local coresets, get a global coreset; the size goes up multiplicatively by s.
- We show a two round procedure with communication only the true size of a global coreset of dataset S.

Coresets

Def: An ϵ-coreset for a set of pts S is a set of points $\tilde{S}$ and weights $w: \tilde{S} \to R$ s.t. for any sets of centers $\mathbf{c}$:

$$(1-\epsilon)\text{cost}(S,\mathbf{c}) \leq \sum_{p\in\tilde{S}} w_p \text{cost}(p,\mathbf{c}) \leq (1+\epsilon)\text{cost}(S,\mathbf{c})$$

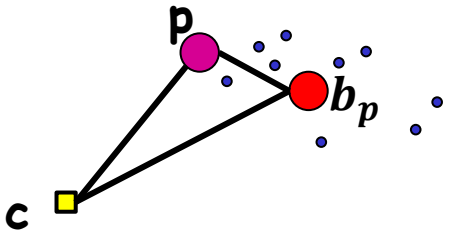


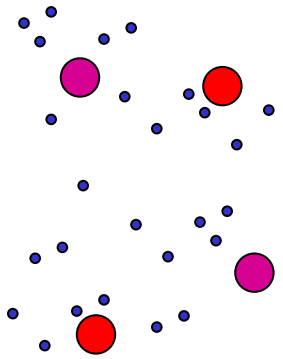
Centralized Coresets of size $O(kd/\epsilon^2)$ [Feldman-Langberg'11]

1. Find a constant factor approx. B, add its centers to coreset
2. Sample $O(kd/\epsilon^2)$ pts according to their contribution to the cost of that approximate clustering B. Add them in too.

Key idea (proof reinterpreted):

- Can view B as rough coreset, with $b \in B$ weighted by size of Voronoi cell.
- If p has closest pt $b_p \in B$, then for any center c, $|cost(p,c) - cost(b_p,c)| \leq \|p - b_p\|$ by triangle inequality.
- So, penalty $f(p) = cost(p,\mathbf{c}) - cost(b_p,\mathbf{c})$ for p satisfies $f(p) \in [-cost(p,b_p), cost(p,b_p)]$.
- Motivates sampling according to $cost(p,b_p)$.

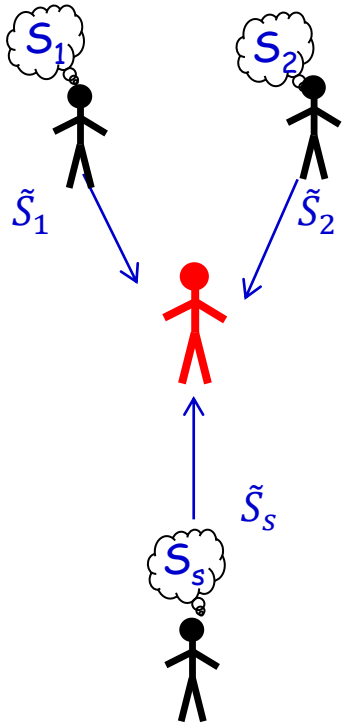




Distributed Clustering

Key fact: $\tilde{S}_i$ is coreset for S_i, then $\cup_i \tilde{S}_i$ is coreset for $\cup_i S_i$.

1. Each player finds coreset of size $O(kd/\epsilon^2)$ on their own data using centralized method.
2. Then they all send local coresets to the center.



For s players, total communication is $O(skd/\epsilon^2)$.

Can we do better?

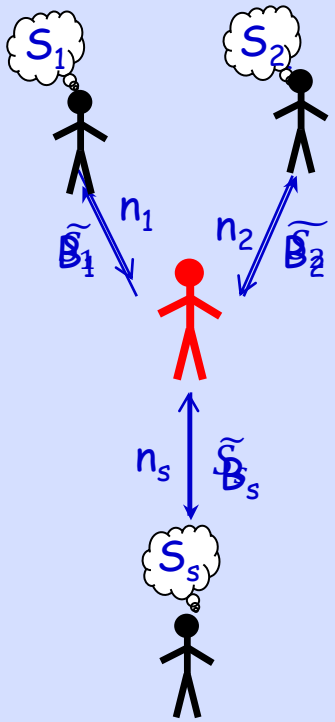
Distributed Coresets [Balcan-Ehrlich-Liang, NIPS 2013]

Key idea: in distributed case, show how to do this using only local constant factor approx.

1. Each player i, finds a local constant factor approx. B_i and sends $\text{cost}(B_i, P_i)$ and the centers to the center.

2. Center samples $n = O(kd/\epsilon^2)$ times $n = n_1 + \cdots + n_s$ from multinomial given by these costs. Sends n_i to player i.

3. Each player i sends n_i points from P_i sampled according to their contribution to the local approx.



For s players, total communication is only $O\left(\frac{kd}{\epsilon^2} + sk\right)$.

Open questions (Learning and Clustering)

- Efficient algorithms in noisy settings; handle failures, delays.
- Even better dependence on $1/\epsilon$ for communication efficiency for clustering via boosting style ideas.
    - Can use distributed dimensionality reduction to reduce dependence on d. [Balcan-Kanchanapally-Liang-Woodruff, NIPS 2014]
- More refined trade-offs between communication complexity, computational complexity, and sample complexity.