LEADING LOG MODEL

Pagels + Tomboulis, Matinyan + Savvidy: conformal trace anomaly ⇒

$$\mathcal{L}(\mathcal{F}) = \frac{1}{8} b_0 \mathcal{F} \log(\mathcal{F}/e\chi^2) + \text{"smaller" term}, \quad \mathcal{F} = E^2 - B^2$$

Review with all references: Adler + Piran Rev Mod Phys 56, 1 (1984)

FIELD EQUATION FORM OF MODEL

$$\vec{\nabla} \cdot \vec{D} = j_0 = Q\left[\delta^3(x - x_1) - \delta^3(x - x_2)\right] = Q\,\delta(x)\delta(y)\left[\delta(z-a) - \delta(z+a)\right]$$

$$|x_1 - x_2| = R = 2a$$

$$\vec{D} = \varepsilon(E)\,\vec{E}$$

$$\varepsilon(E) = \frac{1}{2} b_0 \log\left(\frac{E}{\chi}\right) \qquad b_0 = \frac{1}{8\pi^2}\left(11 - \frac{2}{3} N_f\right)$$

NAIVE ATTEMPT

$$\vec{E} = -\vec{\nabla}\psi$$

NUMERICALLY UNSTABLE

↖ scalar potential

TO LOCATE FREE BOUNDARY, GO TO FLUX FUNCTION Φ

- USE CYLINDRICAL COORDINATES

  $\rho = (x^2+y^2)^{1/2}$ $\qquad$ $\phi = \tan^{-1}(y/x)$

$$\vec{D} = -\frac{1}{2\pi}\,\vec{\nabla}\phi \times \vec{\nabla}\Phi = -\frac{\hat{\phi}}{2\pi\rho} \times \vec{\nabla}\Phi = \vec{\nabla}\times\left[\frac{\hat{\phi}}{2\pi\rho}\,\Phi\right]$$

- $\vec{\nabla}\cdot\vec{D} = 0$ $\qquad$ FLUX THROUGH CAP $= \Phi$

- $\Phi = 0,\ \rho = 0 \qquad |z| > a$

  $\Phi = Q,\ \rho = 0 \qquad |z| < a$

  $\Phi \to 0$ as $\rho^2 + z^2 \to \infty$

- $\vec{\nabla}\cdot\left[\sigma(\rho, \vec{\nabla}\Phi)\,\vec{\nabla}\Phi\right] = 0$ $\qquad$ NEW DIFFERENTIAL EQUATION

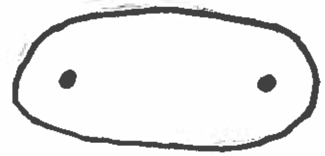
HERE $\sigma(\rho, \vec{\nabla}\Phi) = \dfrac{1}{\rho^2\,\varepsilon(D)}$ $\qquad$ $D = \dfrac{|\vec{\nabla}\Phi|}{2\pi\rho}$

(Fichera equation
Degenerating elliptic
↓
parabolic)

RESULTS

THE MODEL CONFINES FLUX IN AN ELLIPSOID OF REVOLUTION

(large R limit)



(Adler + Piran Phys Lett B)

NUMERICAL AND ANALYTIC RESULTS AGREE AT LARGE, SMALL R

SMALL R CAN DEVELOP MODEL IN A PERTURBATION EXPANSION

(Adler, Nucl Phys B)

$$V_{static} = \frac{1}{2}\int d^3x\, d^3x' \frac{J_0(x) J_0(x')}{4\pi |x-x'|} - \text{Coulomb self-energies}$$

$$\xrightarrow[R\to 0]{} -\frac{\alpha^2}{4\pi R \frac{1}{2} b_0} \frac{1}{\log W_R} + \cdots \qquad W_R = \frac{1}{\Lambda_\ell^2 R^2}$$

$$\Lambda_\ell = 2.52\, K^{1/2}$$

RESULTS, CONTINUED

LARGE R (Lehmann + Wu Nucl Phys B)

MAGIC: $\rho'(\Phi^0, z')$ IS SEPARABLE

$$\Phi = \Phi^0(\underbrace{\rho/R^{1/2}}_{\rho'}, \underbrace{z/R}_{z'}) + \cdots$$

THIS GIVES

$$\Phi^0 = Q\left[1 - \frac{1}{2}\left(\frac{\pi b_0}{2Q}\right)^{1/2} \frac{a\rho^2 \kappa^{1/2}}{\lambda^2 - z^2}\right] \qquad a = \frac{1}{2}R$$

$$V_{static} \underset{R\to\infty}{\longrightarrow} \kappa Q R + Q^{3/2}\frac{2}{3}\left(\frac{2}{\pi b_0}\right)^{1/2} \kappa^{1/2} \log(\kappa^{1/2} R) + \cdots$$

EXTRAPOLATION FROM "ASYMPTOTIC FREEDOM" SCALE TO "INFRARED SLAVERY" SCALE IS SEMIQUANTITATIVELY ACCURATE, BETTER THAN A FACTOR OF 4.

SO THE LEADING LOG MODEL CAPTURES ESSENCE OF CONFINEMENT