

1. What is wrong with the following ‘proof’ that if  $S$  is a body in  $\mathbb{R}^3$  then  $|S|^2 \leq |S_{12}||S_{23}||S_{31}|$ : if we let  $T = S \times S \subset \mathbb{R}^6$ , then  $|S|^2 = |T| \leq |T_{12}||T_{34}||T_{56}| = |S_{12}||S_{31}||S_{23}|$ .
2. Let  $A \subset \mathcal{P}(X)$  be a set system whose projection onto each of the sets  $\{1, 2, 3\}, \{2, 3, 4\}, \dots, \{n-2, n-1, n\}, \{n-1, n, 1\}, \{n, 1, 2\}$  has size less than 8. Explain why when  $n$  is a multiple of 3 it is trivial that  $|A| \leq 7^{n/3}$ , and then prove that in fact this holds for every value of  $n$ .
3. Let  $Y_1, Y_2, \dots, Y_r \subset [n]$  be sets that do *not* form a uniform cover of  $[n]$ . Show that knowledge of  $|S_{Y_1}||S_{Y_2}| \dots |S_{Y_r}|$  does not imply any upper bound on  $|S|$ .
4. Let  $A$  be a family of graphs on  $n$  vertices such that the intersection of any two members of  $A$  has no isolated vertices. Show that, for  $n$  even,  $A$  cannot be larger than the family of all graphs containing a given matching.
5. Let  $S$  and  $T$  be bodies in  $\mathbb{R}^n$ , and let  $B$  and  $C$  be boxes verifying the Bollobás-Thomason box theorem for  $S$  and  $T$  respectively. Give an example with  $S \subset T$  such that  $B$  and  $C$  cannot be chosen to satisfy  $B \subset C$ .
- <sup>+</sup>6. Let  $A$  be a family of graphs on  $n$  vertices such that the intersection of any two members of  $A$  has a Hamilton cycle. How large can  $A$  be?