The Work of Lars Brink

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Lars Brink was a cherished friend and colleague whose fundamental contributions, especially to supersymmetry and string theory, will be long remembered.

Lars first made his mark with a series of important contributions to string theory in 1973. It was known at this time that string theory is consistent only in a critical dimension (26 for bosonic strings and 10 for fermionic strings, that is strings that describe fermions as well as bosons) and only if a certain restriction is placed on the assumed mass of the lightest mode of the string. The meaning of these restrictions was quite mysterious. With Holger Nielsen, Brink removed much of the mystery by computing the ground state energy of a string at the quantum level, that is, by computing the Casimir energy of the vibrations along a string. In this way, they computed the mass of the lightest string state as a function of the assumed spacetime dimension D. Then, they could also determine D by a simple criterion related to consistency of the spectrum. The first excited string state was a spin one state that had to be massless because after imposing the gauge constraints and equivalences, it only has transverse excitations. Once the ground state energy was known, requiring the first excited state to be massless led directly to the result $D = 26$ (or $D = 10$ for the model with fermions). Computing the Casimir energy along the string was a novelty in 1973, but nowadays, half a century later, it is one of the basic ideas that we teach to students. Of course, the calculation of Brink and Nielsen has been generalized to other cases discovered later, such as strings on orbifolds.

Also in 1973, Lars Brink wrote a series of important papers with David Olive aimed at improving the understanding of one-loop amplitudes in string theory. Much was already known at the time, but some key ingredients were not understood, in part because the covariant approach to worldsheet computations was not yet available and in part because of technical difficulties in understanding fermions. In a pair of papers, Brink and Olive improved the understanding of the physical state projection operator for the bosonic string theory in the critical dimension, and went on to calculate the planar one-loop diagram for open strings in a way that exhibited its unitarity. To make unitarity manifest, they used the Feynman tree theorem to construct the one-loop diagrams. This computation was state of the art until the following decade, when new methods were introduced involving Faddeev-Popov ghosts and BRST symmetry. Their use of the Feynman tree theorem anticipated by several decades the development of on-shell methods to analyze gauge and gravitational amplitudes.

In the same year, Brink and Olive wrote two other important papers, with co-authors. One of the striking properties of string theory is the existence of tree-level closed string poles in certain one-loop open string diagrams. Because one of the closed string states has the properties of a graviton, this can be described by saying that open-string theory, at the quantum level, unavoidably leads to gravity; the appearance of these poles thus underlies the whole modern interest in string theory. Accordingly, the couplings of closed strings to open strings were of fundamental interest. Brink and Olive with Joel Scherk explored these couplings and analyzed their physical consistency. In another paper, Brink, Olive and Scherk with Claudio Rebbi improved the understanding of tree level scattering amplitudes involving fermionic states. This is a difficult topic with the methods available at that time.

In 1976, with Paolo Di Vecchia and Paul Howe, Brink reformulated fermionic string theory in terms of a covariant worldsheet action with worldsheet bosons and fermions and with local supersymmetry on the worldsheet. This had tremendous advantages over previous formulations and is the modern way that the theory (or at least its Ramond-Neveu-Schwarz version, as opposed to the Green-Schwarz and Berkovits versions discovered in the 1980's and 1990's) is usually presented. Brink, Di Vecchia, and Howe explored the field theory limit of this construction – actions for spinning particles – in two further papers, one of them co-authored with Stanley Deser and Bruno Zumino.

Also in 1976, Brink was one of a large group of authors (M. Ademollo, L. Brink, A. D'Adda, R. D'Auria, E. Napolitano, S. Sciuto, E. Del Giudice, P. Di Vecchia, S. Ferrara, F. Gliozzi, R. Musto, R. Pettorino, and J. Schwarz) who discovered the string theory with $N = 2$ supersymmetry.

However, the most important paper that Brink wrote in 1976, and possibly the most important paper of his career, was on a seemingly different topic, though a topic that before long was to tie up with string theory in a dramatic fashion. This is the paper that Brink wrote with Scherk and John Schwarz on the construction of supersymmetric Yang-Mills theories. Since the discovery of spacetime supersymmetry a couple of years earlier, and given the central role of gauge theory in the Standard Model of particle physics, there was at that time great interest in the construction of supersymmetric Yang-Mills theories. A number of the basic examples had been constructed. But the maximally supersymmetric version of Yang-Mills theory was not yet known. Brink and his co-authors had the then radical idea that theories with extended supersymmetry could be best understood by starting in a spacetime dimension $D > 4$ and then ultimately "reducing" the theory to 4 dimensions by assuming the fields to be independent of $D - 4$ coordinates. They soon discovered that the maximum dimension of super Yang-Mills theory was $D = 10$. The new theory that they discovered in ten dimensions, when reduced to four dimensions, becomes the maximally supersymmetric gauge theory in four dimensions, usually called $\mathcal{N} = 4$ super Yang-Mills theory. Study of this new theory in the 47 years since its discovery has led to a host of important discoveries, of which perhaps the three most important are electric-magnetic duality, duality with gravity, and integrability in the planar limit. In addition, the origin of the theory in $D = 10$, which happens to be the critical dimension of the fermionic string, was an extremely important hint towards further developments, involving spacetime supersymmetry in string theory and its relations to supersymmetric gauge theory and gravity at low energies. The work of Brink, Scherk, and Schwarz was also part of the inspiration for the discovery a couple of years later of eleven-dimensional supergravity and its reduction to maximal ($\mathcal{N} = 8$) supergravity in four dimensions by Eugène Cremmer, Bernard Julia, and Scherk.

In the late 1970's and early 1980's, string theory was in eclipse, because it had been apparently superseded by QCD as a theory of strong interactions, and because of rapid and exciting developments in quantum field theory. Lars Brink was one of the very few physicists, along with Michael Green and John Schwarz, who continued working on string theory in those years. In particular, Brink, Green, and Schwarz wrote a very important paper in 1982 on one-loop amplitudes in string theory. This was the first paper that computed completely consistent, unitary, and tachyon-free string loop amplitudes. To do such computations, they drew on a variety of insights from previous work, including the Gliozzi-Olive-Scherk projection that is needed to get spacetime supersymmetry, and a refinement of it for closed strings due to Green and Schwarz. Brink, Green, and Schwarz showed that one-loop string amplitudes reduce at low energies to amplitudes of ten-dimensional super Yang-Mills theory and supergravity, confirming the potential of string theory to unify gravity with quantum mechanics and with conventional particle forces. In addition to computing consistent loop amplitudes in ten dimensions, they made a toroidal compactification of six "extra" dimensions and computed loop diagrams in an effective four-dimensional world. Though previous authors had speculated about getting a four-dimensional theory by compactification of a ten-dimensional supersymmetric string theory, this paper by Brink, Green, and Schwarz is the first one in which that program was actually carried out. Their paper is perhaps also the first paper in which formulas related to the T-duality of toroidal compactification are visible, though this was not a primary theme in the paper. T-duality became very important in subsequent developments.

Brink, Green, and Schwarz went on in 1983 to do important work on light cone gauge in string theory and on superstring field theory.

Almost as soon as Brink, Scherk, and Schwarz had constructed $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in four dimensions, it became clear that at leading non-trivial order this theory has quantum conformal symmetry (the theory is ultraviolet finite and the one-loop beta function vanishes). Schwarz and Murray Gell-Mann conjectured that this conformal invariance might persist to all orders and possibly as an exact statement. Brink became fascinated by this conjecture. In 1982, Brink wrote two important papers with Olof Lindgren and Bengt Nilsson formulating $\mathcal{N} = 4$ super Yang-Mills in terms of light cone fields and using this to demonstrate ultraviolet finiteness and thus quantum conformal invariance to all orders of perturbation theory. This was the first proof of quantum conformal invariance in this theory. Several other arguments were discovered later and the result is now understood to be true exactly (not just in perturbation theory).

Light cone formulations of gauge and gravitational theories were one of Lars Brink's main interests in the later part of his career and he wrote many memorable papers on these subjects.

I can remember Lars Brink expressing to me, around 1990, the dream that there would be some sort of important application of $\mathcal{N} = 4$ super Yang-Mills theory, which at the time still seemed like a sort of curiosity, or at least a highly idealized theory, far removed from real physics. I am sure he was pleased with the many discoveries about this theory that were made in the 1990's and later and with the central role that it came to play in many aspects of theoretical physics.

The work of Lars Brink has had and will continue to have a lasting influence on the development of theoretical physics.