Given any bounded multiplicative function $f : \mathbb{N} \to \mathbb{D}$, a deep conjecture of Elliott predicts cancellations of the form

$$\lim_{x\to\infty} \frac{1}{x} \sum_{n\leq x} f(n+h_1)f(n+h_2)\dots f(n+h_k) = 0,$$

for all distinct shifts $h_i \in \mathbb{N}$, unless it is ``close" to the modulated Dirichlet character in an appropriate sense. Partial progress towards this conjecture has had numerous consequences, including solution of the Erdős discrepancy problem, progress on the (logarithmic) Chowla and Sarnak's conjectures, and many others.

In this talk I will report on some new developments towards this conjecture and present several applications in number theory, ergodic theory and combinatorics. This is based on a joint work with A. Mangerel and J.Teräväinen.