

TEN YEARS SINCE THE IAS SYMPLECTIC DYNAMICS SPECIAL YEAR

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I was asked to write this impact statement in lieu of John Mather, who unfortunately passed away in 2017. The special year **Symplectic Dynamics** lead by John Mather (distinguished visiting professor) and Helmut Hofer was not the typical program. Usually long-term thematic programs at mathematical research institutes evolve around well-established themes, but this was not the case here. Even the name picked for the program did not really exist before the program. The main goal of the program was to start an interaction between researchers in the fields of dynamical systems and symplectic geometry with the hope that these activities would lead in the long run to a new field “Symplectic Dynamics” with highly integrated ideas and methods from both disciplines. There were some indications that such a field should exist but nevertheless the goal was ambitious. As it turned out the special characteristics of the Institute for Advanced Study (IAS) turned out to be important for the success. Programs at the IAS, compared to other research institutes, are smaller since by design they only involve one third of the activities. In addition the whole set-up fosters interactions of the program with other mathematical areas. The following description focuses on the contributions in this new area, but there were also other important contributions.

Looking back, the program lead to a formation of a core group of researchers who were able to start this new field with spectacular results. This meant that the special year created a community which was very important for the activities which followed. Having this community, focussed several activities and attracted graduate students and researchers and established the “Symplectic Dynamics Community” which is growing ever since. The special year itself produced some breakthroughs, for example Bramham’s work on pseudo-rotations settling questions going back to Anosov and Katok about zero entropy systems in low dimensions. Work by Hryniewicz, Salamao and collaborators on transversal foliations leads to extremely powerful tools to understand the finer structure of low-dimensional Hamiltonian systems. Other important activities at that time have been concerned with Arnold Diffusion with activities around V. Kaloshin and Ke Zhang. During the program a major effort was made on proving Arnold diffusion for fiber convex Hamiltonians and the work settled a 50 year old conjecture by Arnold for convex Hamiltonians of three degrees of freedom. This work is the culmination of two decades of efforts of several mathematicians with John Mather leading the way to develop what is now called weak KAM theory and variational methods. Mather’s and Fathi’s contribution were very important. Since the program, a certain amount of activity in the area of symplectic dynamics has continued at IAS and contributed to further important developments around C^0 -symplectic geometry (Seyfaddini) and the simplicity conjecture (Cristofaro-Gardiner). Another outcome is the theory of *feral curves* by J. Fish and H. Hofer, which solves a problem by M. Herman, which was stated in Herman’s 1998 ICM address in Berlin, in dimension four, by showing that regular compact Hamiltonian energy surfaces in \mathbb{R}^4 can never have a minimal flow.

Ten years after the program, the field of Symplectic Dynamics has flourished. Symplectic methods have allowed to solve long-standing problems in dynamical systems. Questions of a dynamical nature irrigate the field of symplectic topology. A large number of conferences have been organized around this topic. Between April and July 2021 a thematic trimester on Symplectic Topology, Contact Topology and Interactions was organized at the Institut Henri Poincaré in Paris, and questions pertaining to Symplectic Dynamics were central to the program. Beyond any doubt, the 2011-2012 program at IAS was instrumental in all these recent developments and fulfilled its goal beyond all expectations. It precipitated the birth of Symplectic Dynamics and established it as an important new mathematical field of research.