PCMI 2023 | *Instructor: Sandy Irani*
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Quantum Hamiltonian Complexity: Day 5

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Stoquastic Hamiltonians

1. Consider the Heisenberg anti-ferromagnetic Hamiltonian on a 2D lattice:

$$\sum_{(i,j)} X_i \otimes X_j + Y_i \otimes Y_j + Z_i \otimes Z_j$$

   (a) Is this Hamiltonian stoquastic in the standard basis?

   (b) Now consider a change of basis in which a Z operator is applied to every other vertex in a checkerboard pattern. Show that this Hamiltonian when expressed in the new basis is stoquastic.

   (c) Generalize this observation to show that the Heisenberg anti-ferromagnetic on any bipartite graph is stoquastic.

2. Is the Toric code Hamiltonian stoquastic? Is there a similar transformation to the one in the previous question that will make it stoquastic?

3. Now consider the Heisenberg anti-ferromagnet model applied to each edge of a 4-cycle.

   (a) The ground state for this Hamiltonian is:

$$\frac{1}{\sqrt{12}}\left[|1100\rangle + |0110\rangle + |0011\rangle + |1001\rangle - 2|0101\rangle - 2|1010\rangle\right]$$

   Even though this state is non-positive, verify that when the state is expressed in the basis from question 1, that it is indeed a positive state.

   (b) What is the ground energy for this state? Is it frustration-free?

   (c) Determine the operator G for this Hamiltonian. Use this operator to describe the transition probabilities for the resulting Markov chain. Note that you may need to scale H first so that each term has norm at most 1.

   (d) Verify that the amplitudes given for the ground state correspond to the stationary distribution for the Markov chain.

4. Now consider a variation on the Heisenberg anti-ferromagnetic model with matrix:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Our Hamiltonian for this question will consist of this term applied to every pair of particles along a cycle with 8 vertices.

(a) Is this system frustration free?

(b) Express the transition probabilities for this Markov Chain. (Verify first that the normal of each term is at most 1).

(c) Characterize the set of ”good” and ”bad” strings for this Hamiltonian.

(d) Characterize the set of strings reachable by the Markov chain from the following three possible starting strings:

  i. 00000000

  ii. 00001000

  iii. 01000100

(e) Give an example of a ”good” string that is not in the support of a groud state for this Hamiltonian.

(f) What is the dimension of the ground space for this Hamiltonian? Give a basis for the ground space.