PCMI 2023 *Instructor: Sandy Irani*
TA: Kunal Marwaha

Quantum Hamiltonian Complexity: Day 1

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topics: Introduce Local Hamiltonian Problem, complexity class QMA, local Hamiltonian contained in QMA, boosting completeness and soundness, SAT is NP-hard

1. It was mentioned in lecture that if the gap between the completeness c and soundness s for a QMA verifier is at least $|c - s| \geq 1/\text{poly}(n)$, then is can be amplified efficiently so that $c > 1 - 1/\exp(n)$ and $s < 1/\exp(n)$. Prove this claim.

2. Consider a one-qubit Hamiltonian $H = \frac{1}{3}X + \frac{1}{2}I$. Give a two-qubit operator O such that when O is applied to $|\phi\rangle \otimes |0\rangle$ and the second bit is measured in the standard basis, then the probability of measuring 0 is $\langle\phi|H|\phi\rangle$.

3. Since every Hamiltonian operator H is a Hermitian matrix, it can be expressed as $H = \sum_j \alpha_j \Pi_j$, a real-valued linear combination of orthogonal projectors. Suppose that each Π_j acts non-trivially on at most two qubits. Given copies of a state $|\psi\rangle$, describe a procedure for estimating the energy of $|\psi\rangle$ using only two-qubit measurements. How would the error in your estimation behave as a function of the number of two-qubit measurements?

4. What's the difference between a language and a promise class?

5. Suppose in the local Hamiltonian problem, the local terms H_i do not have bounded norm. Is this problem still in QMA? Why or why not?

6. Consider the Constraint Satisfaction Problem with 4 variables and constraints $\{x_1 \wedge x_2 \wedge x_3, \neg x_1 \wedge x_4, x_2 \wedge x_4\}$. Write this as a local Hamiltonian problem. What is its ground state?

7. The MAX-CUT problem is a classical problem which takes as input a graph G and asks for a way to partition the vertices into two sets in order to maximize the number of edges whose endpoints are in different sets. Phrased as an optimization problem one asks for the maximum number of edges that can be obtained by any partition of the vertices. Describe how to formulate the MAX-CUT problem as an instance of classical Local Hamiltonian.

8. Suppose that a QMA verifier on input x with n bits requires a quantum witness with m qubits. Suppose that for any NO instance x with n bits, the verifier will accept with probability at most s on input x using any witness state $|\phi\rangle$. Now suppose that the verifier is given a $t \cdot m$ qubit state $|\psi\rangle$. The verifier runs t times using input x and each time uses a different m qubits of $|\psi\rangle$ as the witness. Argue that the probability of acceptance on each run of the verifier is at most s, regardless of the previous acceptances.