Quantum Hamiltonian Complexity

Part III

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Recap: The Local Hamiltonian Problem

Input:

$H_1, \ldots, H_r$:

Hermitian positive semi-definite matrices
operating on k qudits of dimension d
with bounded norm $\|H_i\| \leq 1$.
n qudits in the system.

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Two real numbers E and $\Delta \geq 1/\text{poly}(n)$

Output:

Is the smallest eigenvalue of $H = H_1 + \cdots + H_r \leq E$
or are all eigenvalues $\geq E + \Delta$?

Recap: The class QMA (Quantum Merlin Arthur)

NP

A problem is in NP if there is a polynomial time Turing Machine M such that on input x, where $|x| = n$:

If $x \in L$, then there is a witness y such that $M(x, y)$ accepts.

If $x \notin L$, then for every y, $M(x, y)$ rejects.

$|y| \leq \text{poly}(x)$

Boolean Satisfiability is NP-complete

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QMA

A *promise* problem is in QMA if there is a poly-sized uniform **quantum** circuit family $\{C_n\}$ such that on input x, where $|x| = n$:

If $x \in$ YES, then there is a **quantum** witness $|\phi\rangle$ such that $\text{Prob}[C_n(x, |\phi\rangle) = 1] \geq 2/3$.

If $x \in$ NO, then for every $|\phi\rangle$, $\text{Prob}[C_n(x, |\phi\rangle) = 1] \leq 1/3$.

$|\phi\rangle$ has poly(n) qubits.

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Is there a state whose energy (according to H) is less than *E*?

$\langle \Phi | H | \Phi \rangle \leq E$?

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Guarantee:

There exists $|\Phi\rangle$ such that $\langle\Phi|H|\Phi\rangle \leq E$

OR

For all $|\Phi\rangle$, $\langle\Phi|H|\Phi\rangle \geq E + \Delta$

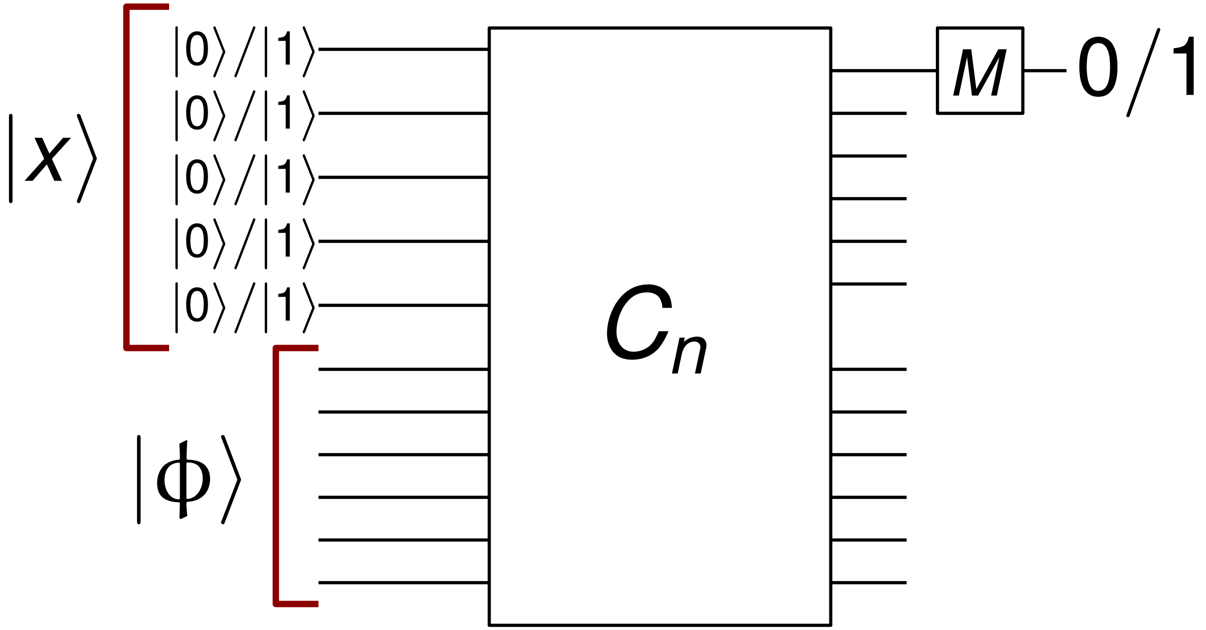
$\Longrightarrow$

Showed a measurement whose outcome = 1 with probability $\propto \langle\Phi|H|\Phi\rangle$.

Recap: Local Hamiltonian is QMA-hard

Start with a generic language L in QMA

Is $x \in L$?



Is there a quantum state $\phi\rangle$
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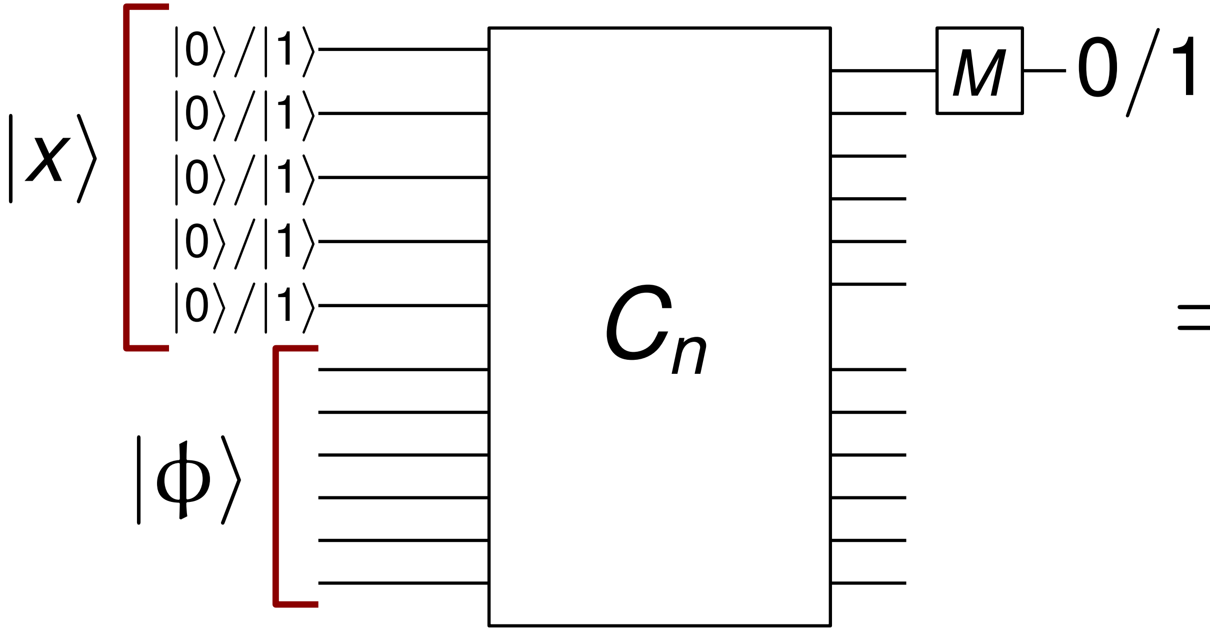

$\Rightarrow$ k-Local Hamiltonian: (H_x, E, Δ)

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Is the ground energy of H_x $\leq E$ or $\geq E + \Delta$?

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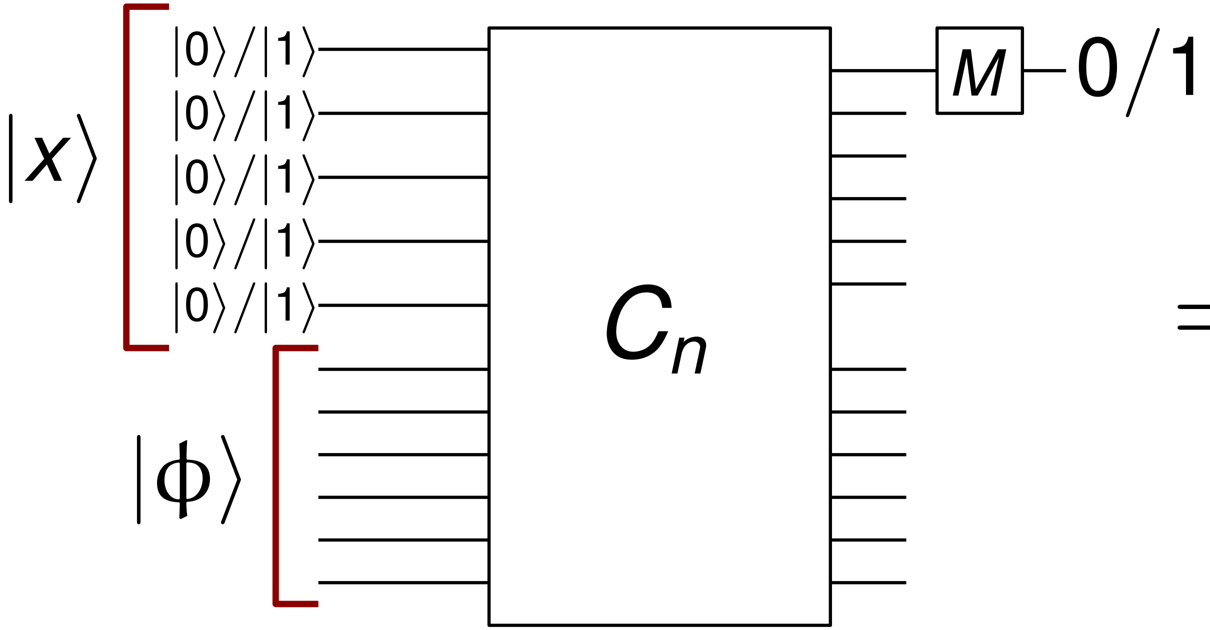

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$$H_t = \frac{1}{2}\left[I \otimes |t\rangle\langle t| + I \otimes |t-1\rangle\langle t-1| + U_t \otimes |t\rangle\langle t-1| - U_t^\dagger \otimes |t-1\rangle\langle t| \right]$$

$$H_{prop} = \sum_{t=1}^{T} H_t$$

Ground State:

$$\frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} U_t U_{t-1} \cdots U_2 U_1 |x\rangle |\xi\rangle \otimes |t\rangle$$

Spectral Gap:

$$\geq \frac{1}{2(T+1)^2}$$

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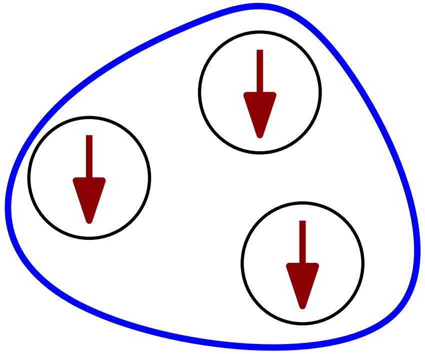
$$H = H_{prop} + H_{init} + H_{out}$$

Local Hamiltonian Variations



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where each H_a acts on at most k qudits

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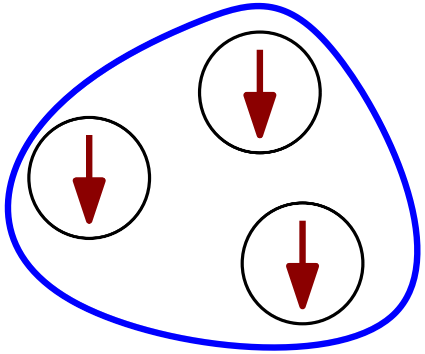

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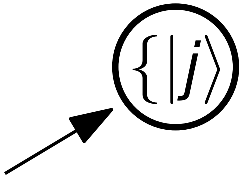

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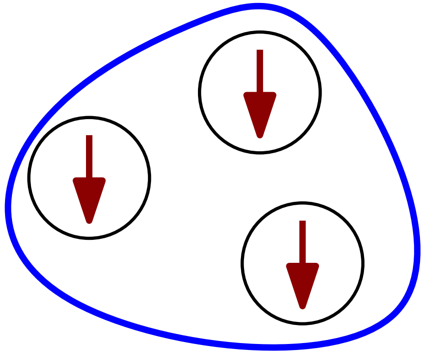

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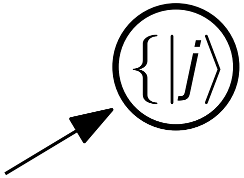
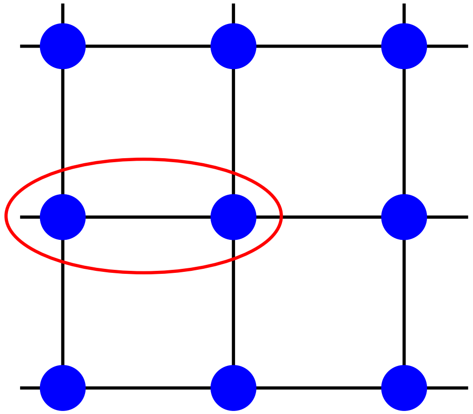


$\{|0\rangle, |1\rangle, \ldots, |d-1\rangle\}$

Geometry



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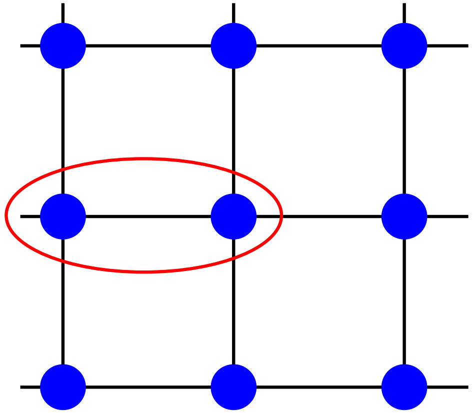

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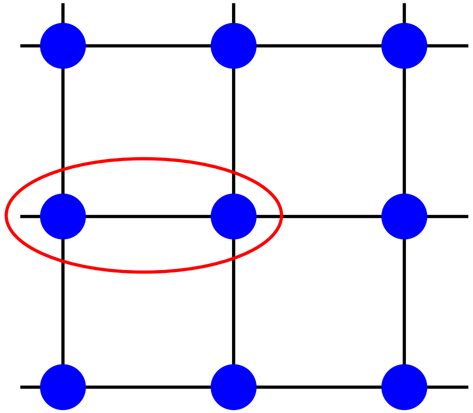

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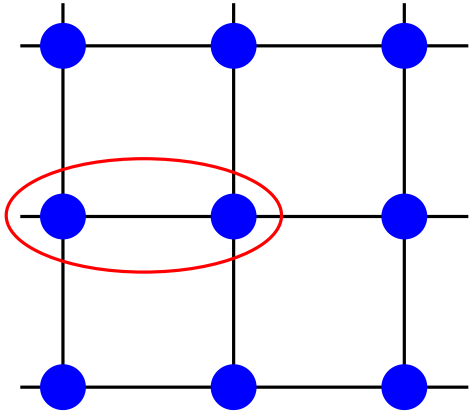

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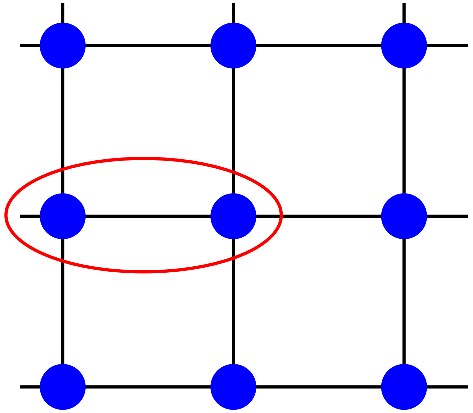

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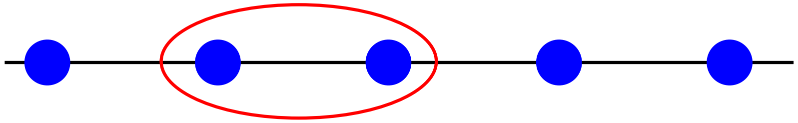

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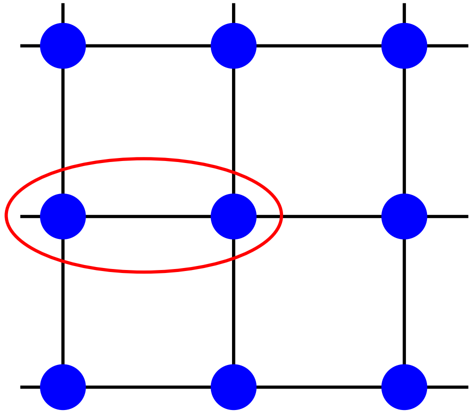

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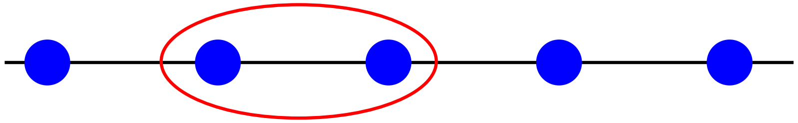

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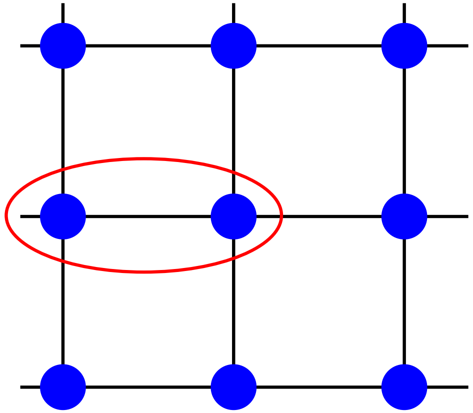

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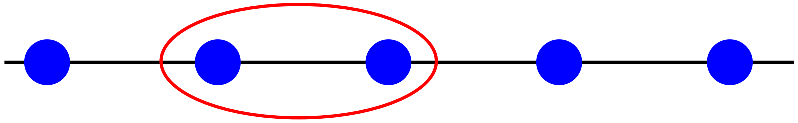

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Final state will be close to the ground state of H_{final} if speed of transition is

$$\Omega(\ \|H_{final} - H_{start}\| / \underline{\Delta(H(t))}^{2+\delta}\ )$$

$\Delta(H)$: Spectral gap of H

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Final measurement to determine result of computation

The Adiabatic Model

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Adiabatic computation may be more robust against certain kinds of errors.
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What is the spectral gap of the intermediate Hamiltonians?

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  Yes - [van Dam, Mosca, Vazirani]

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$H_{final} = H_{prop}$

Hamiltonian whose ground state is the computation state for Quantum Circuit C with input x.

[Aharonov, van Dam, Kempe, Landau, Lloyd, Regev 2004]

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$$\underbrace{|00\cdots00\rangle}_{\text{Computation}}\underbrace{|00\cdots00\rangle}_{\text{Clock}}$$

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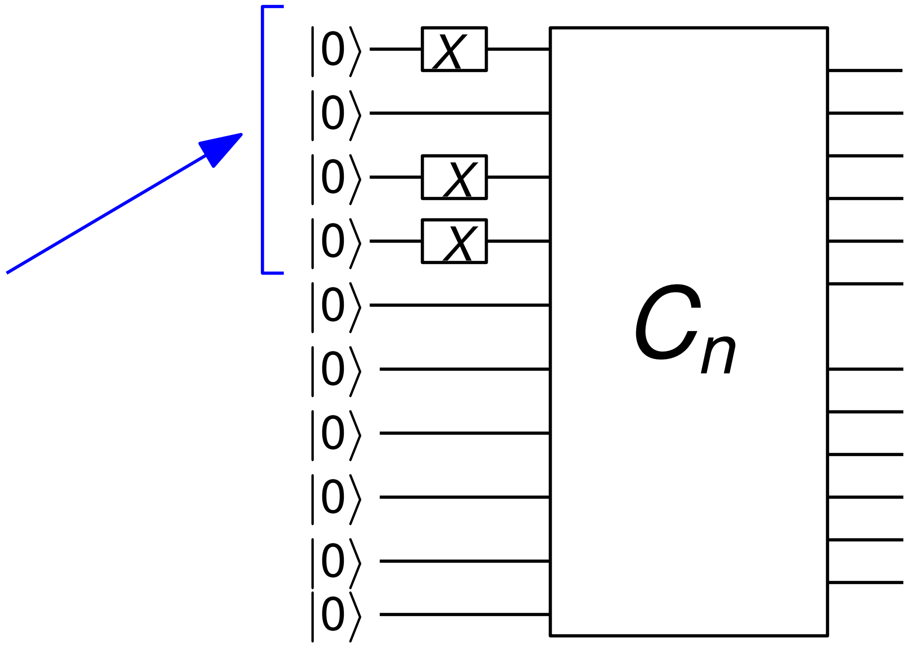

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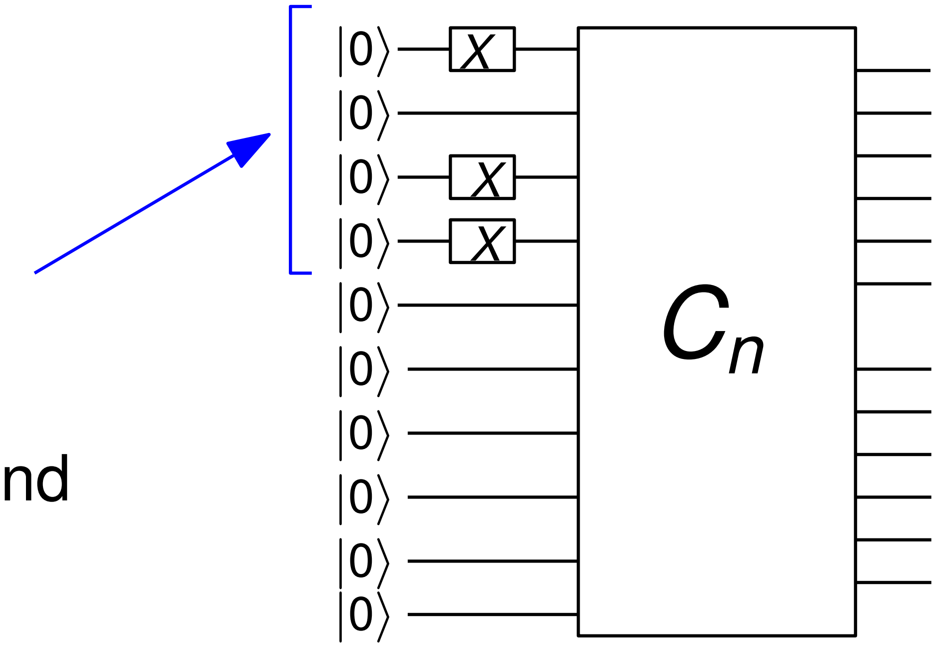

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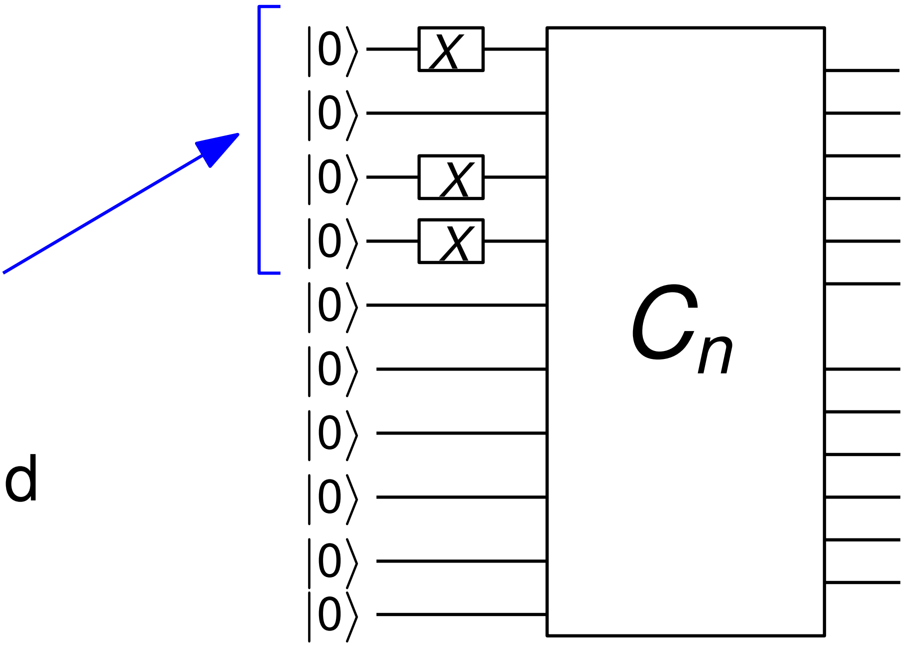

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Measure:

$$|T\rangle\langle T|_{clock} \quad \text{then} \quad |1\rangle\langle 1|_{out}$$

Probability to measure the clock in state T is $\frac{1}{T+1}$

Lower Bound Spectral Gap

$$H_{start} = \begin{bmatrix} 0 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

$$H_{final} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & & & & & & \\ -\frac{1}{2} & 1 & -\frac{1}{2} & & & & & \\ 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & & & & \\ & & & \cdot & & & & \\ & & & & \cdot & & & \\ & & & & & \cdot & & \\ & & & & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ & & & & & -\frac{1}{2} & 1 & -\frac{1}{2} \\ & & & & & & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

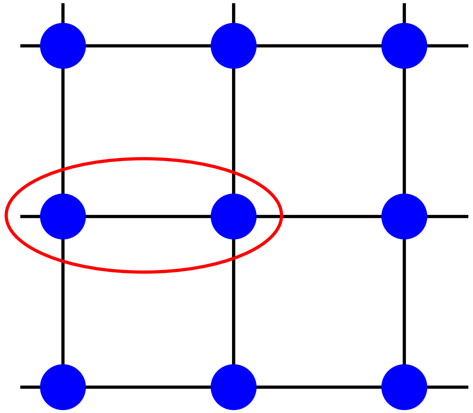
Spectral gap of:

$$(1-s)H_{start} + sH_{final} \text{ for } s \in [0,1] \text{ is } \geq \frac{1}{2(T+1)^2}$$

QMA-complete Problems

5-local 2-state Hamiltonian is QMA-Complete [Kitaev 1995]

2-dimensional 2-local 6-state Hamiltonian is QMA-complete
[Aharonov, van Dam, Kempe, Landau, Lloyd, Regev 2004]

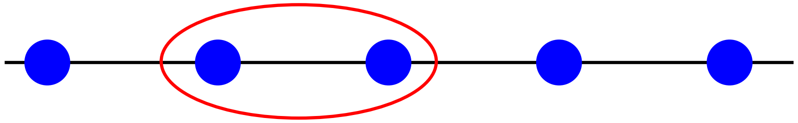


2-local 2-state Hamiltonian is QMA-complete
[Kempe, Kitaev, Regev 2005]

2-dimensional 2-local Hamiltonian is QMA-complete
[Oliveira Terhal 2008]

1-dimensional 12-state Hamiltonian is
QMA-complete

[Aharonov, Gottesman, Irani, Kempe, 2009]



2D Local Hamiltonian Reduction

Kitaev Construction:

$$\frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} \underbrace{|\psi_t\rangle}_{\text{Computation Qubits}} \underbrace{|1^{t+1}0^{T-t}\rangle}_{\text{Clock Qubits}}$$

2D Local Hamiltonian Reduction

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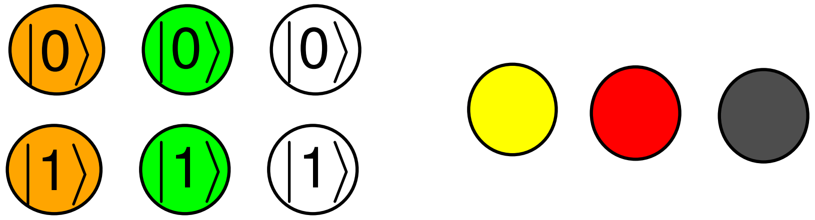
$$\frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} \underbrace{|\psi_t\rangle}_{\text{Computation Qubits}} \underbrace{|1^{t+1}0^{T-t}\rangle}_{\text{Clock Qubits}}$$

The "Clock" is distributed throughout the entire quantum system:

State space for a particle:

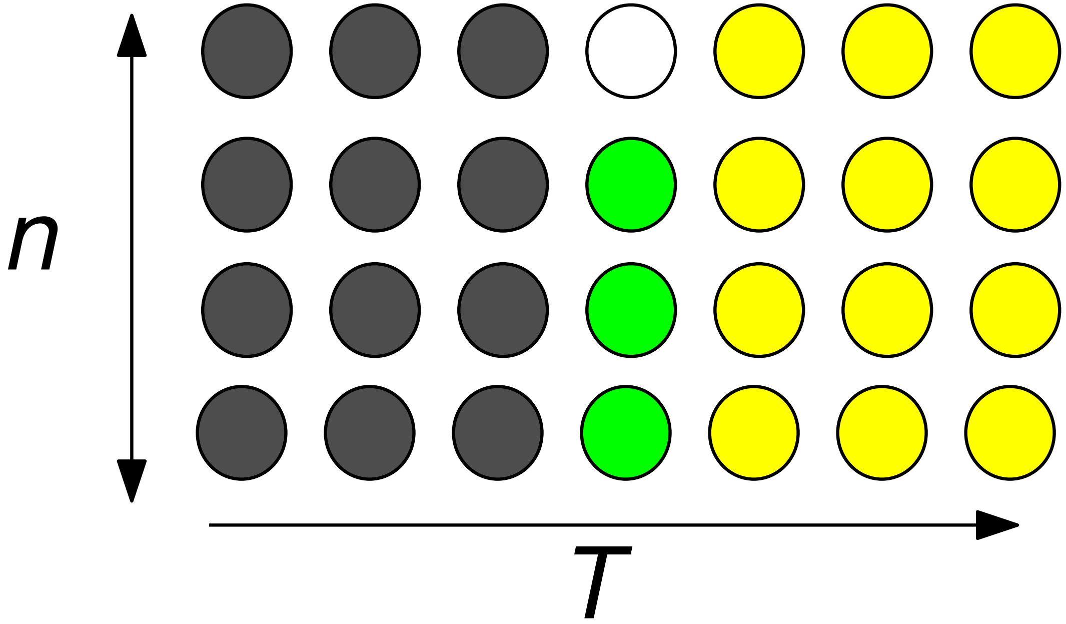
$\{|0\rangle, |1\rangle\} \otimes \{|$🟠$\rangle, |$🟢$\rangle, |$○$\rangle\}$

$\cup \; \{|$🔴$\rangle, |$⚫$\rangle, |$🟡$\rangle\}$



2D Local Hamiltonian Reduction, cont.

Clock state is a pattern of colors on the 2D grid of particles:



2D Local Hamiltonian Reduction, cont.

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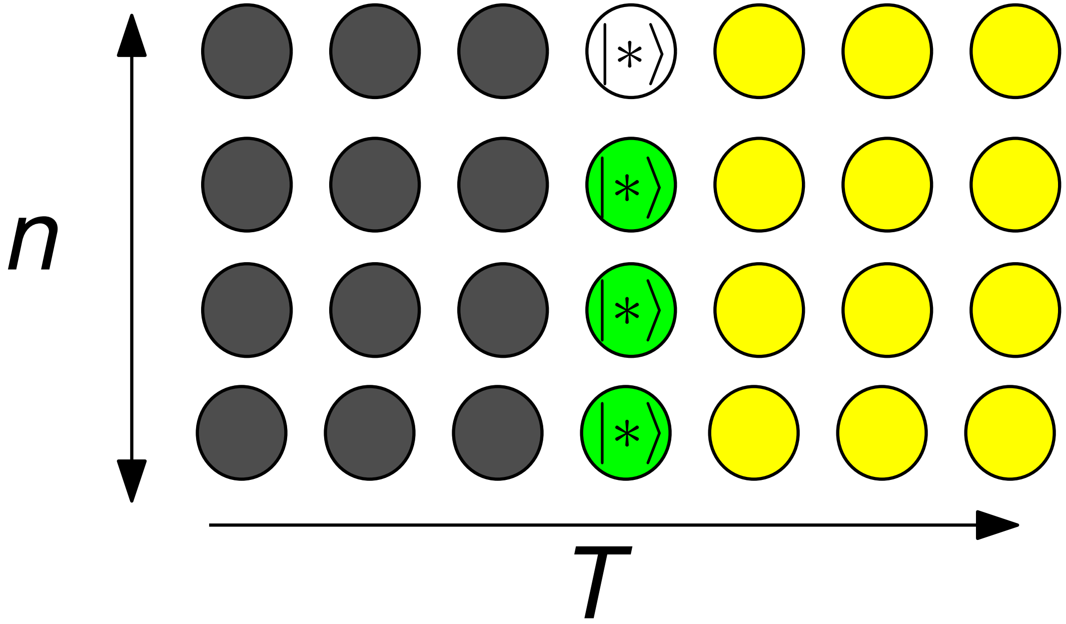
Some particles have a computation bit embedded in their state.



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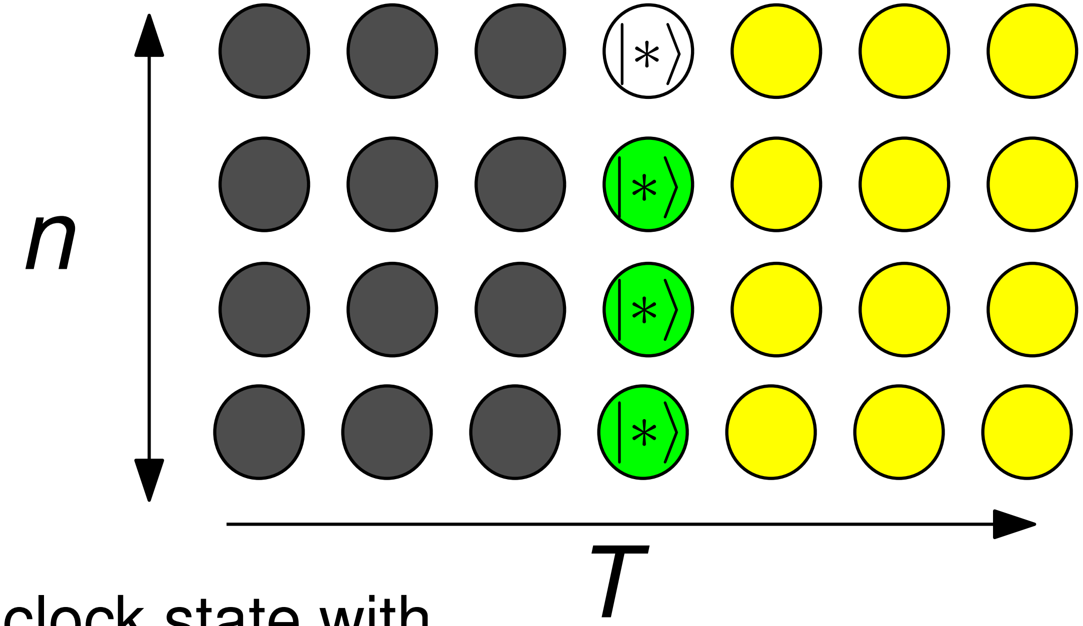
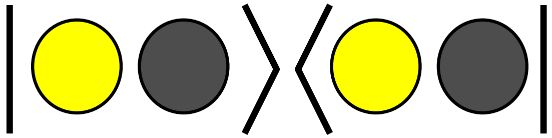
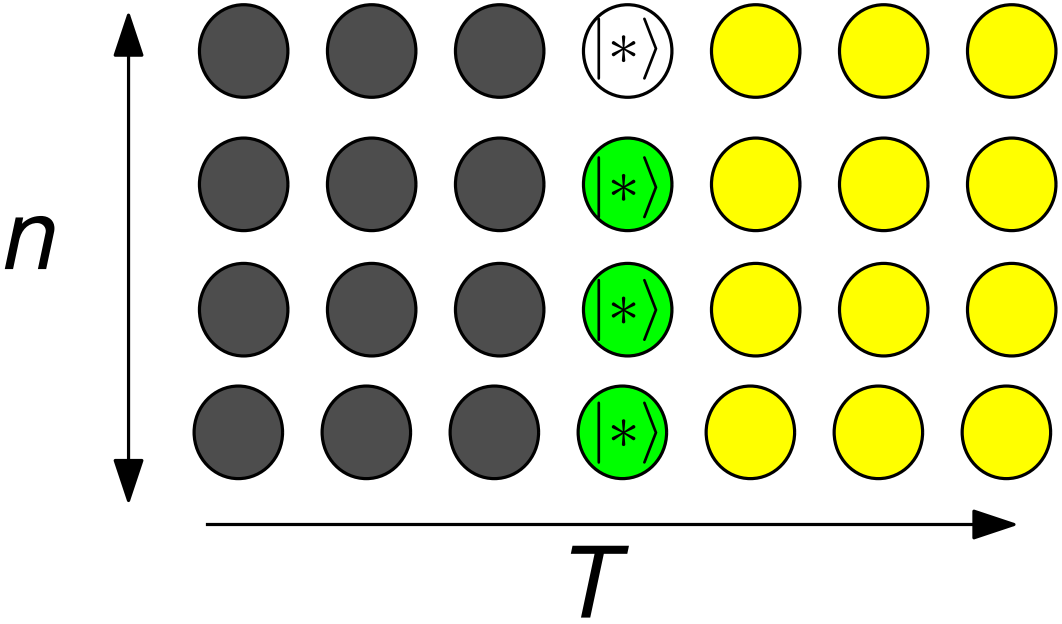


Enforce valid clock state with
"forbidden"
local configurations:



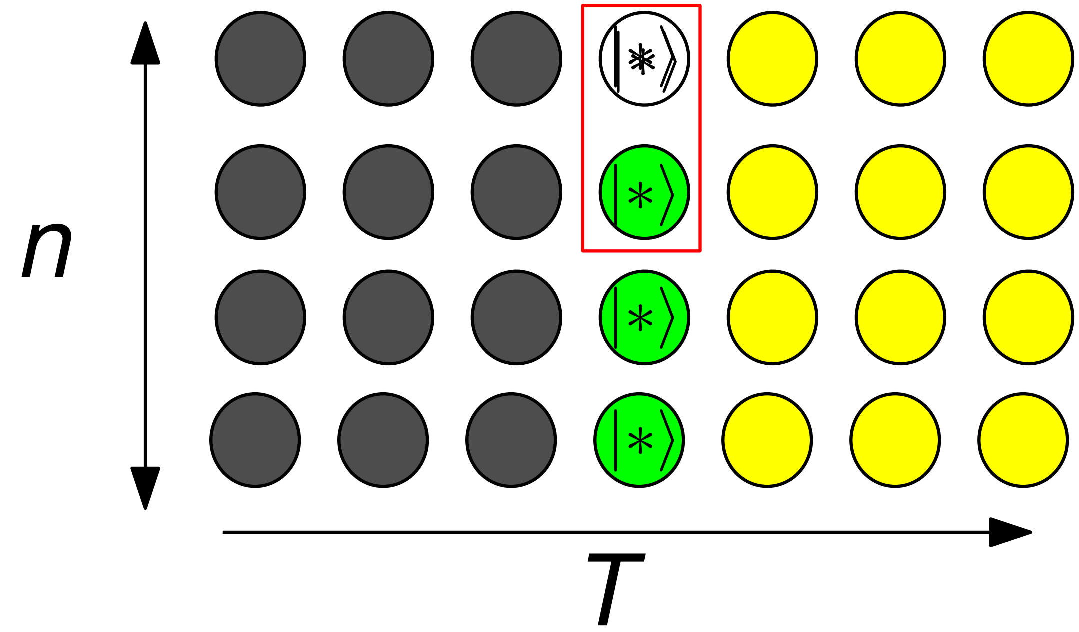
2D Local Hamiltonian Reduction, cont.

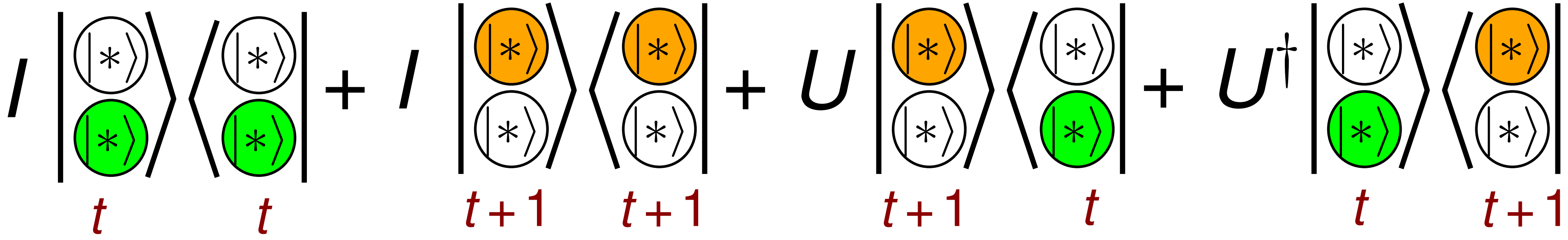
Advancing the clock and implementing gates:



2D Local Hamiltonian Reduction, cont.

Advancing the clock and implementing gates:

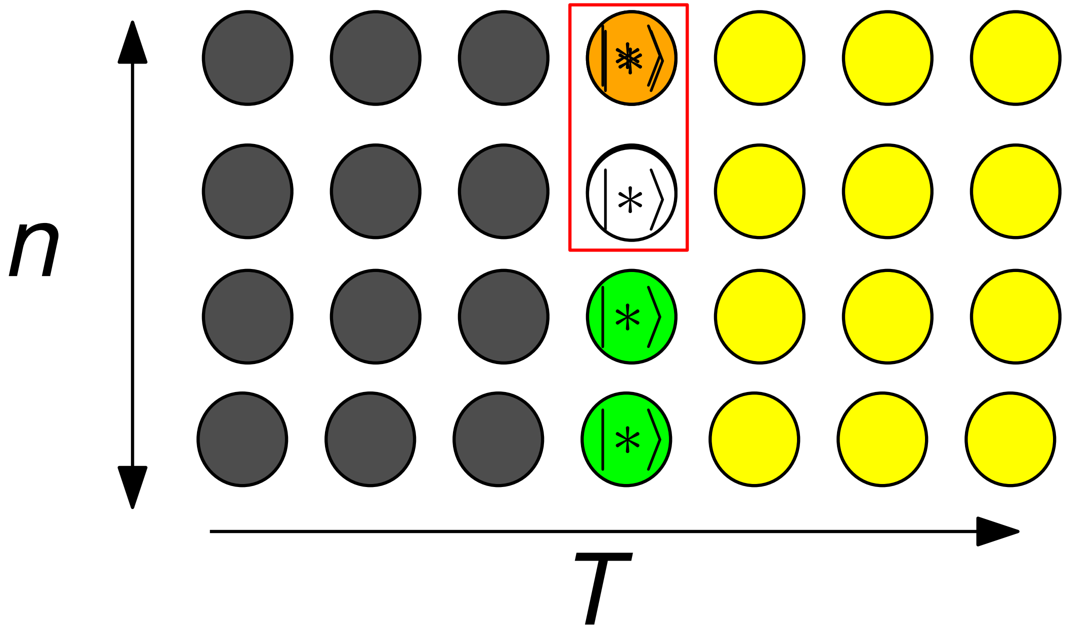


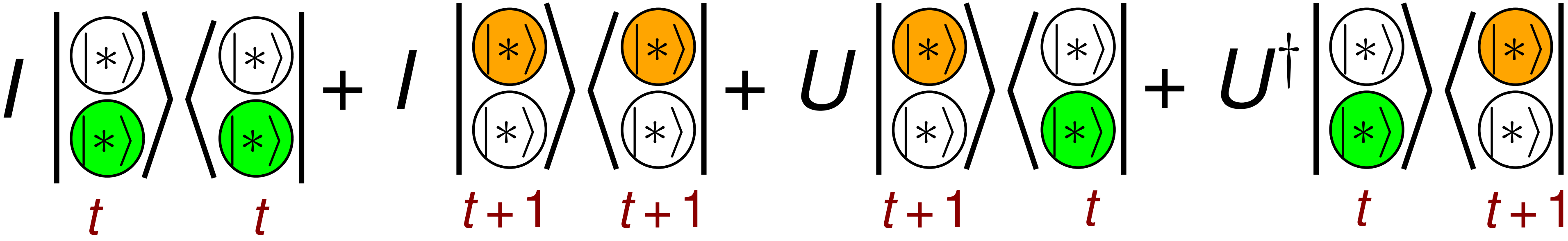


Applied to two particles in □

2D Local Hamiltonian Reduction, cont.

Advancing the clock and implementing gates:





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Clock Configuration Graph

Need to ensure at most one propagation term applied to each valid clock state.

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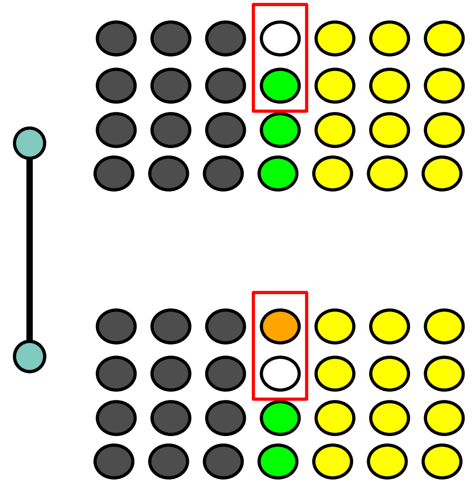
Vertices: Standard basis of clock states

Edge (x, y) if a propogation term converts x to y



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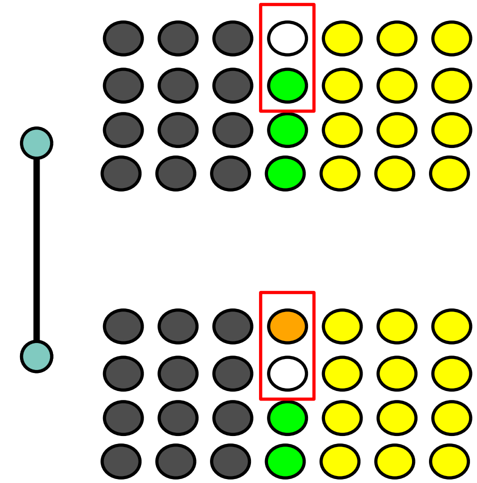

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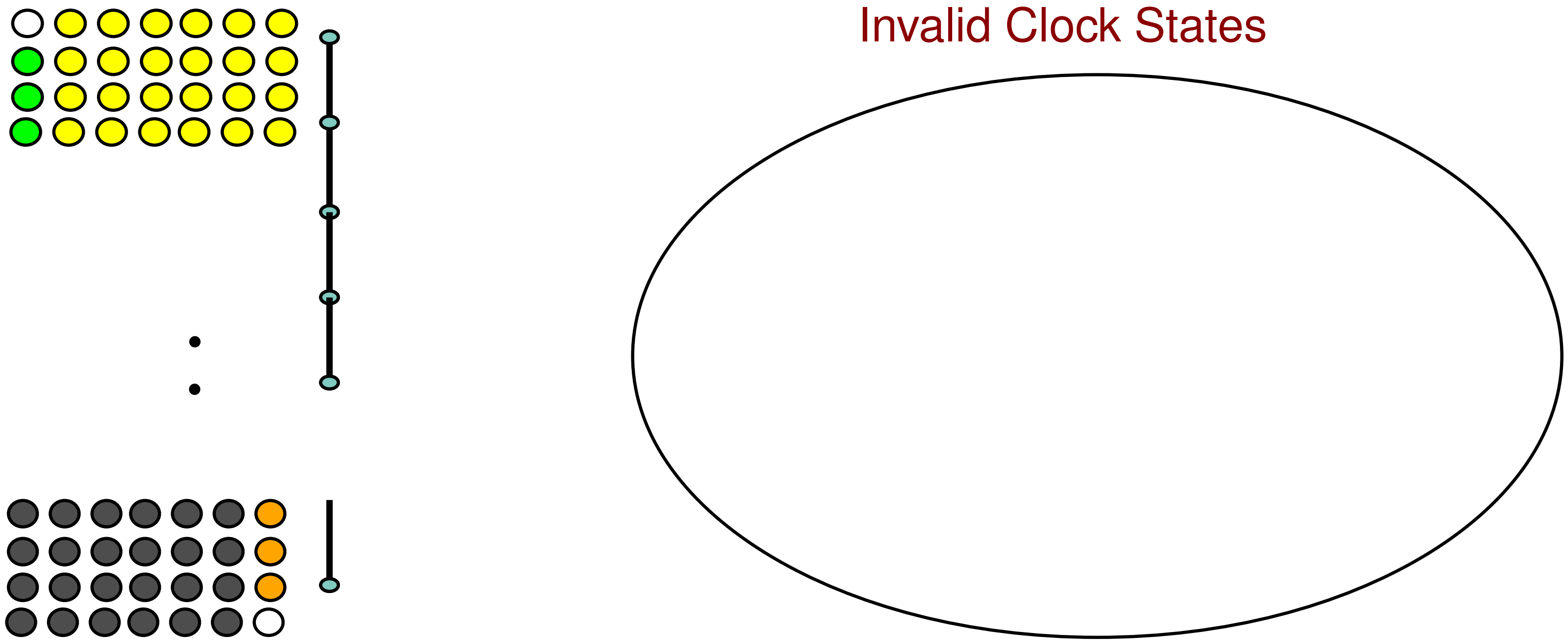
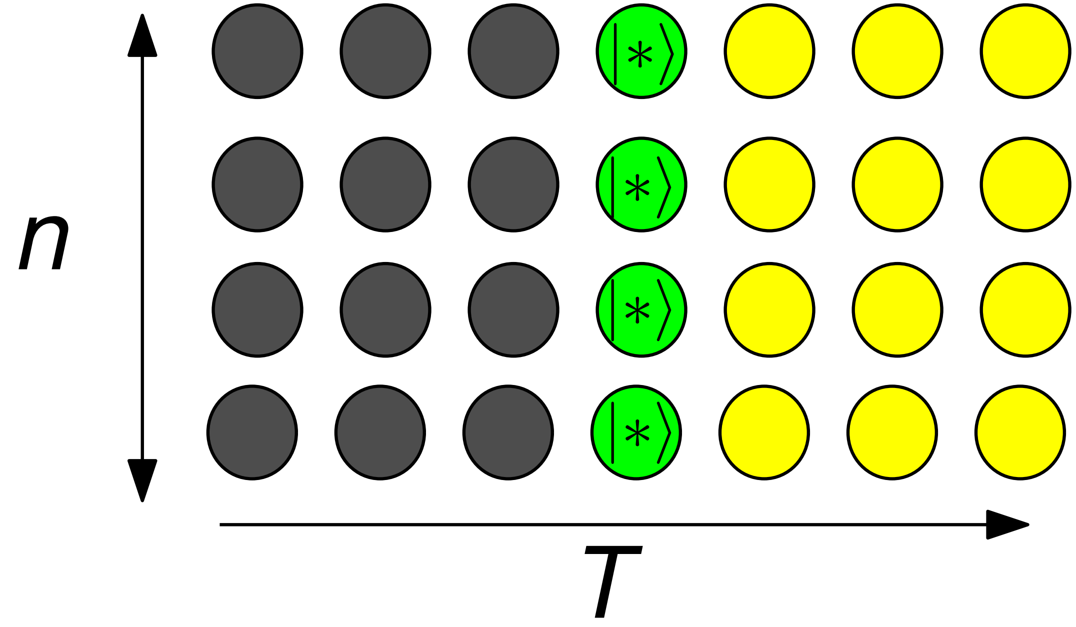


2D Local Hamiltonian Reduction



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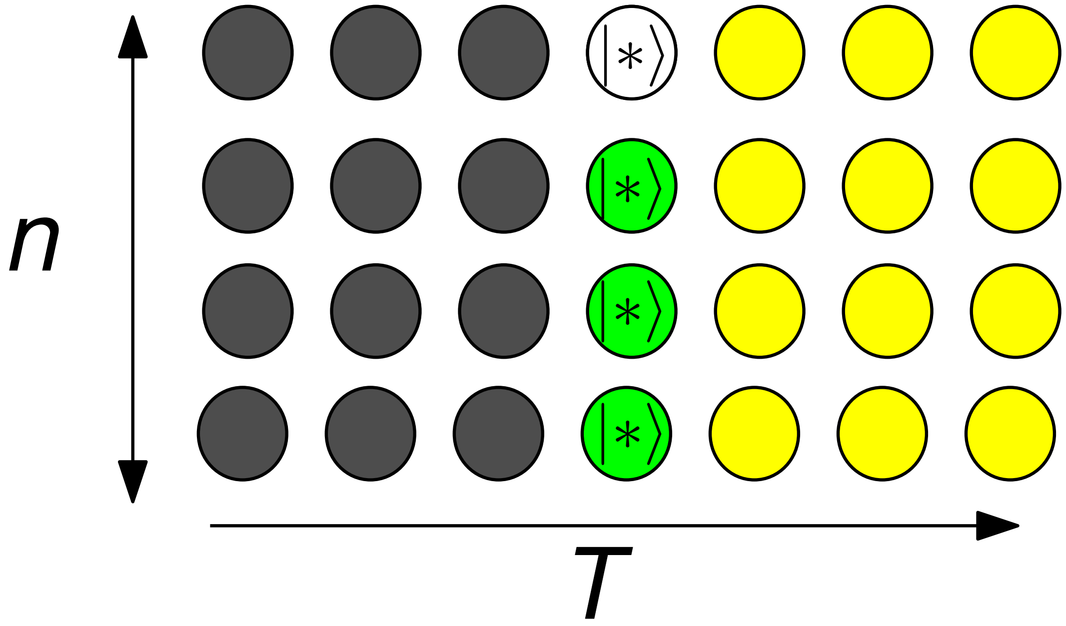

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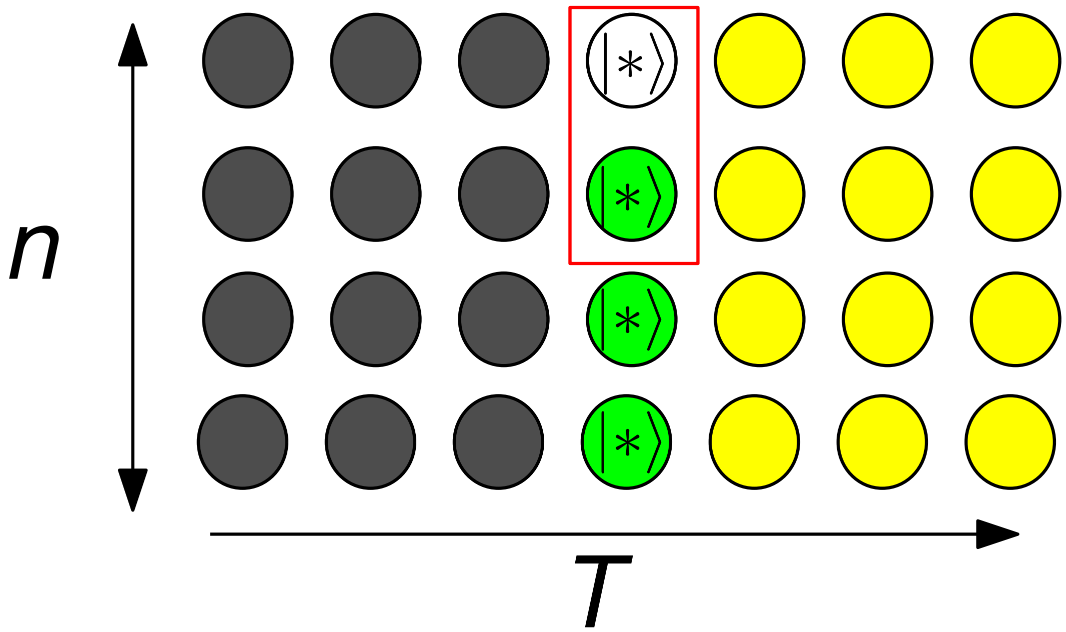

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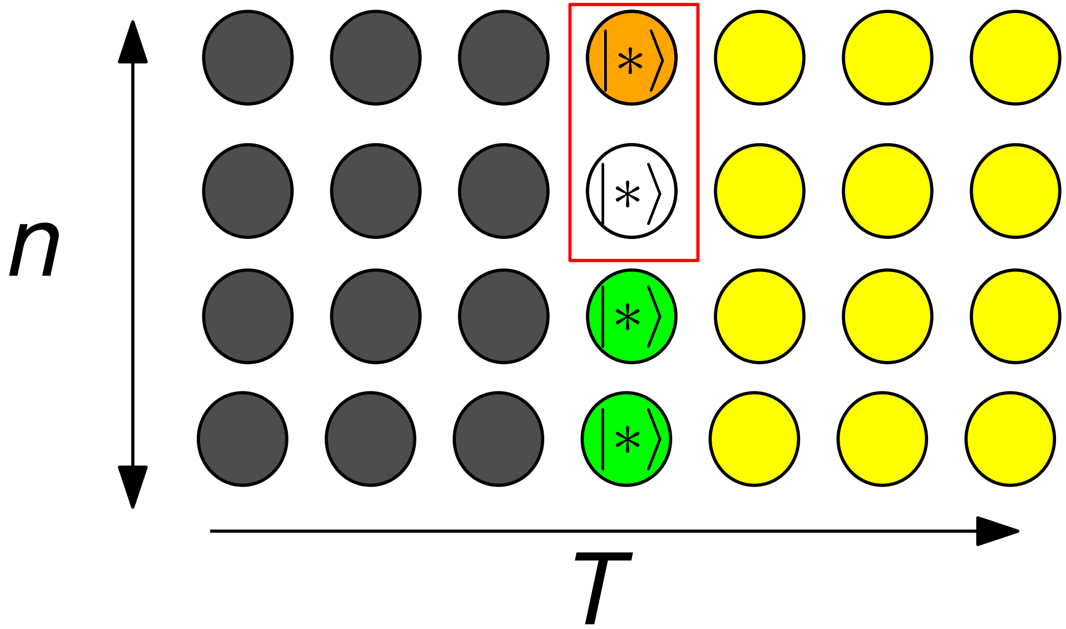

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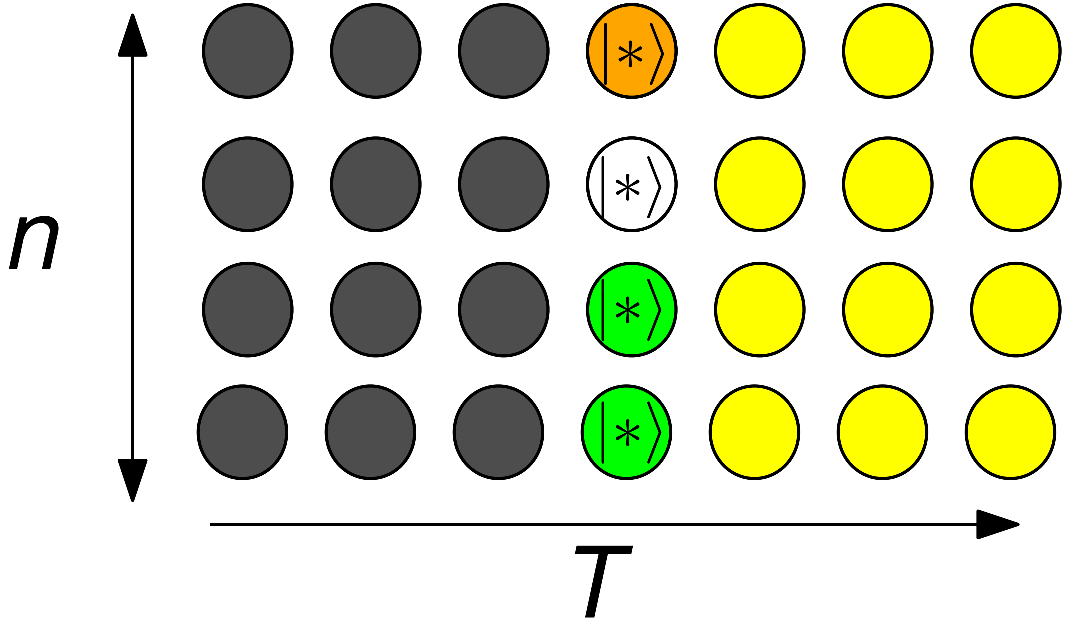

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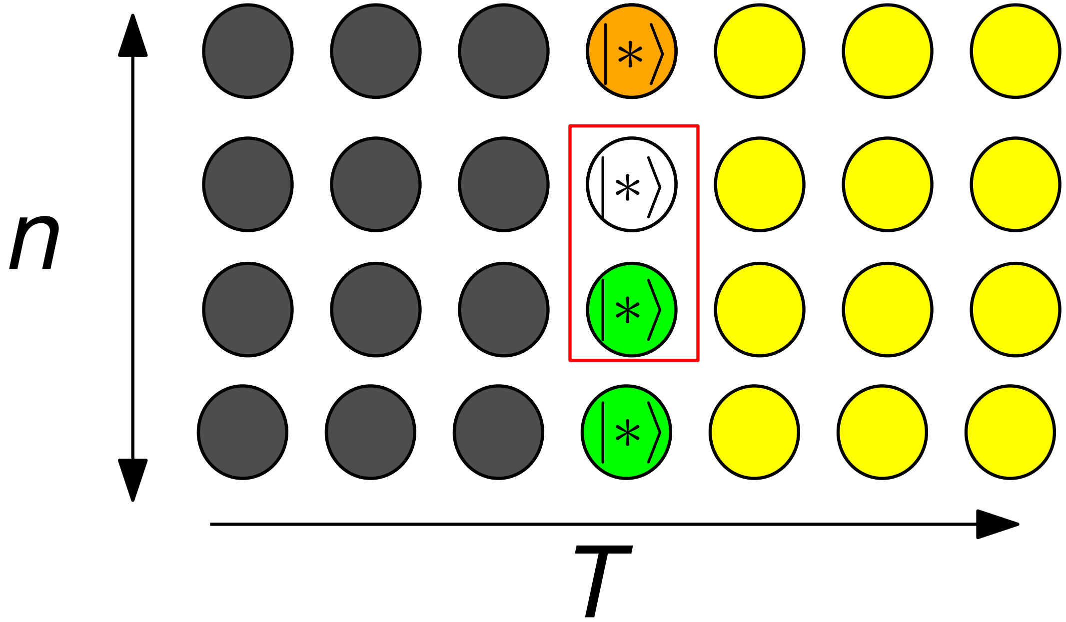

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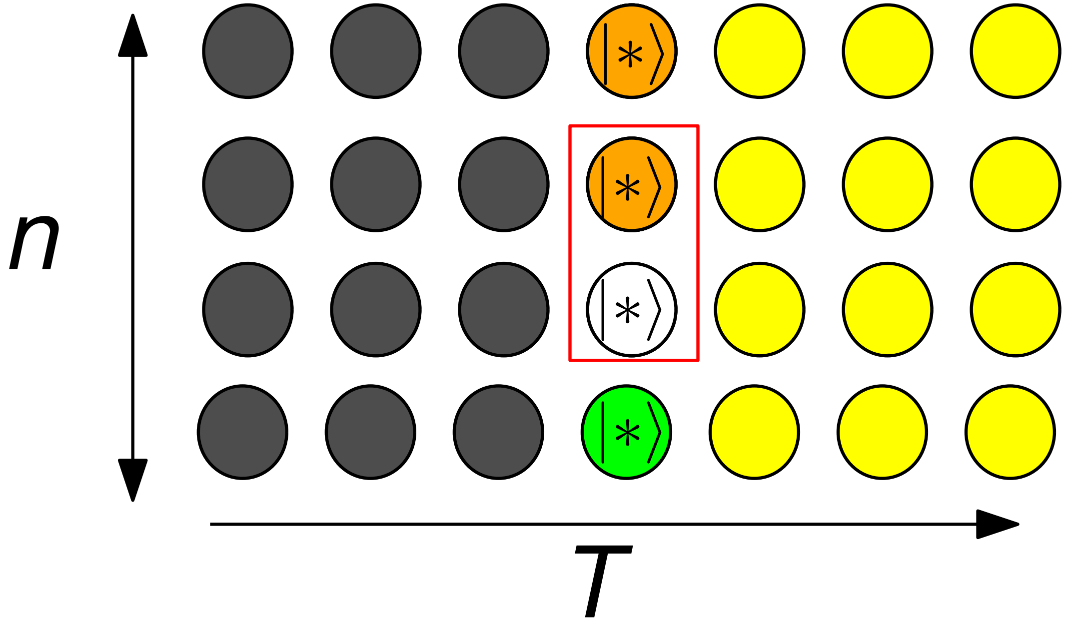

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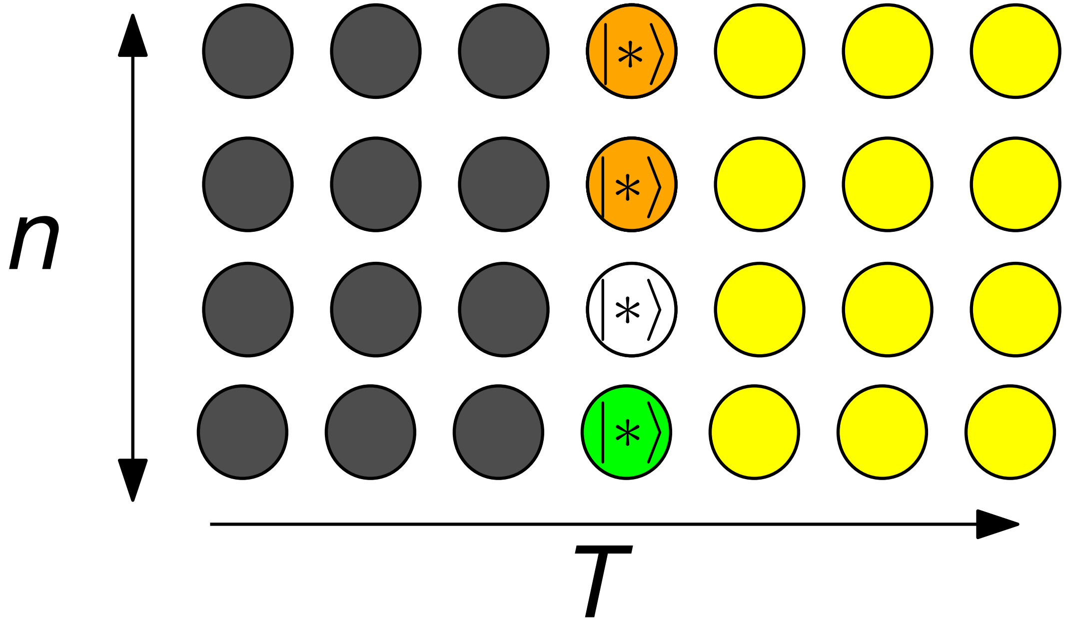

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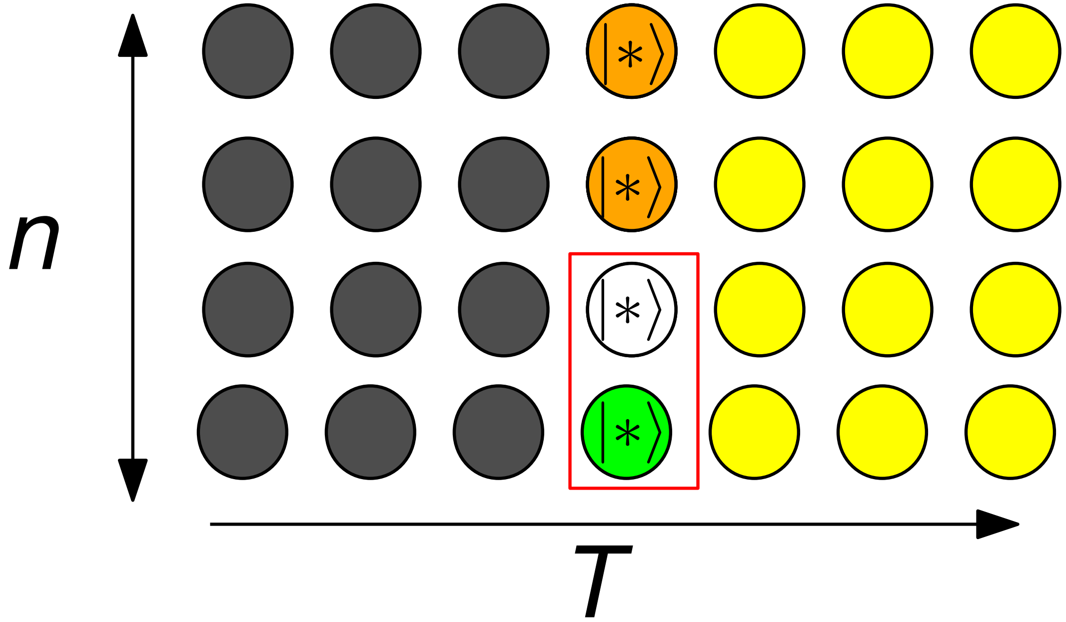

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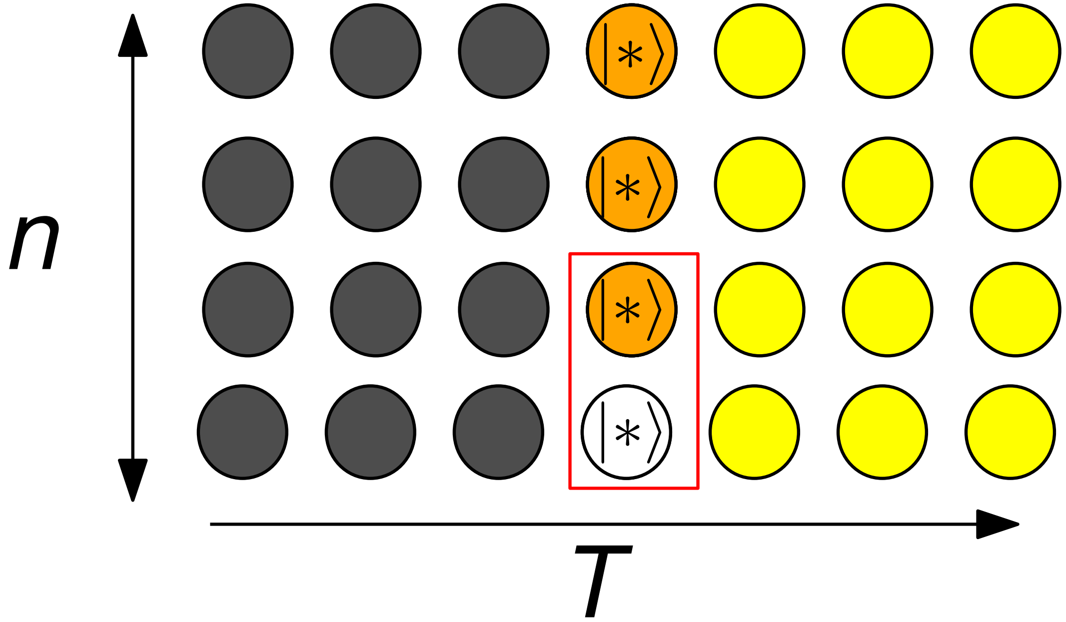

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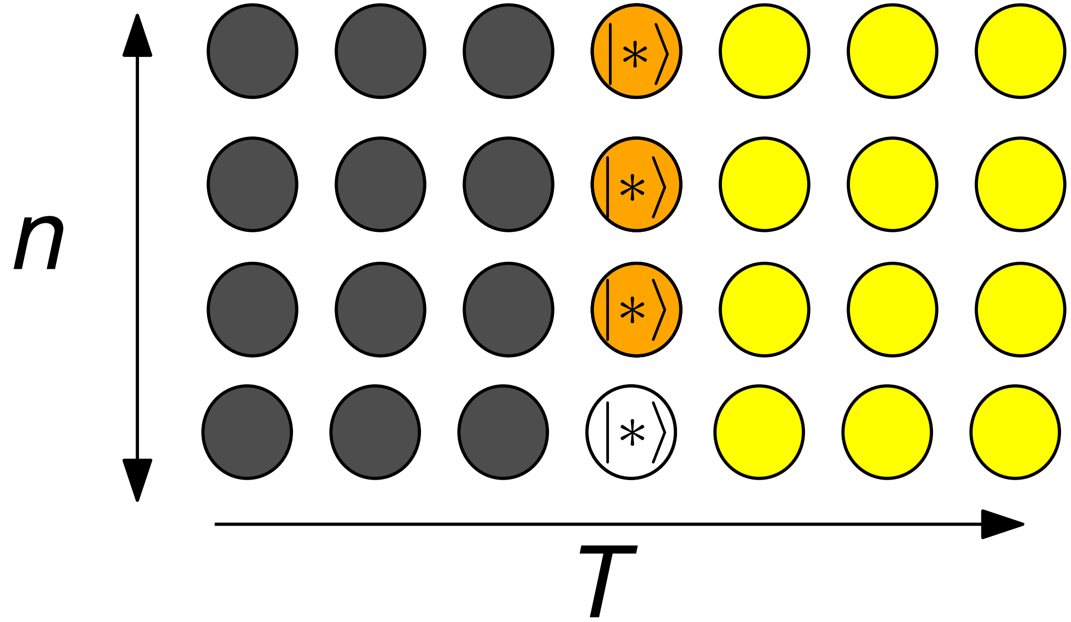

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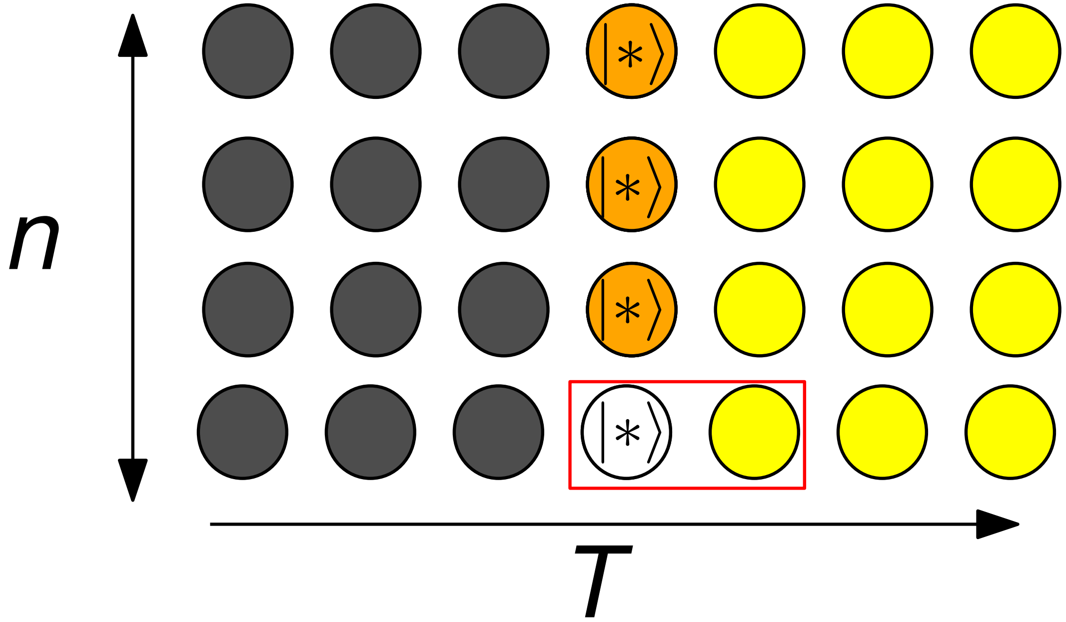

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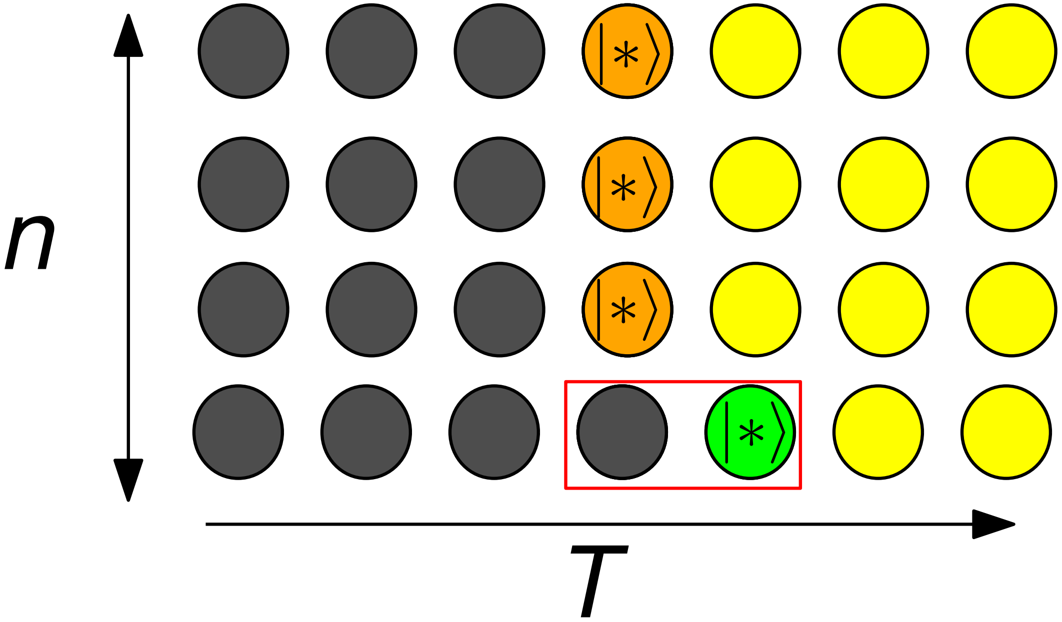

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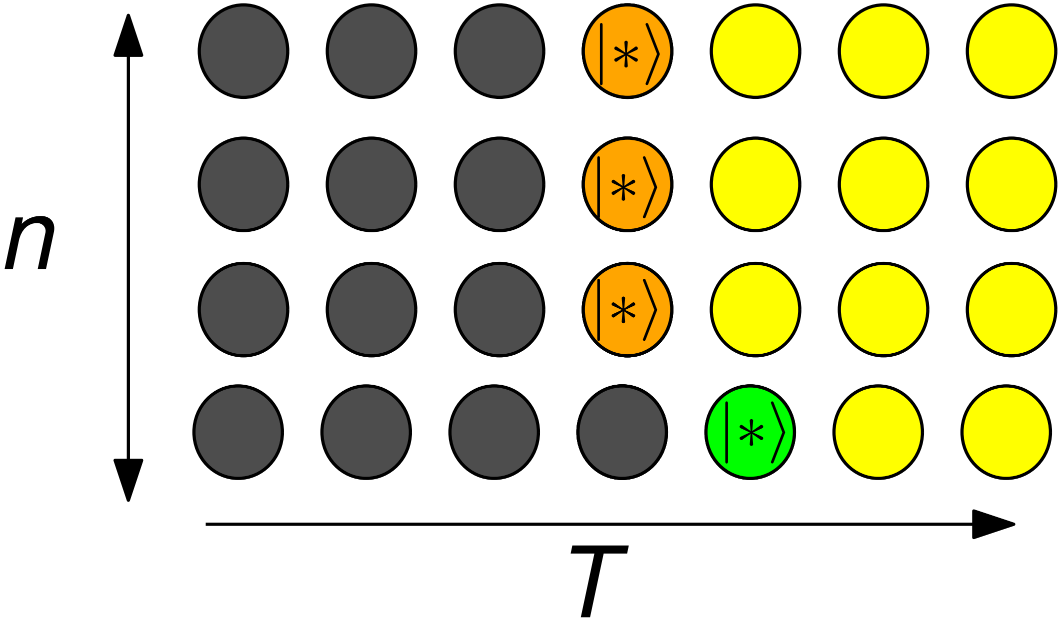

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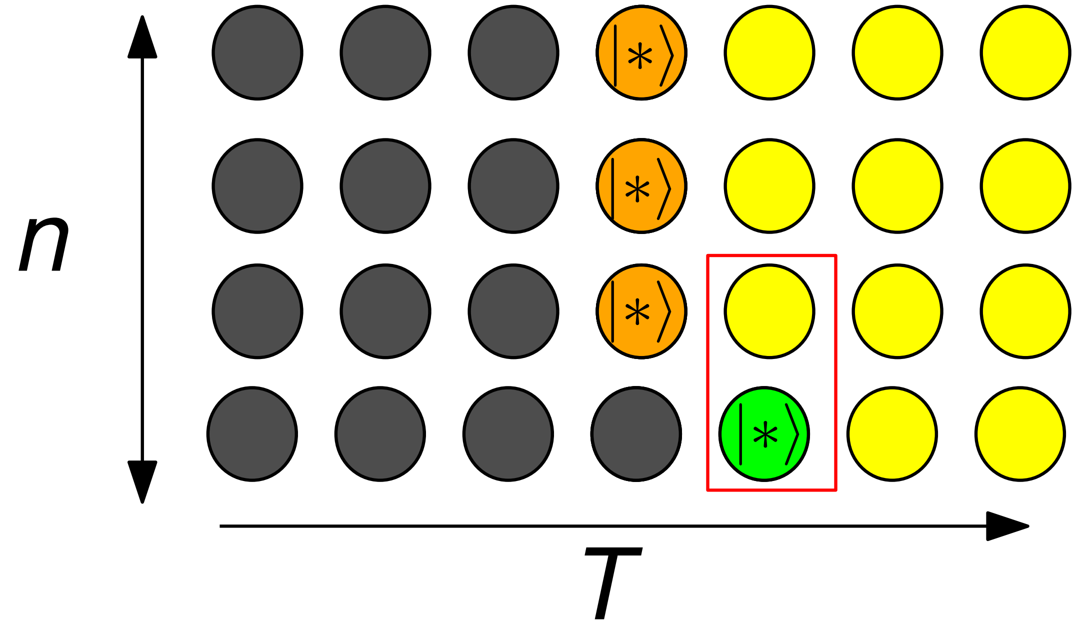

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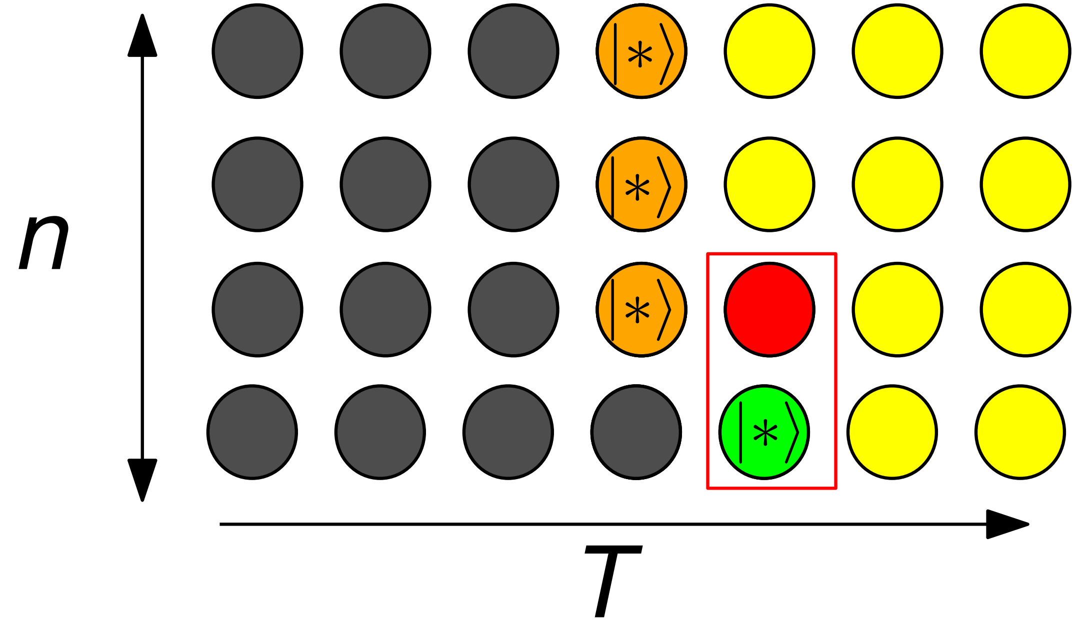

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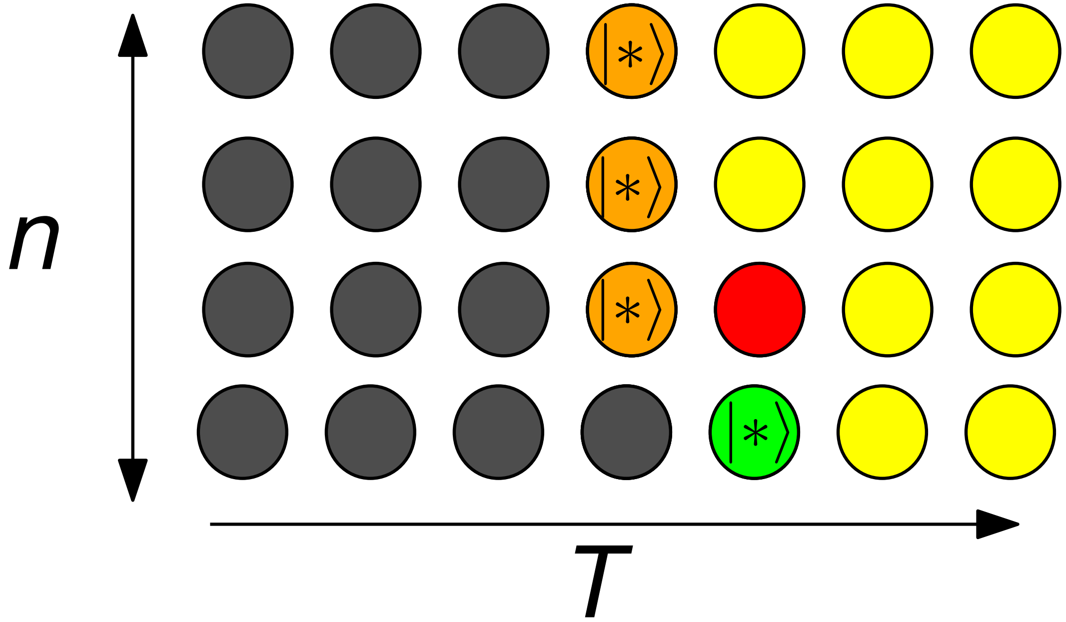

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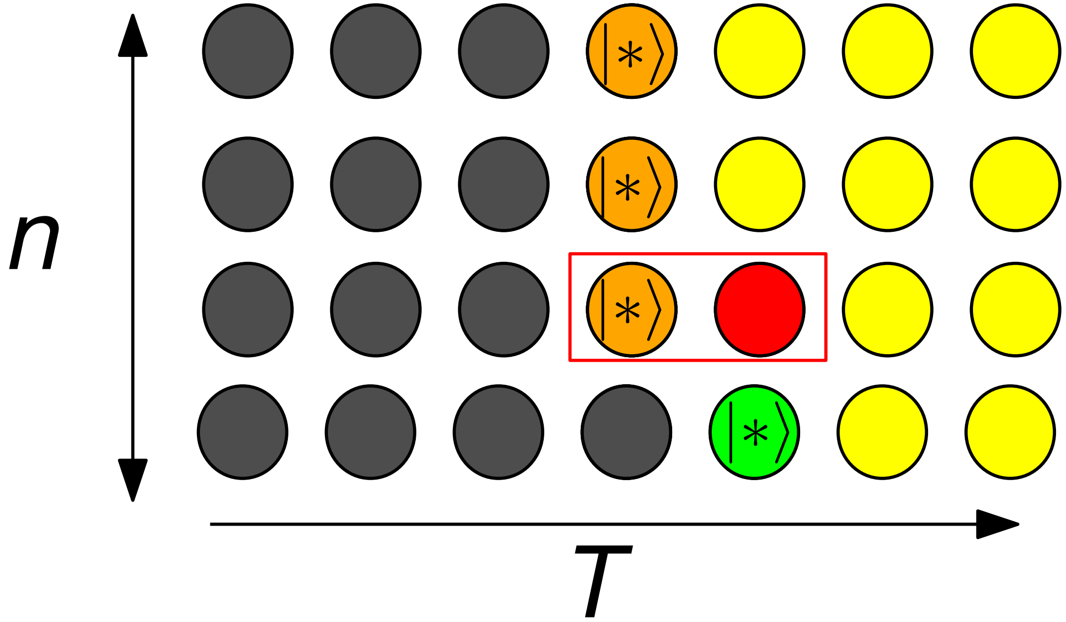

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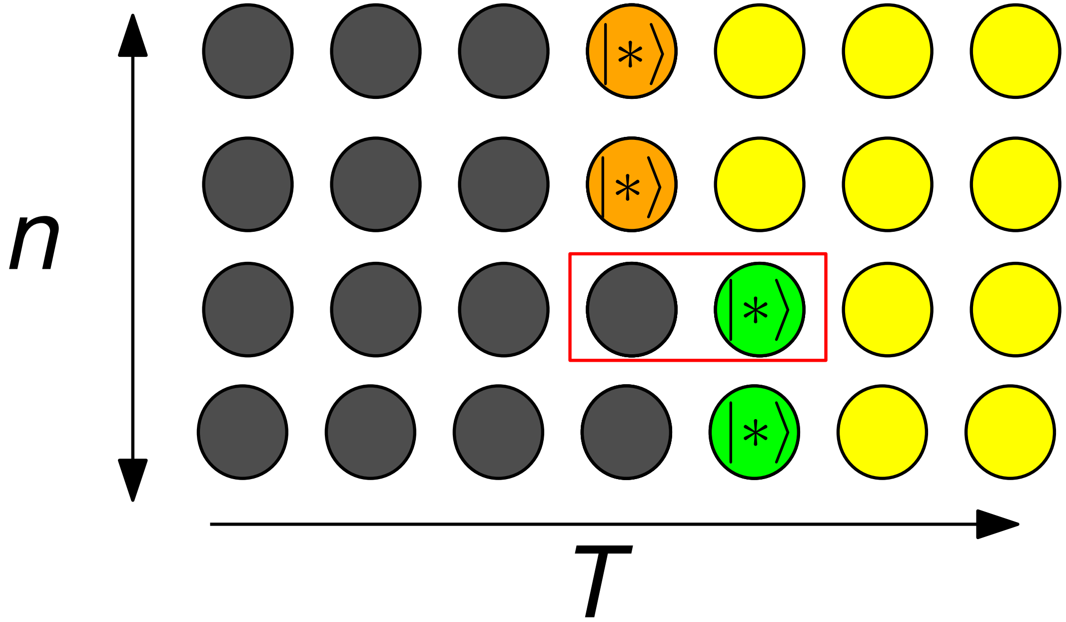

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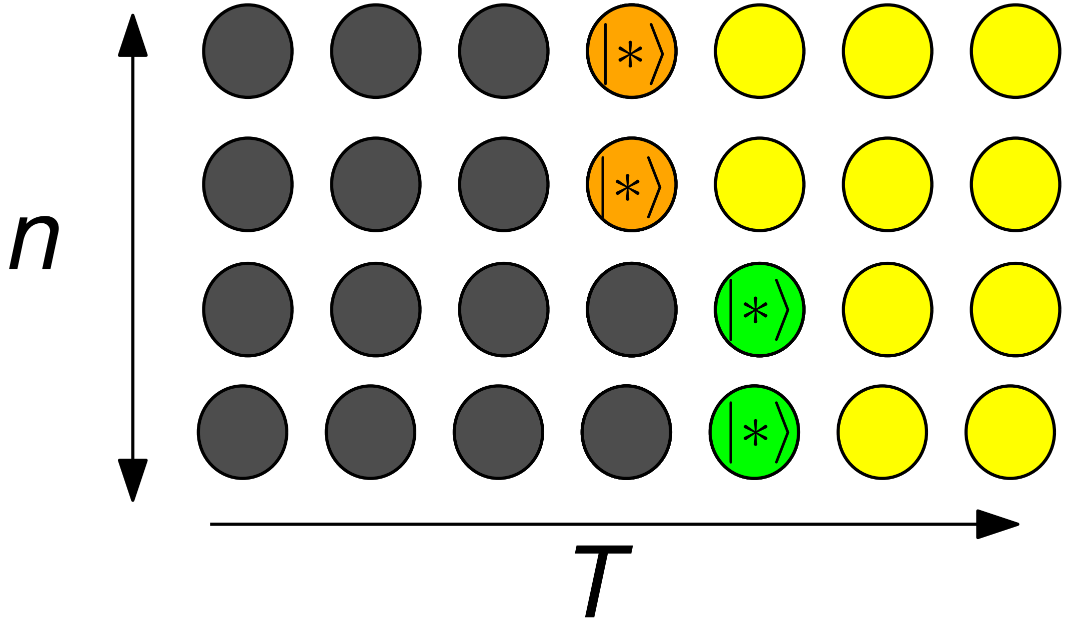

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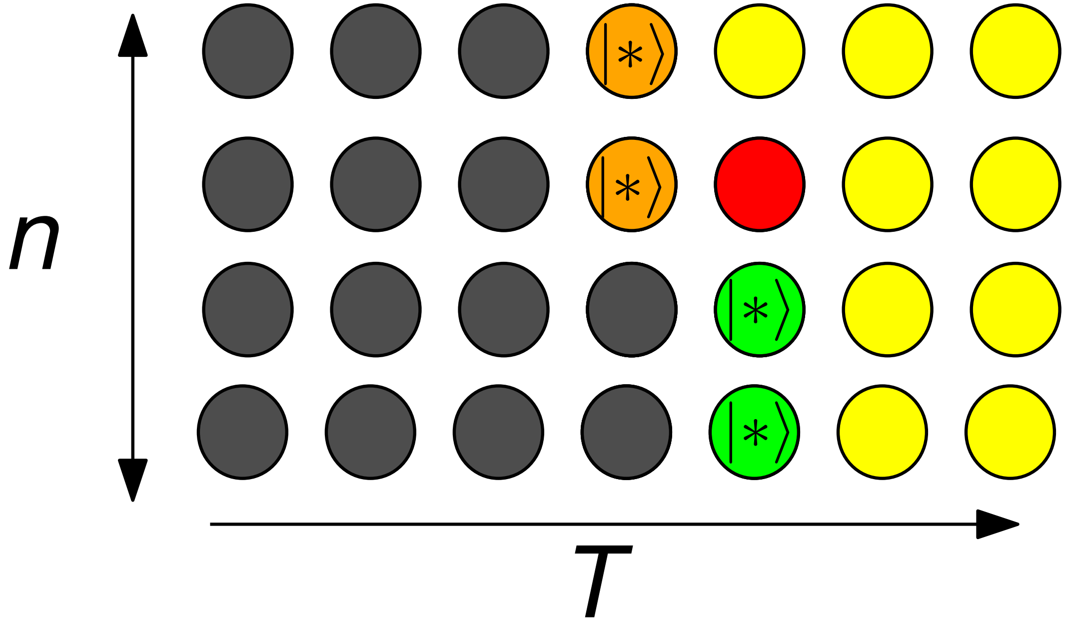

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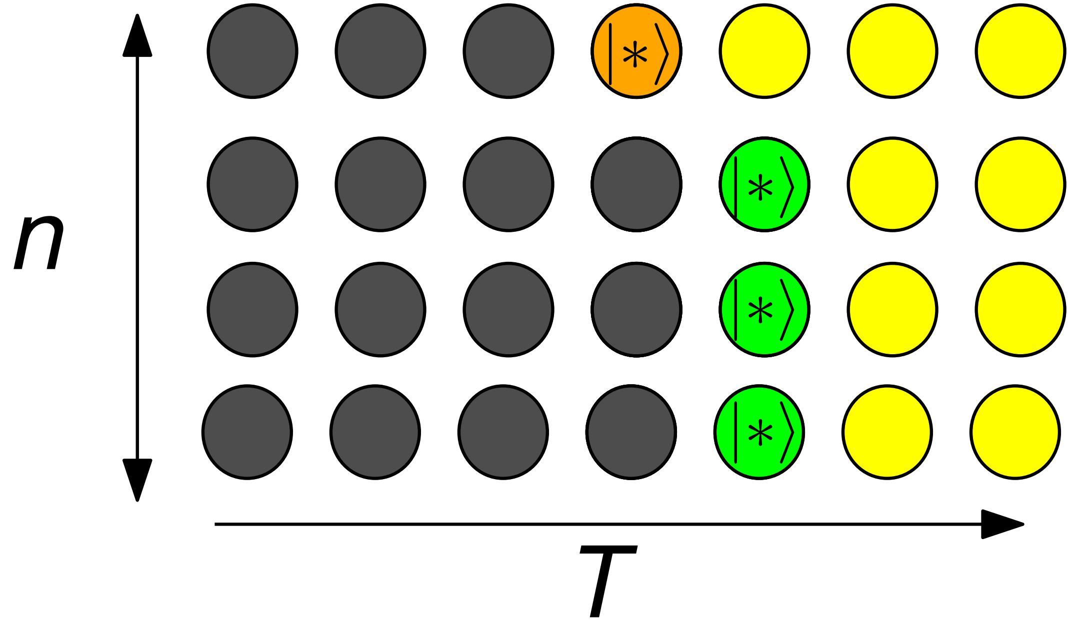

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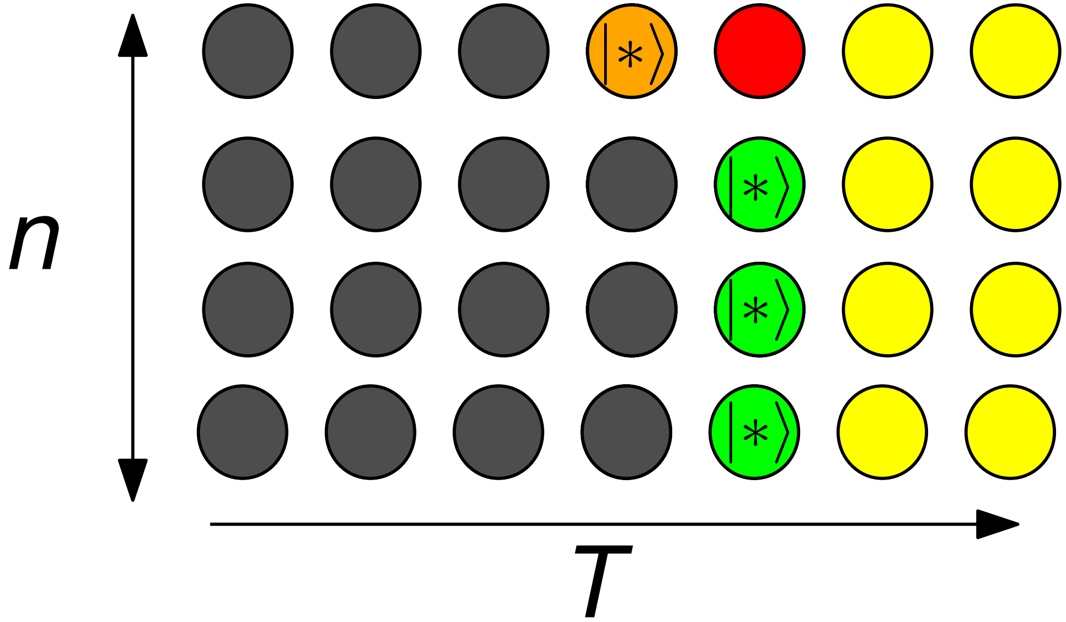

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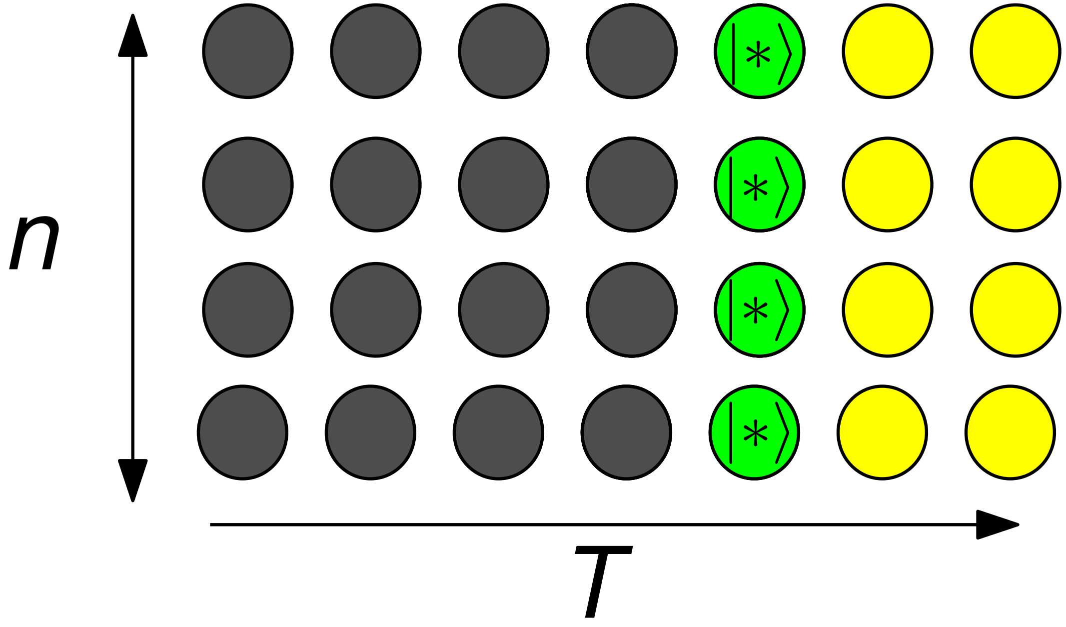
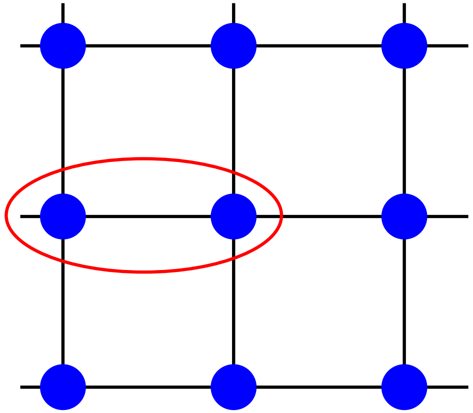


QMA-complete Problems

5-local 2-state Hamiltonian is QMA-Complete [Kitaev 1995]

2-dimensional 2-local 6-state Hamiltoanian is QMA-complete
[Aharonov, van Dam, Kempe, Landau, Lloyd, Regev 2004]



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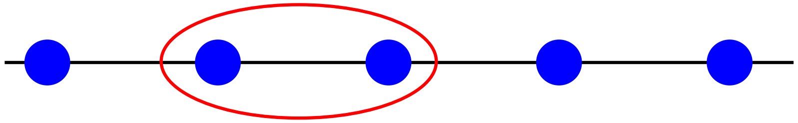
1-dimensional 13-state Hamiltonian is QMA-complete
[Aharonov, Gottesman, Irani, Kempe, 2009]



Improved to 8-state
[Hallgren, Nagaj, Narayanaswami 2013]

1-Dimensional Local Hamiltonian

Classical Methods:

DMRG (Density Matrix Renormalization Group) [White 1992]

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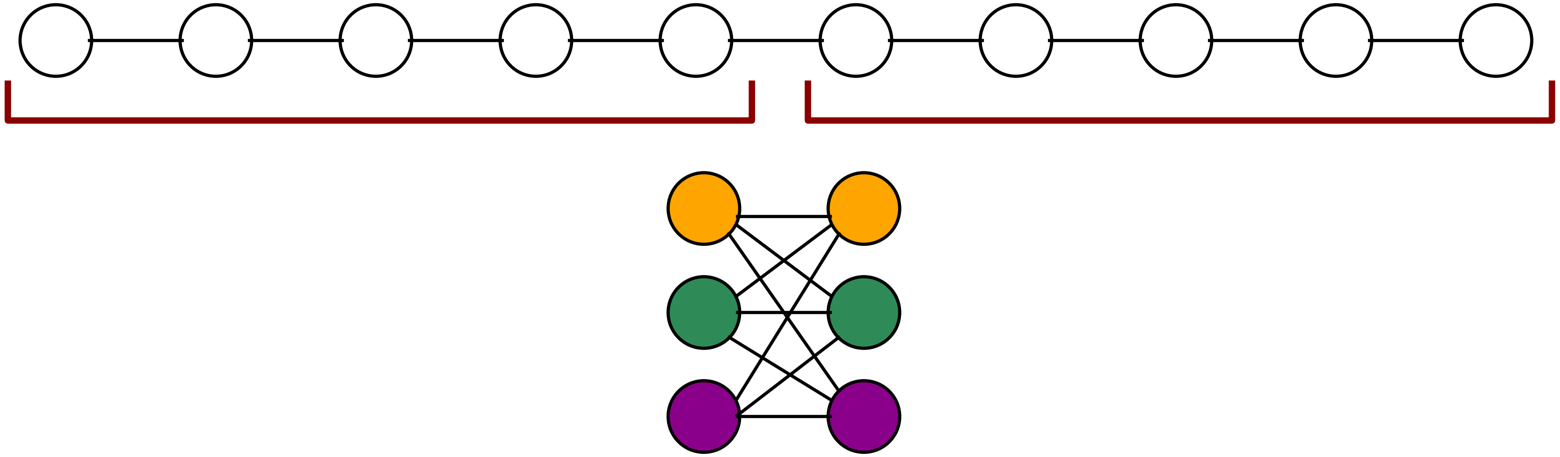
The Classical Analog:

1D MAX-2-SAT with d-state variables is in P:



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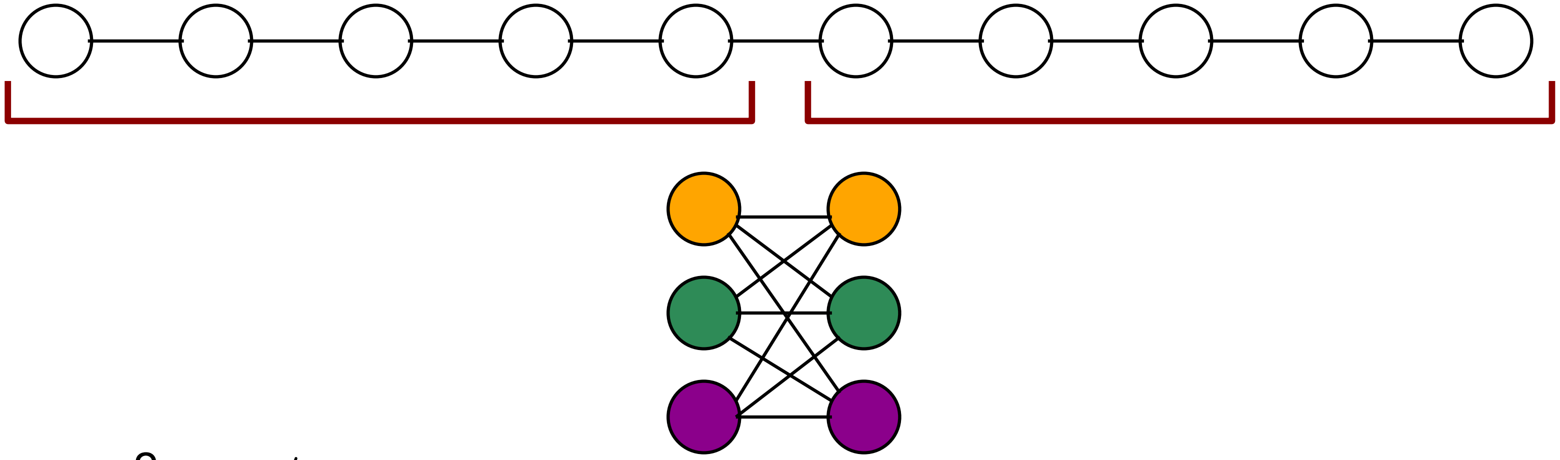

$$T(n) = 2d^2 T(n/2) + O(1)$$

$$\Rightarrow$$

$$T(n) = O(n^{\log(2d^2)})$$

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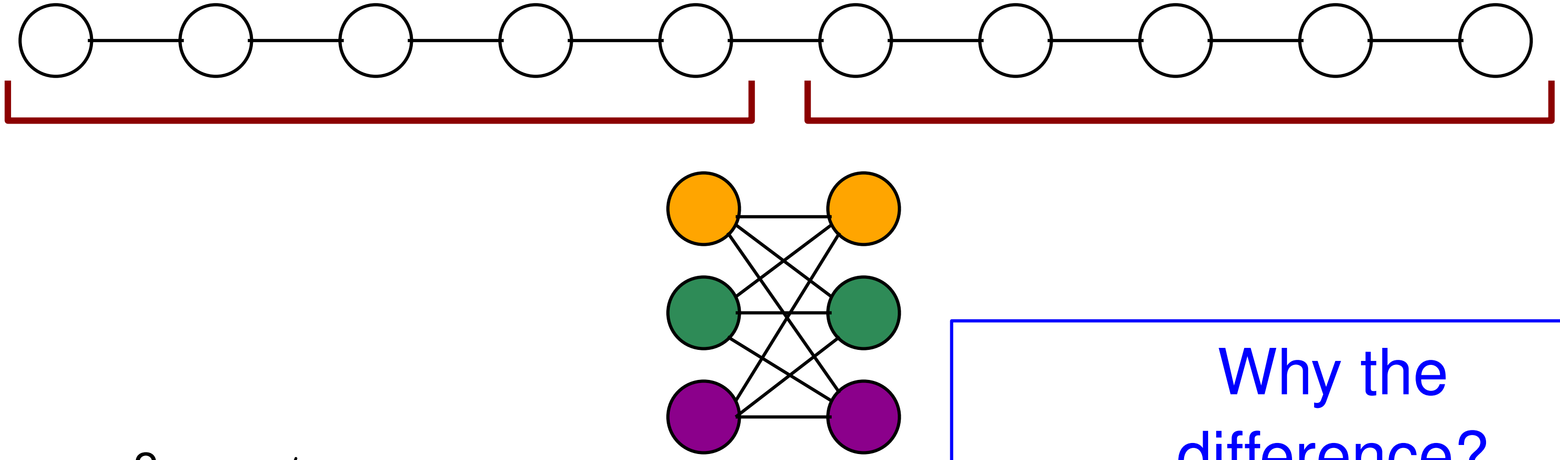

$$T(n) = 2d^2 T(n/2) + O(1)$$

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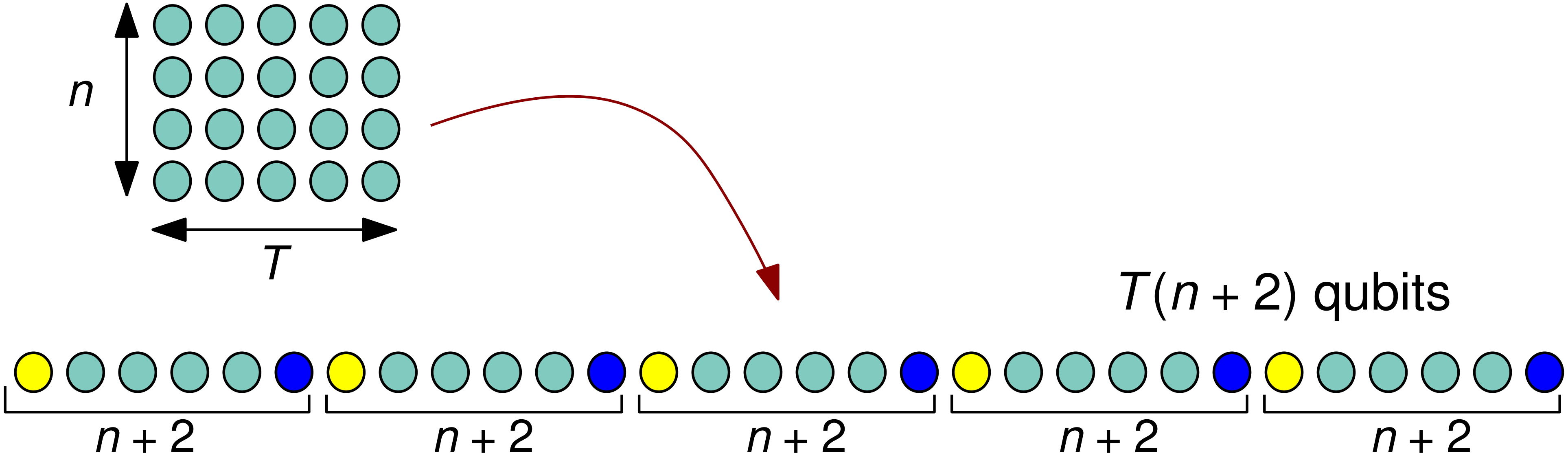
Why the difference?

$$\frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |\psi_t\rangle |1^{t+1}0^{T-t}\rangle$$

1D Local Hamiltonian



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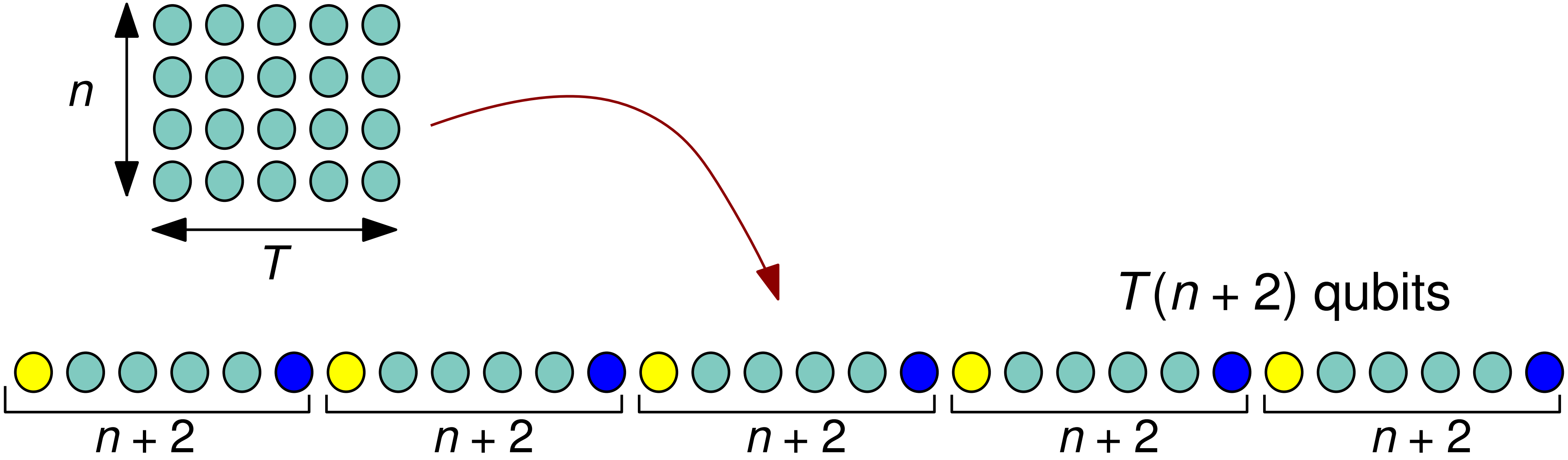

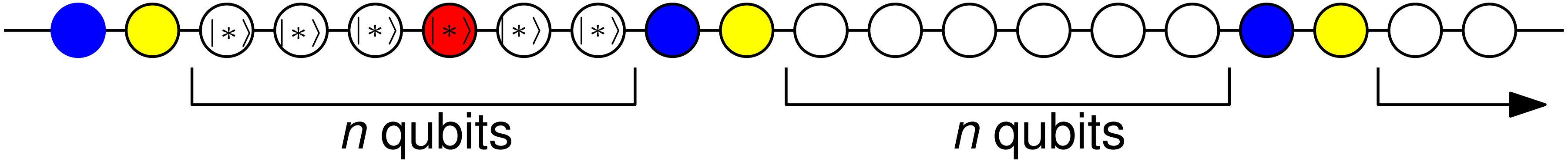

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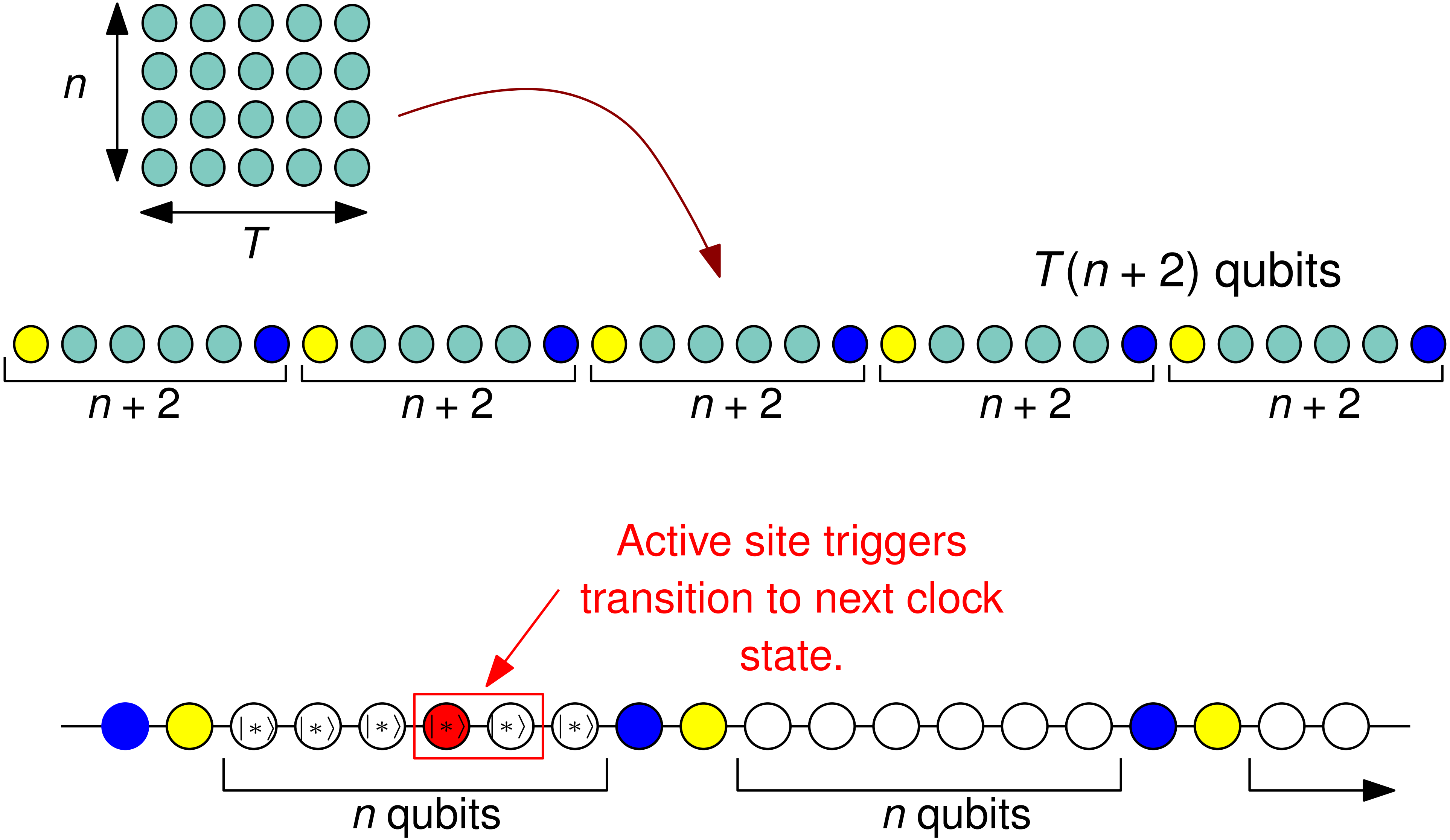
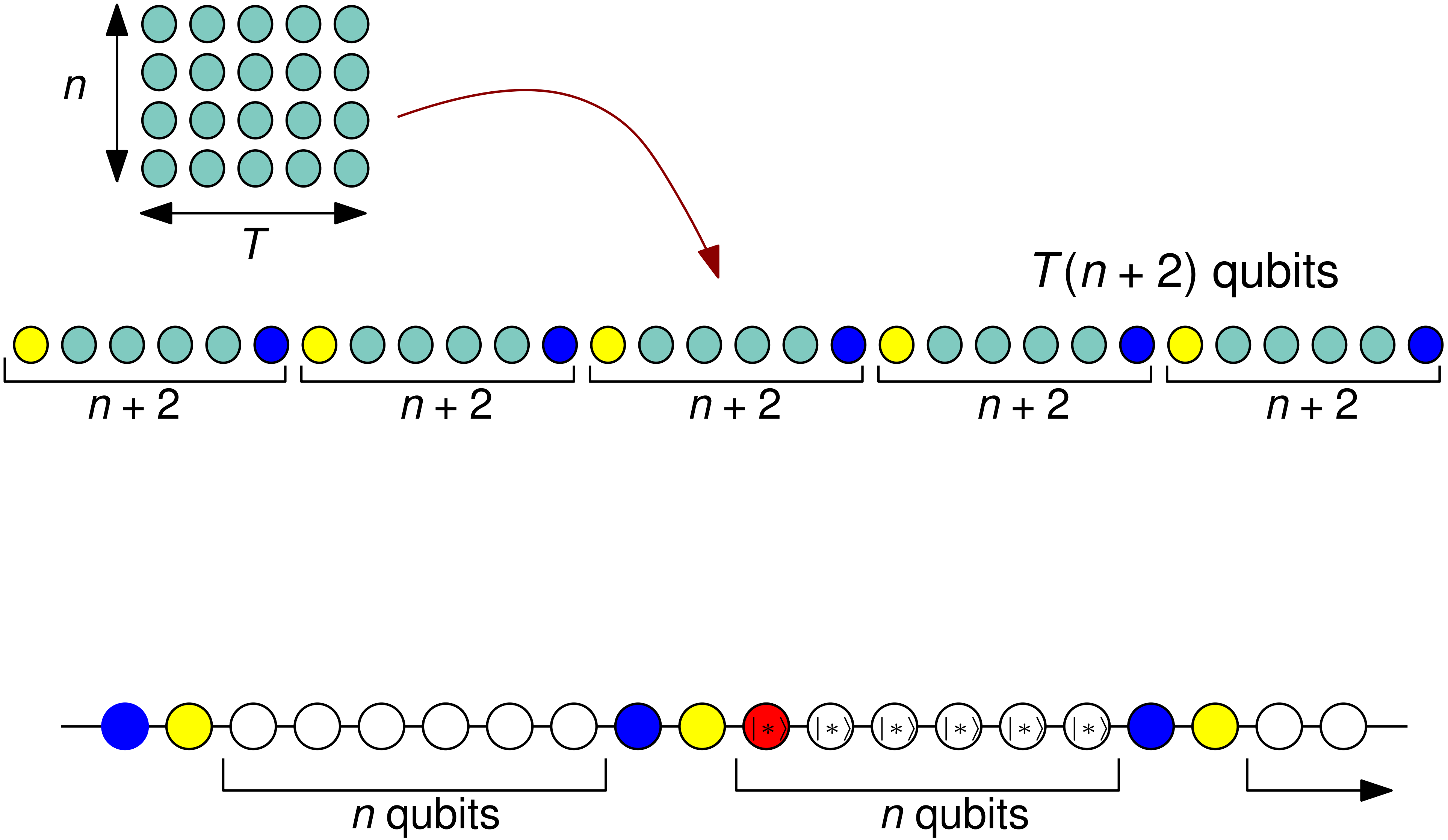


Active site triggers transition to next clock state.

1D Local Hamiltonian



Clairvoyance Lemma

1D clock: can't eliminate all invalid clock states with a local term

Configuration Graph:

Vertices: Standard basis of clock states

Edge (x, y) if a propagation term converts x to y

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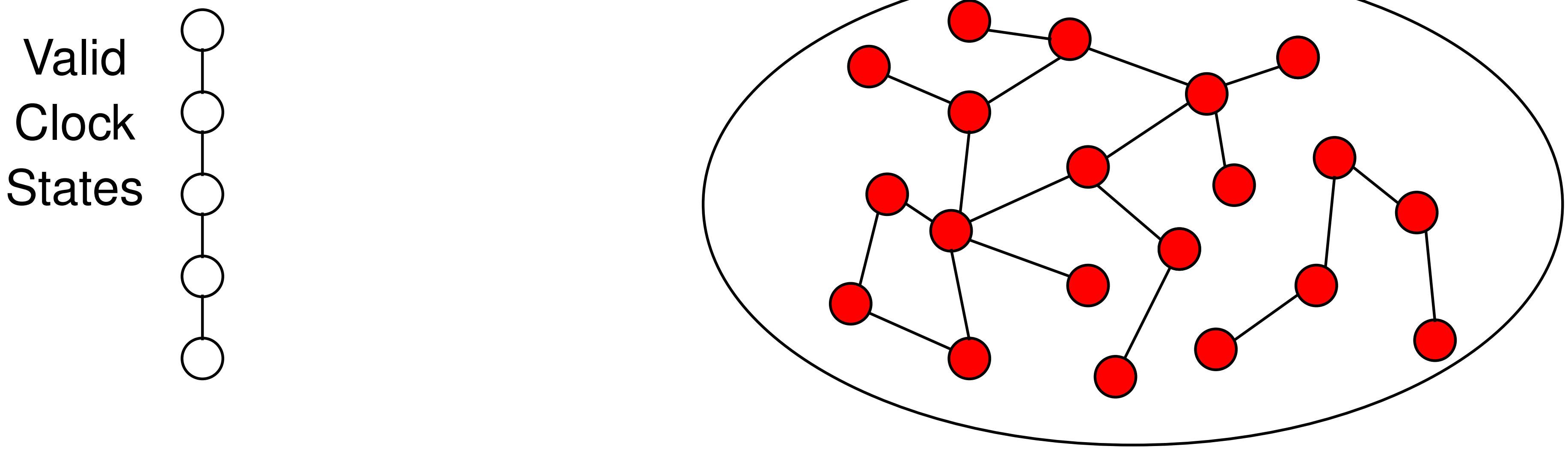

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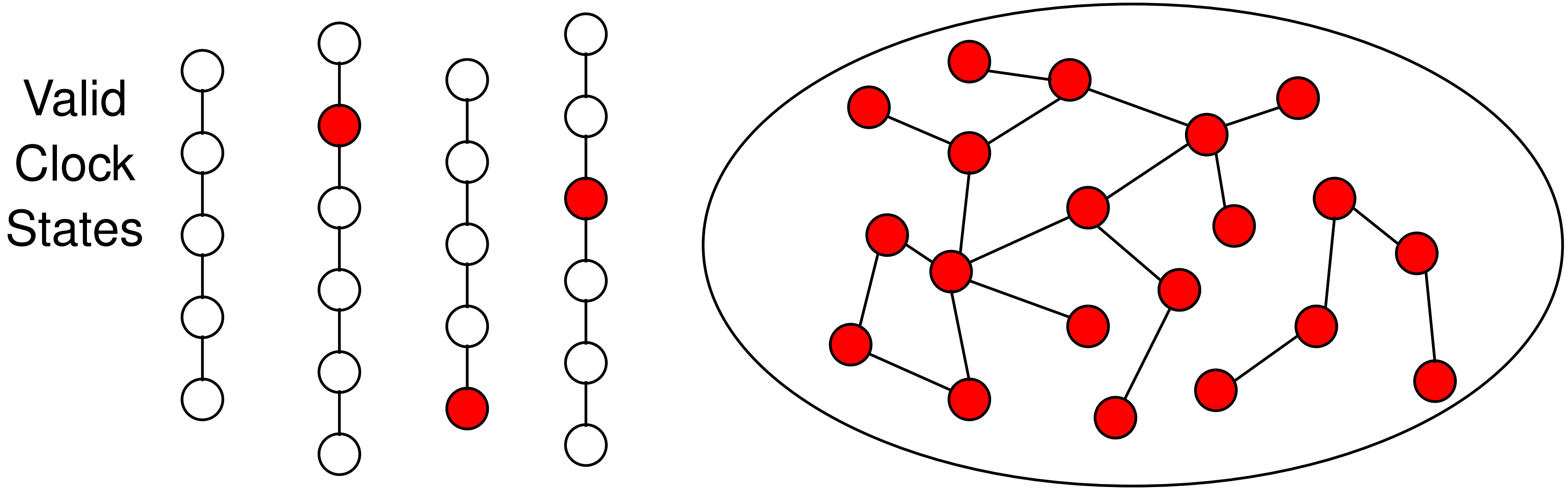

Clairvoyance Lemma

Need to lower bound lowest eigenvalue of:

$$\begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & & & & & \\ -\frac{1}{2} & 1 & -\frac{1}{2} & & & & \\ 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & & & \\ & & & \ddots & & & \\ & & & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ & & & & -\frac{1}{2} & 1 & -\frac{1}{2} \\ & & & & & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Clairvoyance Lemma

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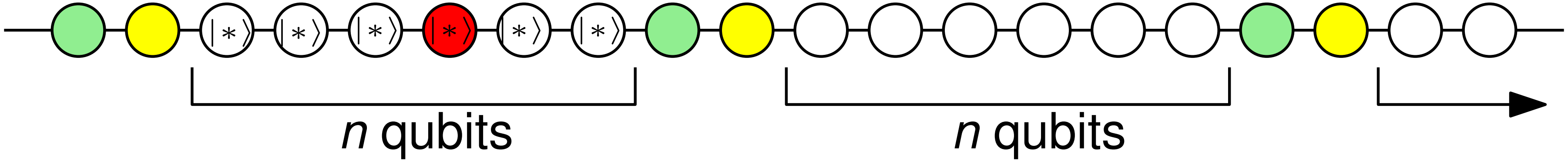
$$\begin{bmatrix} 0 &&&&& \\ & 0 &&&& \\ && 1 &&& \\ &&& \ddots && \\ &&&& 0 & \\ &&&&& 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} &&&&&& \\ -\frac{1}{2} & 1 & -\frac{1}{2} &&&&& \\ 0 & -\frac{1}{2} & 1 & -\frac{1}{2} &&&& \\ &&& \cdot &&&& \\ &&&& \cdot &&& \\ &&&&& \cdot && \\ &&&& -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ &&&&& -\frac{1}{2} & 1 & -\frac{1}{2} \\ &&&&&& -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$\Omega(1/K^3)$, where K is the length of the chain

Need to upper bound the length of the ”invalid” chains

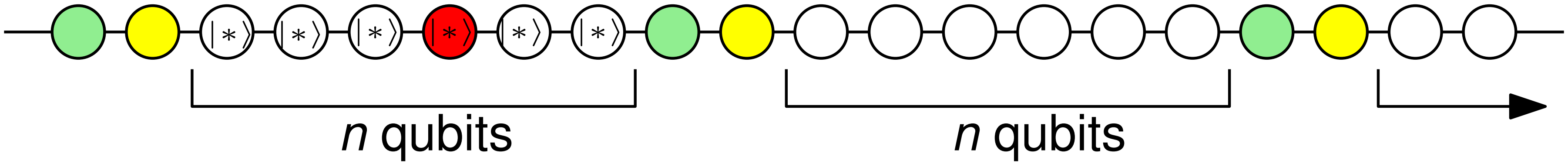
1D Local Hamiltonian



[AGIK]: 12 states per particle

[Narayanaswami, Hallgren]: 9 states per particle

1D Local Hamiltonian



[AGIK]: 12 states per particle

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Hamiltonian: sum of terms on each neighboring pair.

Terms are position-dependent. (Very non-physical!)

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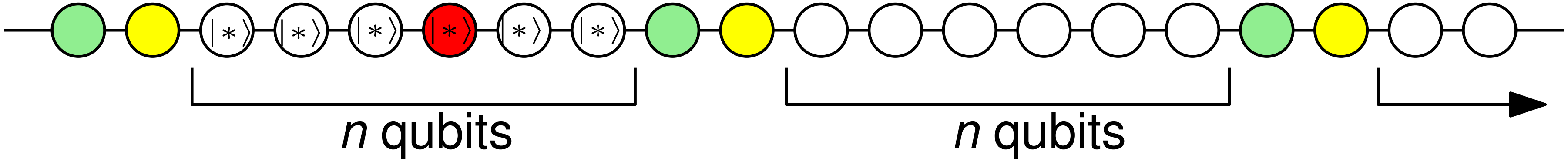

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Hamiltonian: sum of terms on each neighboring pair.

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In most systems of physical interest:

The Hamlitonian describing the energy of the system is the same for each pair of neighboring particles.

Translational Invariance

1) Input: (d, n, h_1, h_2)

Translational Invariance

Particle/Variable dimension

1) Input: (d, n, h_1, h_2)

Number of Particles/Variables

Translational Invariance

Constraints to be applied in each dimension

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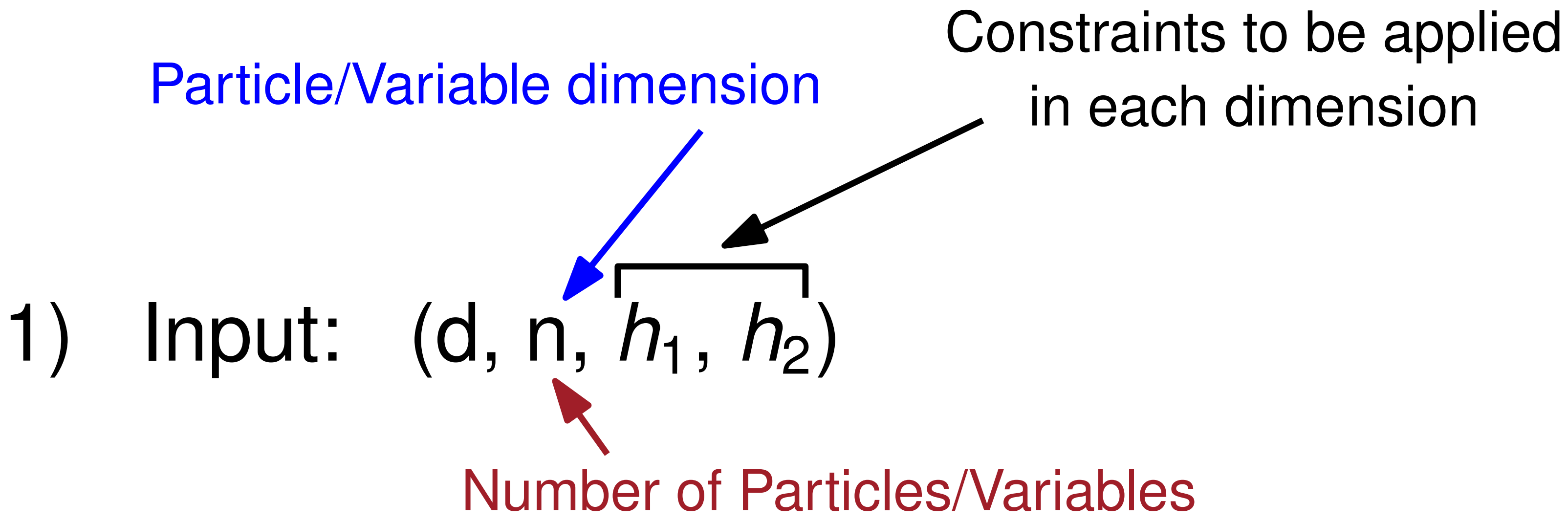
Translational Invariance



Particle/Variable dimension

Constraints to be applied in each dimension

1) Input: $(d, n, \overbrace{h_1, h_2})$

Number of Particles/Variables

2) Fixed Problem Parameters: (d, h_1, h_2)

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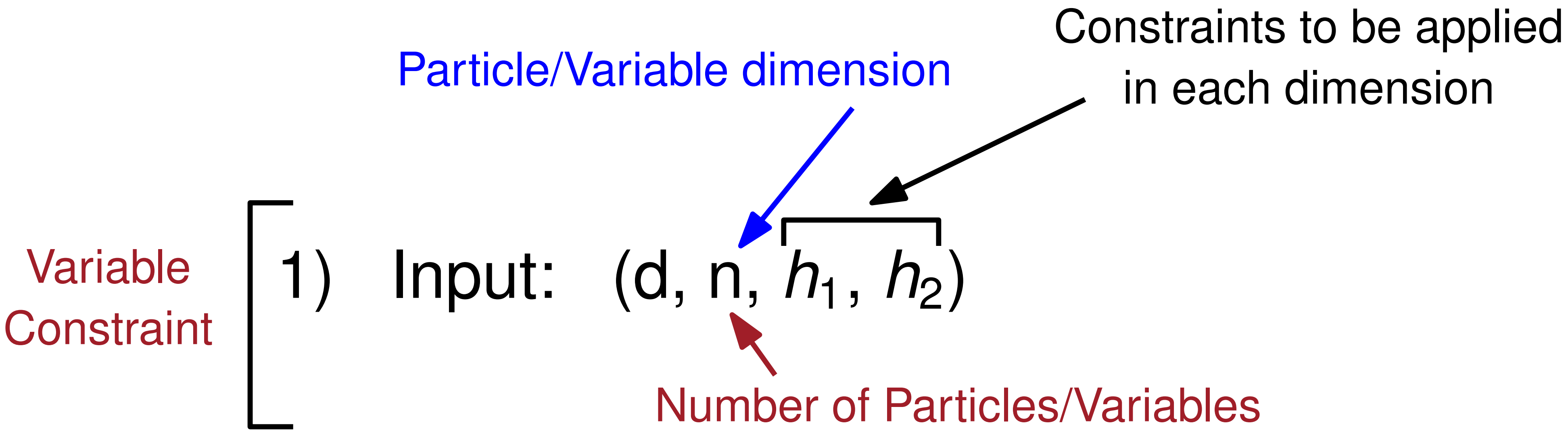
Translational Invariance



Constraints to be applied in each dimension

Particle/Variable dimension

Variable Constraint

1) Input: $(d, n, \overbrace{h_1, h_2})$

Number of Particles/Variables

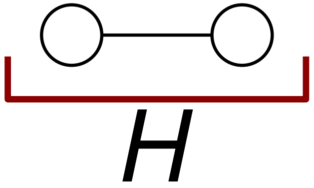
Fixed Constraint

2) Fixed Problem Parameters: (d, h_1, h_2)

Input: n

Translational Invariance

How hard is it to find ground states of translationally invariant quantum systems?



Problem parameters:

Hamiltonian term H on two d-dimensional particles

Fixed $2^d \times 2^d$ matrix.

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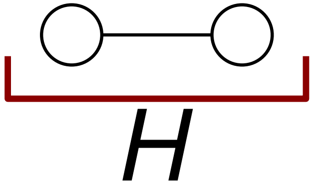

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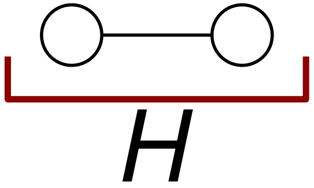

Hamiltonian term H on two d-dimensional particles

- Fixed $2^d \times 2^d$ matrix.
- Two polynomials $p(N)$ or $q(N)$.

Problem input: N (the number of particles in the system)

Output:

When H is applied to every pair of neighboring particles in a line of n particles, is the ground energy

$$\leq p(N) \quad \text{OR} \quad \geq p(N) + \frac{1}{q(N)}?$$

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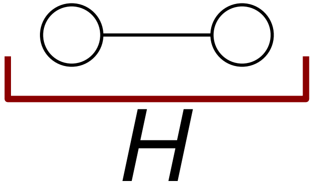

Hamiltonian term H on two d-dimensional particles

Fixed $2^d \times 2^d$ matrix.

Two polynomials $p(N)$ or $q(N)$.

Problem input: N (the number of particles in the system)

log N bits

(Note the size of the input is now logarithmic in the size of the system)

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Translationally Invariant Local Hamiltonian

1-Dimensional Translationally Invariant Local Hamiltonian is QMA_{EXP}-complete. [Gottesman, Irani, 2010]

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QMA

$L \in$ QMA if there is a
poly-sized uniform
quantum circuit family $\{C_n\}$:

If $x \in L \Rightarrow \exists |\phi\rangle$
Prob$[C_n(x, |\phi\rangle) = 1] \geq 2/3$.

If $x \notin L \Rightarrow \forall |\phi\rangle$
Prob$[C_n(x, |\phi\rangle) = 1] \leq 1/3$.

$|\phi\rangle$ has poly(n) qubits.

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EXP-time quantum
Turing Machine V

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Instance $x \Rightarrow N$ size of the system

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$$\boxed{\begin{array}{c}\exists|\psi\rangle \text{ such that prob} \\ V(x,|\psi\rangle) \text{ accepts} \geq 2/3\end{array}} \Rightarrow \boxed{\begin{array}{c}H \text{ on } N\text{-particle chain has} \\ \text{ground energy} \leq p(N)\end{array}}$$

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Translationally Invariant Local Hamiltonian

Ground State of *H* is ”computation state” encoding a *process*:

1) ”Count” the number of particles and write the number in binary on the tape.
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Contents of the tape are a binary counter.
Start with 0 and continually increment the counter.

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$|x| \approx \log N$

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M_{BC} can be made quantum. [Bernstein-Vazirani]

Translationally Invariant Local Hamiltonian

Ground State of H is ”computation state” encoding a *process*:

1) Simulate M_{BC} for N steps.

2) Simulate V for N steps using output of M_{BC} as input to V.

where N is the length of the chain.

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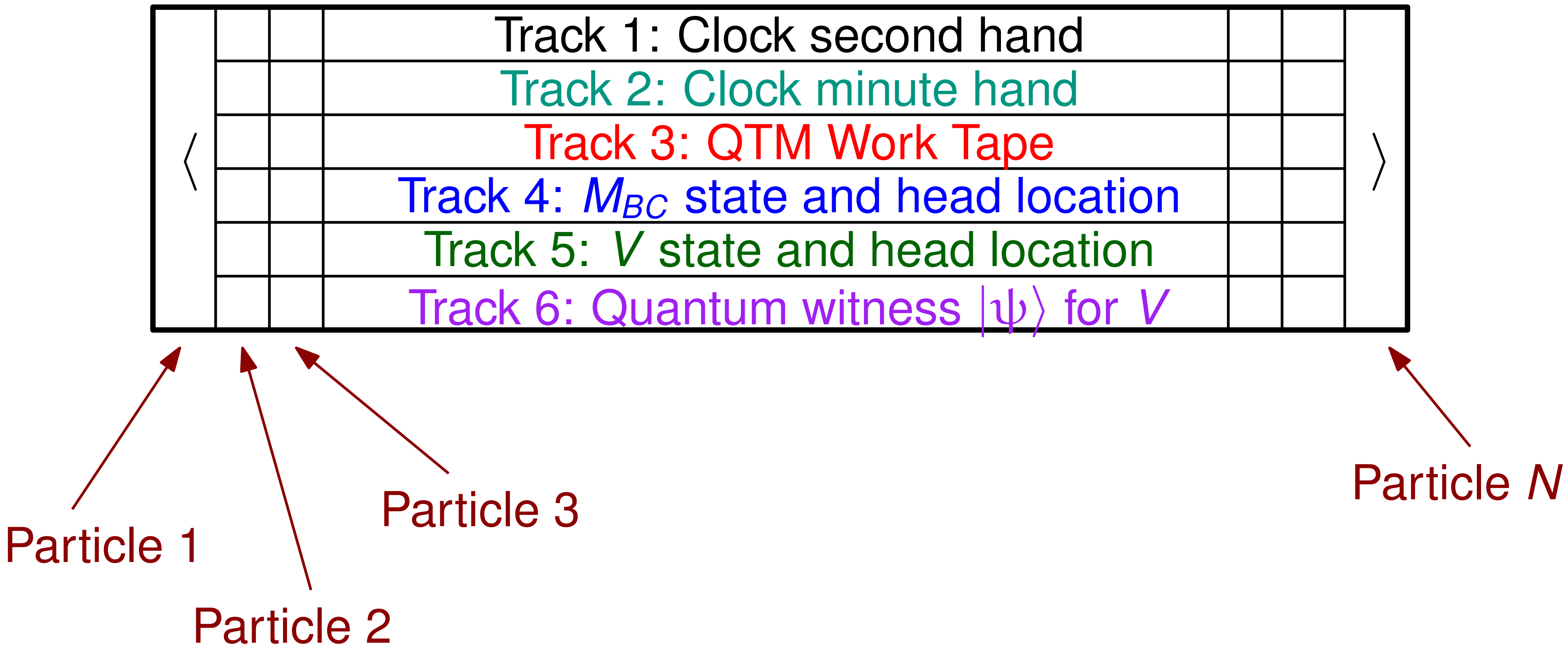
where N is the length of the chain.

Need a clock that counts the number of particles in the chain twice.

Each ”tick” of the clock triggers a step of a QTM.

Translationally Invariant Local Hamiltonian



⟨

Track 1: Clock second hand

Track 2: Clock minute hand

Track 3: QTM Work Tape

Track 4: M_{BC} state and head location

Track 5: V state and head location

Track 6: Quantum witness $|\psi\rangle$ for V

⟩

Particle 1

Particle 2

Particle 3

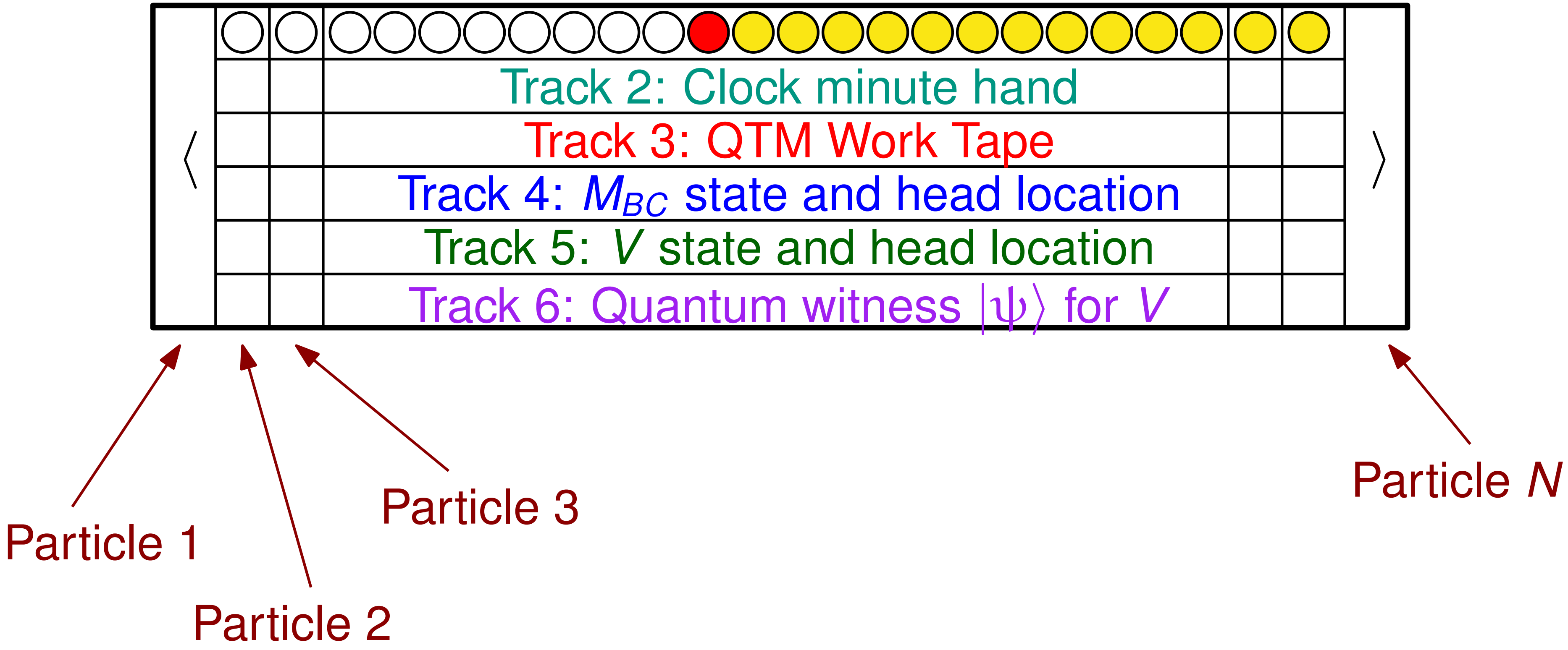
Particle N

Particle states:

6-tuple denoting the state for each track.

OR $\boxed{\langle}$ OR $\boxed{\rangle}$

Translationally Invariant Local Hamiltonian



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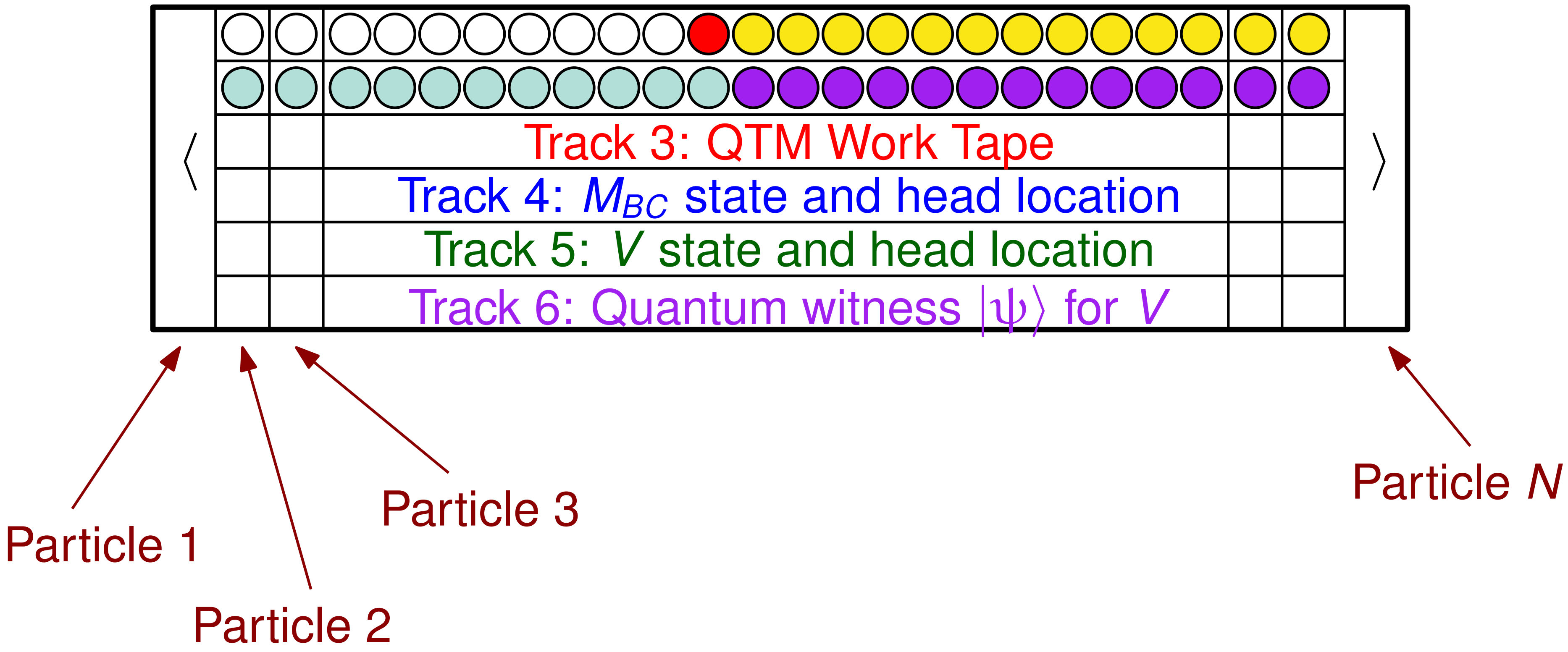

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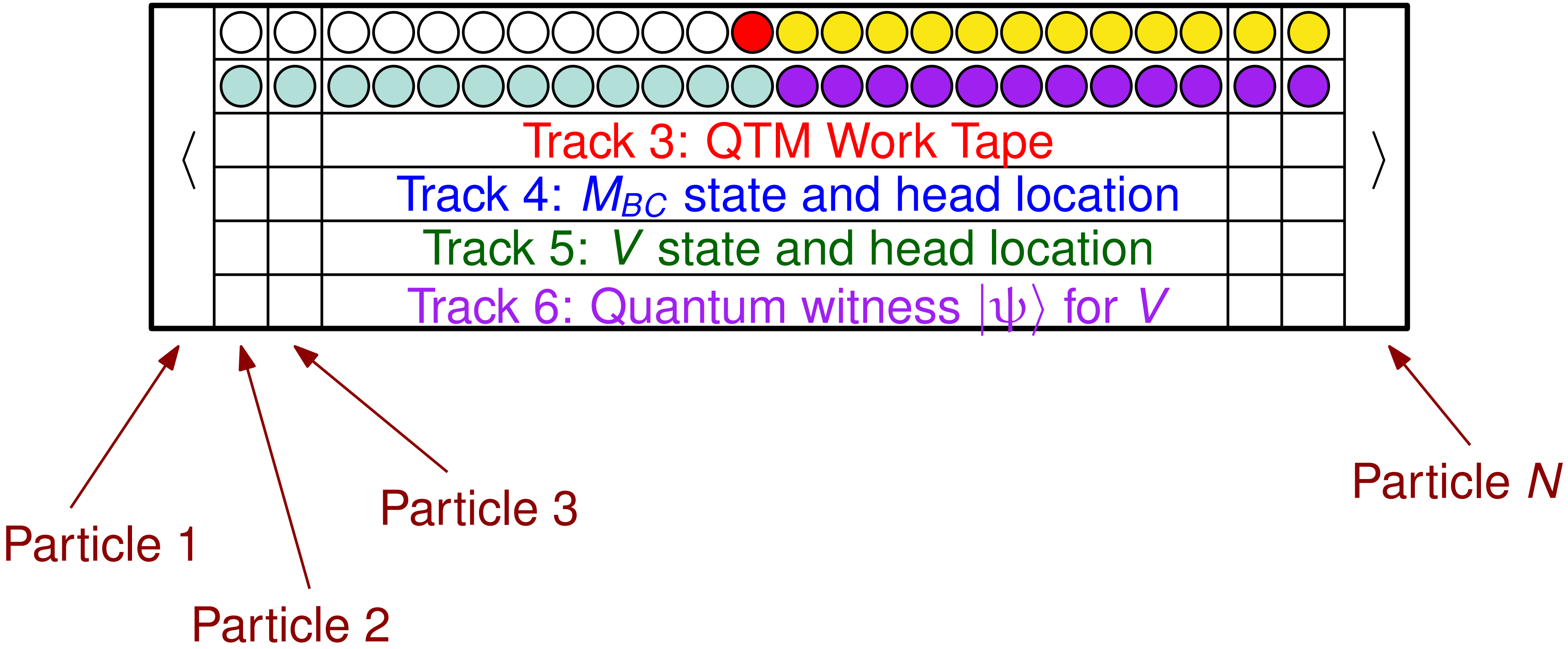

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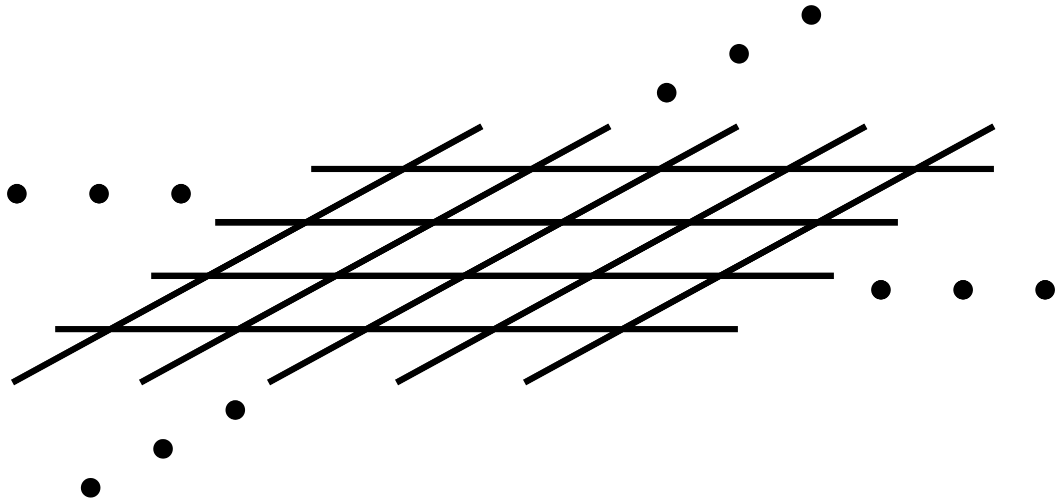
The Thermodynamic Limit



What are the properties of the ground state as $N \to \infty$?

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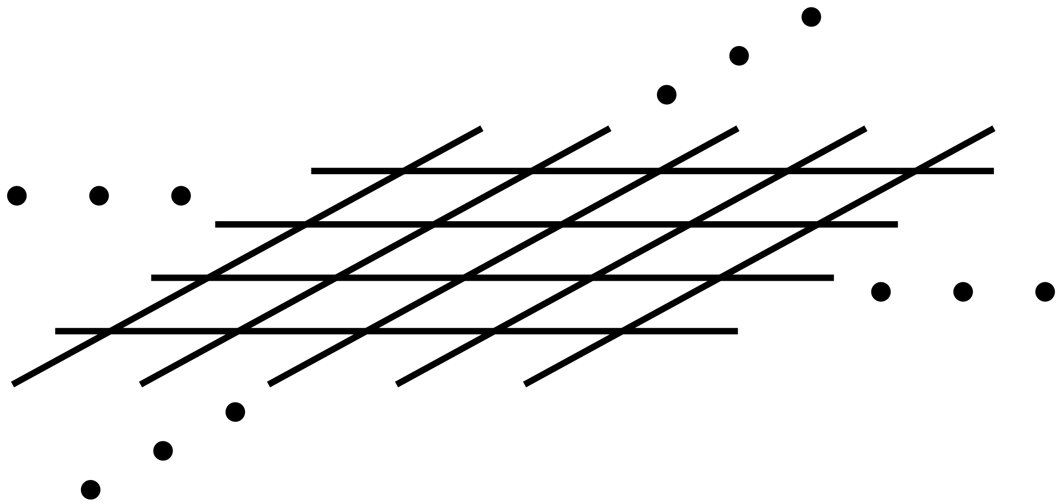

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Each grid dimension has its own term

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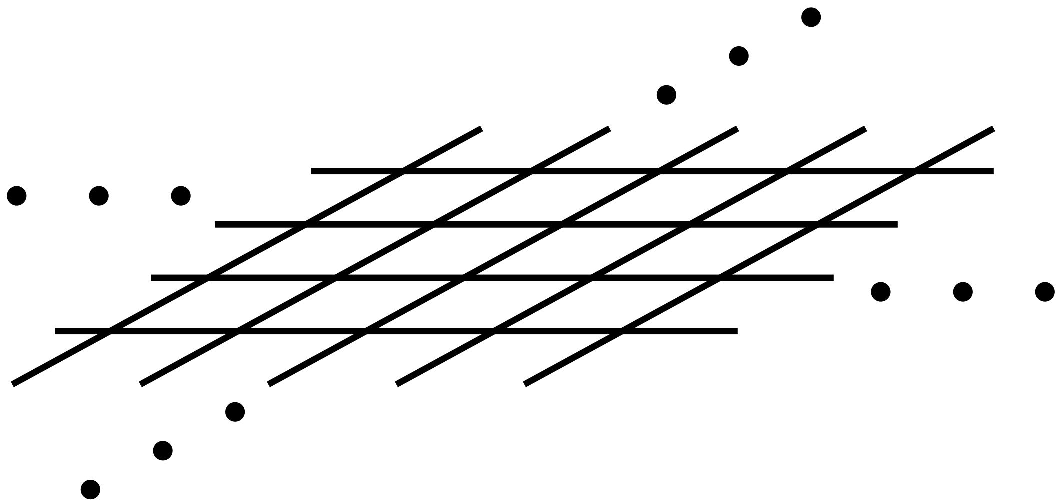

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Translationally-Invariant:

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Ground Energy Density: $H(N)$ Hamiltonian on an $N \times N$ finite grid.

$$\alpha_0 = \lim_{N \rightarrow \infty} \frac{\lambda_0(H(N))}{N^2}$$

(energy per particle)

The Thermodynamic Limit

What is the ground Energy Density (energy per particle) when H is applied to an infinite grid/line?

Input: Hamiltonian term H on two d-dimensional particles. (n bits)

In 2D: $H = (H_{horiz}, H_{vert})$

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Determining the Spectral Gap of H is undecidable.

Is $\Delta \geq 1$ or is H gapless?

[Cubitt, Perez-Garcia, Wolf *Nature*, 2015] ⟵ 2D

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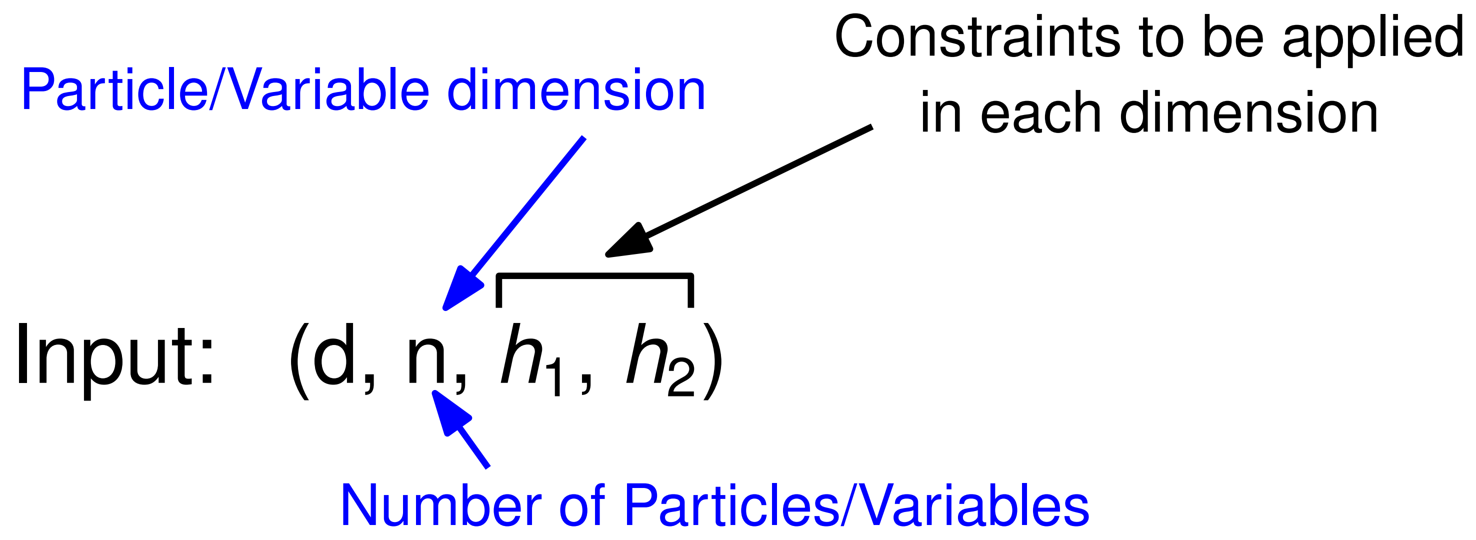
Weaker Version of Translational Invariance

Translational Invariance

Finite
Systems



Particle/Variable dimension

Constraints to be applied
in each dimension

Variable
Constraint

1) Input: (d, n, h_1, h_2)

Number of Particles/Variables

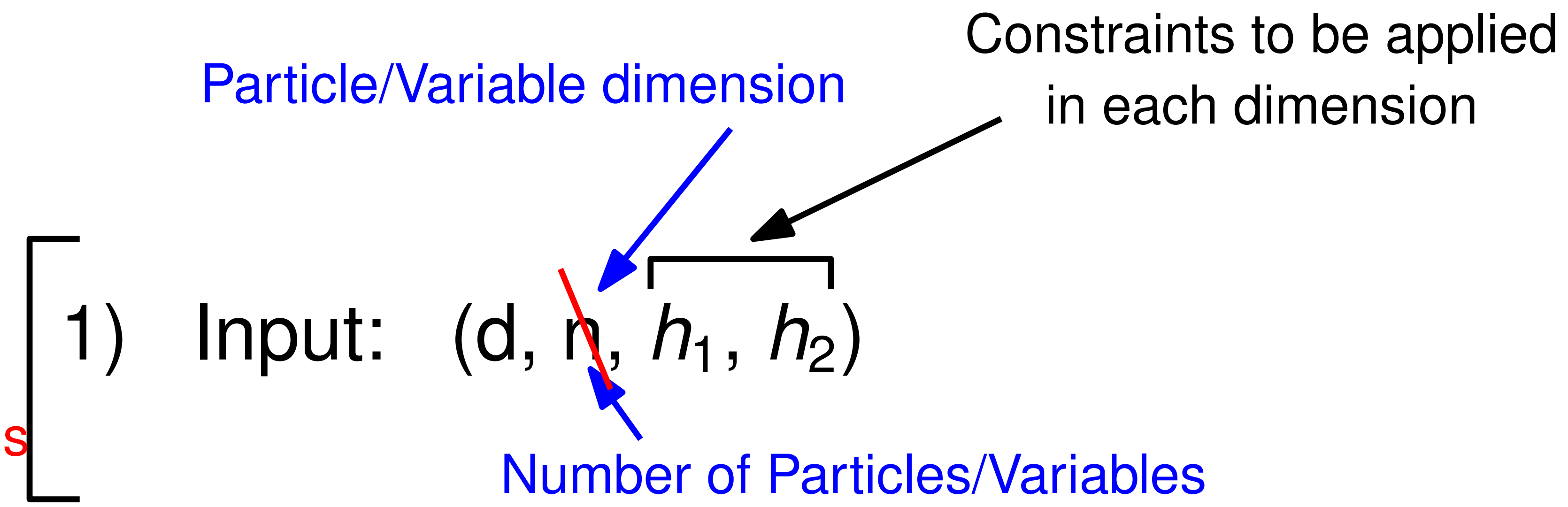
Fixed
Constraint

2) Fixed Problem Parameters: (d, h_1, h_2)

Input: n

Translational Invariance

In Finite
Systems



Particle/Variable dimension

Constraints to be applied
in each dimension

Infinite
family of
Hamiltonians

1) Input: $(d, \cancel{n}, \overbrace{h_1, h_2})$

Number of Particles/Variables

One
Hamiltonian

2) Fixed Problem Parameters: (d, h_1, h_2)

~~Input: n~~

Ground Energy Density = α_0

Function Ground Energy Density (Function-GED)

Function-GED (h_{row}, h_{col})

Input: n (binary number)

Output: α, where $|\alpha - \alpha_0| \leq \frac{1}{2^n}$

$$\alpha_0 = .\underbrace{10111001010001}_{n}0011101101\ldots$$

Function Ground Energy Density (Function-GED)

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Why a function problem?

In order to determine the n^{th} bit, you need to know the first $n - 1$ bits.

Also...more natural?

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Function-GED is contained in $\text{FEXP}^{\text{QMA-EXP}}$

Function-GED is hard for $\text{FEXP}^{\text{NEXP}}$

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FEXP:
Functions
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QMA-EXP:
YES instances can be verified by

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EXP time ~~quantum~~ verifier with
classical

EXP-size ~~quantum~~ witness
classical

Containment

Function-GED is contained in $\mathrm{FEXP}^{\mathrm{QMA\text{-}EXP}}$

Containment

Function-GED is contained in $\text{FEXP}^{\text{QMA-EXP}}$

Oracle language: Decision-GED (h_{row}, h_{col})

Input: $\alpha \in [0, 1]$ specified with n bits

Output: Accept if $\alpha_0 \leq \alpha$

Reject if $\alpha_0 \geq \alpha + \frac{1}{2^n}$

Containment

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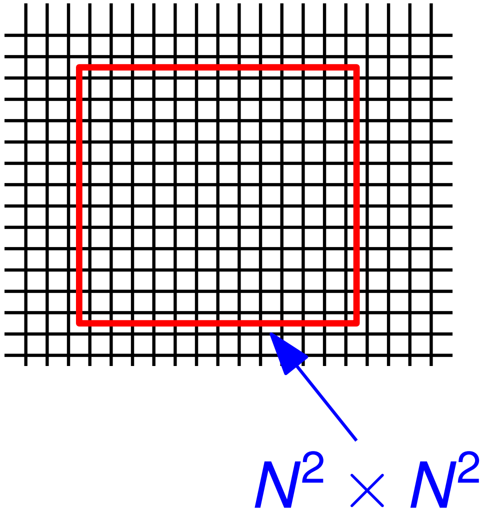

Observation:

The ground energy for an $N^2 \times N^2$ grid is within $\pm O\left(\frac{1}{N}\right)$ of α_0

$\Rightarrow$ Decision-GED $\in$ QMA-EXP

Containment

Function-GED is contained in $\text{FEXP}^{\text{QMA-EXP}}$

Binary Search using Decision-GED

2 queries reduces the interval size by $\frac{1}{2}$

$$\alpha_0 = .\underbrace{10111001010001}_{n}0011101101\ldots$$

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Input : n (log n bits)

Binary Search: $O(n)$ iterations (EXP time)

Query Prescision: $\frac{1}{2^n}$ (Oracle class: QMA-EXP)