PCMI 2023

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Quantum Hamiltonian Complexity: Day 4

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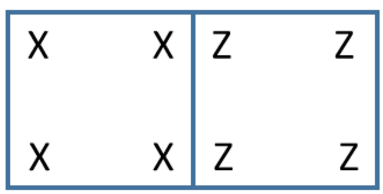
Commuting LH problem

1. Consider the following two operators on three qubits: XZI and $I \otimes |1\rangle\langle 1| \otimes X$.
   (a) Verify that the two operators commute.
   (b) The second qubit is the only qubit which both operate on non-trivially. Give the decomposition of the Hilbert space for the second qubit induced by the two operators as described in the structural lemma. Describe each operator restricted to each subspace in the decomposition.

2. Show that XX and ZZ commute by showing that they are both diagonal in a single basis.

3. Consider the following two terms of the Toric code.



   (a) For each term, express the projector onto the ground state and verify that they still commute.
   (b) You can think of the two qubits where the two terms intersect as a single 4-dimensional Hilbert space. Give the decomposition of this space induced by the two commuting terms as described in the structural lemma. Describe each operator restricted to each subspace in the decomposition.

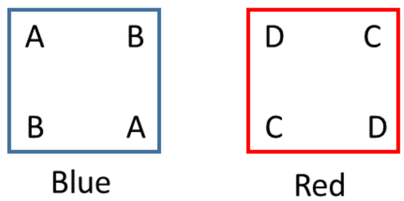
4. Give an example of two operators which intersect on a single qubit and commute in a $(1, 1)$ way.

5. Give an example of two operators which intersect on a single qubit and commute in a (2) way.

6. Consider an instance of CLH on a 2D grid of qutrits. Let

$$
\begin{aligned}
A &= |0\rangle\langle 0| + |1\rangle\langle 2| + |2\rangle\langle 1| \\
B &= |0\rangle\langle 1| + |1\rangle\langle 0| + |2\rangle\langle 2| \\
C &= |2\rangle\langle 2| - |1\rangle\langle 1| \\
D &= |0\rangle\langle 0| - |1\rangle\langle 1|
\end{aligned}
$$

Starting in the upper left corner and going clockwise, the blue faces have terms $-ABAB$ and the red terms are $-DCDC$ as shown:



(a) Verify that the instance is commuting.

(b) Describe the projector onto the ground space for each term. The projector for face i will be called P_i.

(c) Describe a projector for each qutrit for the blue terms $P_{q,B}$ and a projector for each qutrit for the Red terms $P_{q,R}$ and use them to argue that for your chosen operators:

$$\text{Tr}\left[\Pi_q P_{q,B} \cdot \Pi_{i\in\text{Blue}} P_i \cdot \Pi_q P_{q,B} \cdot \Pi_q P_{q,R} \cdot \Pi_{i\in\text{Red}} P_i \cdot \Pi_q P_{q,R}\right] > 0$$