Quantum LDPC codes
Problem session 4

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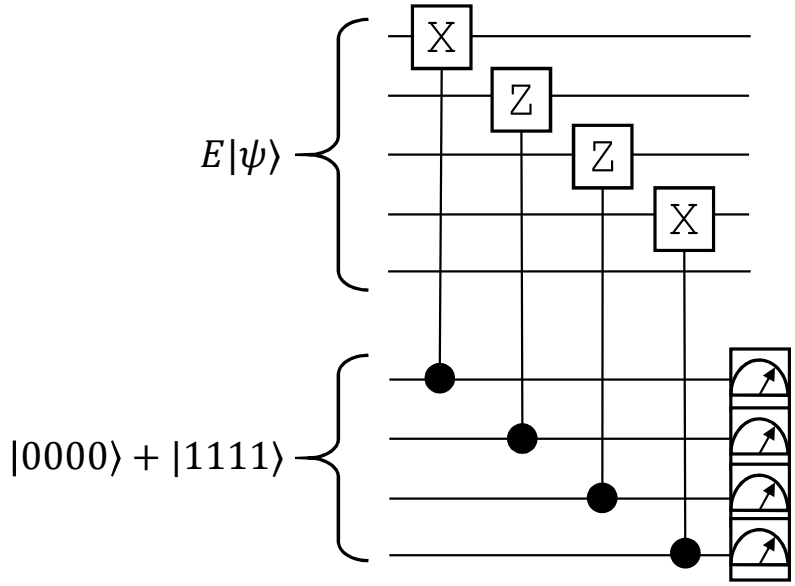
Cayley graph of the Pauli group

- Draw the Cayley graph of the Pauli group
$$G = \{\pm I, \pm iI, \pm X, \pm iX, \pm Y, \pm iY, \pm \mathrm{Z}, \pm iZ\}$$

With $A = \{\pm \mathrm{i}I, X, Z\}$.

FT syndrome extraction circuits

- What is the circuit on the right doing?
- Prove that any single qubit fault propagates to at most one of the top 5 qubits.
- Design a full FT syndrome extraction circuit for the 5-qubit code based on this strategy. You can use only 4 ancilla qubits.
- Design a circuit to prepare the state $|0000\rangle + |1111\rangle$. You circuit can use the following gates:
  - Prepare |0>
  - H
  - CNOT
  - Measure a qubit
- Can you make this preparation circuit FT using more qubits? You



Girth of a graph

The girth of a graph is the length of the shortest cycle in the graph.

Consider a d-regular graph G with girth g.

- What is the girth of the toric code lattice?
- What is the girth of the Tanner graph of the classical Hamming code? Of the Quantum Hamming code?
- Find a bound on the girth as a function of the code distance?
- Let v be a vertex of G. If g>7, how many distinct vertices are at distance 1 from v? at distance 2 from v, at distance 3 from v?
- If G has girth g, find a lower bound on the number of vertices of G.
- Find a lower on the girth as a function of the number of vertices of G.