Quantum LDPC codes
Problem session 3

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LUT decoder

- What is the complexity in number of bit operations to build a LUT table for a stabilizer code with length n.
- Assume that we use a code with length $n = 10{,}000$.
  - How many bits are needed to store a n-qubit Pauli operator?
  - Assume that we want to implement a decoder with 1 GB of memory. How many Pauli errors can we store at most with this memory.
  - What is the cost of storing all weight-1 errors? Weight-2 errors?
  - Find a bound on the maximum weight M that we can use in the LUT construction algorithm.

Classical Peeling decoder

```
Input: syndrome + erased qubits (?).
Output: bit value for each erased qubit.
1. While there exists an erased bit do:
2.          Loop over all checks and
   select a dangling check c.
3.          If there is no dangling check
   return failure.
4.          Use the dangling check to
   correct an erased bit.
5. Return the corrected values.
```

Consider the pseudo code for the peeling decoder.

- What is the worst-case complexity of this algorithm?
- Modify the pseudo-code to reduce the complexity.\
- Can you achieve a linear time complexity?

Repetition code

- Write the pseudo-code for an efficient decoding algorithm for a distance-d repetition code.
- What is the complexity of your decoder.

Bacon-Shor decoding

Consider a quantum state $|\psi\rangle$ on a grid of $\ell \times \ell$ qubits encoded in a stabilizer code with stabilizer generators acting as ZZ on vertical edges.

Assume that $|\psi\rangle$ suffers from a Z error $|\psi\rangle \to E_Z|\psi\rangle$.

- Show that this error E_Z is equivalent to an error acting on the first row of qubits.

We measure the operators S_i such that S_i acts as X on all qubits of columns i and $i+1$. The outcome of S_i is a bit σ_i.

- Design a decoding algorithm to corrects the errors E_Z.
- How many Z error can you correct at most with using this $\ell \times \ell$ grid of qubit.

MW decoder

- Show that A MW decoder corrects all Pauli errors E with weight $|E| \leq \frac{d-1}{2}$.
- Assume that the decoder D find an error with weight at most $m + c$ for some constant c where m is the weight of the correction returned by a MW decoder. What is the max weight w such that D corrects all Pauli errors with weight up to w.
- Consider a classical decoder for a binary symmetric channel (bit flip channel). If the bit flip rate is $p < 0.5$, show that any MLE decoder is a MW decoder and vice-versa.