Quantum LDPC codes
Problem session 2

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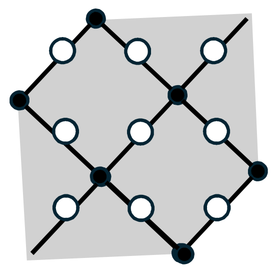
Problem – The rotated surface code



- Construct the stabilizer matrix of the distance-3 surface code.
- What are the parameters [n, k, d] of this code?
- Find a logical basis.
- Generalize this logical basis a distance-d surface code.

Problem - Syndrome extraction

$$\begin{bmatrix} X & I & X & I & X & I & X \\ I & X & X & I & I & X & X \\ I & I & I & X & X & X & X \\ Z & I & Z & I & Z & I & Z \\ I & Z & Z & I & I & Z & Z \\ I & I & I & Z & Z & Z & Z \end{bmatrix}$$

- Design a syndrome extraction circuit for the three X stabilizer generators of the quantum Hamming code.

Your circuit must use a single ancilla qubit that can be reset in the state $|0\rangle$ and it must be made of the following operations: Prepare $|0\rangle$ on the ancilla qubit, H, CNOT, measure a qubit in the Z basis.

- Show that a fault Z before a H gate is equivalent to a fault X after the H gate.
- Consider the gate CNOT(1, 2).
  - Show that a fault Z_2 before the CNOT is equivalent to a fault $Z_1 Z_2$ after CNOT.
  - Show that the fault Z_1 commute with the CNOT.
- Show that a fault Z before a measurement has no effect.
- Assume that we run this circuit with input state $Z_2|\psi\rangle$ where $|\psi\rangle$ is a state of the quantum Hamming code. What is the fault at the end of the circuit?

Problem – 5-qubit code

$$H = \begin{bmatrix} X & Z & Z & X & I \\ I & X & Z & Z & X \\ X & I & X & Z & Z \\ Z & X & I & X & Z \end{bmatrix}$$

- Design a syndrome extraction circuit for the 5-qubit code.

Your circuit must use a single ancilla qubit that can be reset in the state $|0\rangle$ and it must be made of the following gate:

- Prepare $|0\rangle$ on the ancilla qubit,
- H,
- CNOT=CX,
- CY,
- CZ,
- measure the ancilla qubit in the Z basis.

Quantum syndrome measurement

Prove the following proposition.

Prop. Consider a system in the state $E|\psi\rangle$ where $|\psi\rangle \in Q(S)$ and $E \in \mathcal{P}_n$.

- The outcome of the measurement of S_i is $(-1)^{\sigma_i(E)}$ with probability 1.
- The state of the system after measurement is $E|\psi\rangle$.