Quantum LDPC codes
Problem session 2

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Problem – The rotated surface code

• Construct the stabilizer matrix of the distance-3 surface code.

• What are the parameters \([n, k, d]\) of this code?

• Find a logical basis.

• Generalize this logical basis a distance-\(d\) surface code.
Problem - Syndrome extraction

• Design a syndrome extraction circuit for the three X stabilizer generators of the quantum Hamming code. Your circuit must use a single ancilla qubit that can be reset in the state $|0\rangle$ and it must be made of the following operations: Prepare $|0\rangle$ on the ancilla qubit, H, CNOT, measure a qubit in the Z basis.

• Show that a fault $Z$ before a H gate is equivalent to a fault $X$ after the H gate.

• Consider the gate CNOT(1, 2).
  • Show that a fault $Z$ before the CNOT is equivalent to a fault $Z_1Z_2$ after CNOT.
  • Show that the fault $Z_1$ commute with the CNOT.

• Show that a fault $Z$ before a measurement has no effect.

• Assume that we run this circuit with input state $Z_2|\psi\rangle$ where $|\psi\rangle$ is a state of the quantum Hamming code. What is the fault at the end of the circuit?
Problem - 5-qubit code

Design a syndrome extraction circuit for the 5-qubit code.

Your circuit must use a single ancilla qubit that can be reset in the state $|0\rangle$ and it must be made of the following gate:

- Prepare $|0\rangle$ on the ancilla qubit,
- $H$,
- $\text{CNOT}=\text{CX},$
- $\text{CY},$
- $\text{CZ},$
- measure the ancilla qubit in the $Z$ basis.

$$H = \begin{bmatrix}
X & Z & Z & X & I \\
I & X & Z & Z & X \\
X & I & X & Z & Z \\
Z & X & I & X & Z \\
\end{bmatrix}$$
Quantum syndrome measurement

Prove the following proposition.

Prop. Consider a system in the state $E|\psi\rangle$ where $|\psi\rangle \in Q(S)$ and $E \in P_n$.
• The outcome of the measurement of $S_i$ is $(-1)^{\sigma_i(E)}$ with probability 1.
• The state of the system after measurement is $E|\psi\rangle$. 