Quantum LDPC codes
Problem session 1

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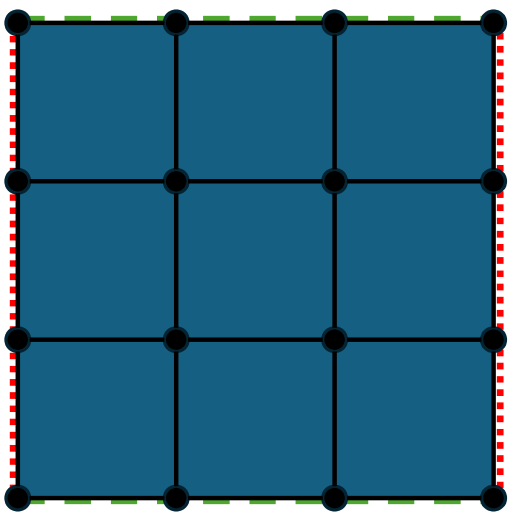
2D toric code



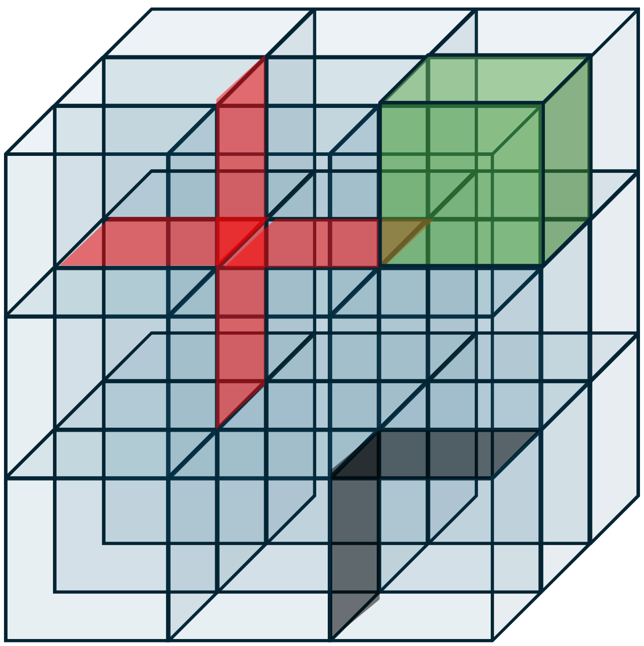
Consider the 3×3 toric code.

- Prove the rank of the group generated by the Z stabilizers if |F|-1.
- Prove the rank of the group generated by the X stabilizers if |V|-1.
- Prove that k=2.
- Find a logical basis.

Consider the $\ell \times \ell$ square cellulation of the torus.

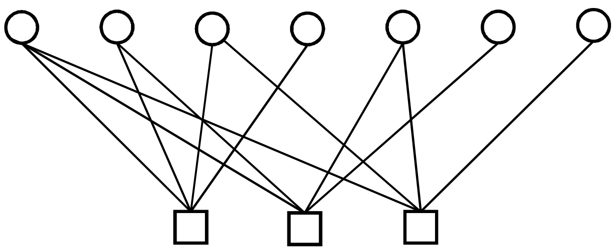
- What are the parameters of the associated Kitaev code?
- What is the limit of $\frac{k}{n}$? $\frac{d}{n}$?

3D toric code



- Find a logical basis for the 3D toric code.

HGP codes



- Compute parameters $[n_1, k_1, d_1]$ of the Hamming code.
- Compute parameters $[n_1^T, k_1^T, d_1^T]$ of the transposed code of the Hamming code

(the transposed code is obtained by swapping bits and checks).

- What are the parameters $[[n, k]]$ of the hypergraph product of the Hamming code by itself.
- What is the lower bound on its distance.

LDPC HGP codes

Suppose that for all n, we can generate classical linear codes such that

- Their parameters are $[n, Rn, \delta n]$ for some constants $R, \delta > 0$,
- Their transposed code contains only the zero vector,
- Their Tanner graph has maximum degree w.

- What are the parameters of the HGP of two such codes?
- What is the maximum weight of a stabilizer generator of this HGP.

Minimum distance of CSS codes

Define the X minimum distance as the minimum weight of a non-trivial logical error in $\{I, X\}^{\otimes n}$.

Define the Z minimum distance as the minimum weight of a non-trivial logical error in $\{I, Z\}^{\otimes n}$.

- Prove that if Q is a CSS code, then $d = \min(d_X, d_Z)$.

Counting logical qubits of a stabilizer code.

Consider a stabilizer group $S = \langle S_1, \dots, S_r \rangle$ with r independent generators. Our goal is to prove that its stabilizer code $Q(S)$ has $k = n - r$ logical qubits.

- Prove that for all $u \in \{0,1\}^r$, the group $S_u = \langle (-1)^{u_1} S_1, \dots, (-1)^{u_r} S_r \rangle$ is a stabilizer group.
- Prove that for all $u \in \{0,1\}^r$, there exists $E_u \in \mathcal{P}_n$ such that for all $i = 1, \dots, r$ we have, $[E_u, S_i] = u_i$.

Hint. Use a generating set of S obtained by applying Gaussian elimination to the S_i.

- Prove that for all $u \in \{0,1\}^r$, $Q(S_u) = E_u Q(S)$.
- Prove that $(\mathbb{C}^2)^{\otimes n} = \bigoplus_u Q(S_u)$.
- Prove that for all $u \in \{0,1\}^r$, $\dim Q(S_u) = 2^{n-r}$.
- Prove that $\dim Q(S) = 2^{n-r}$