Quantum LDPC codes
Lecture 2

Nicolas Delfosse

Microsoft

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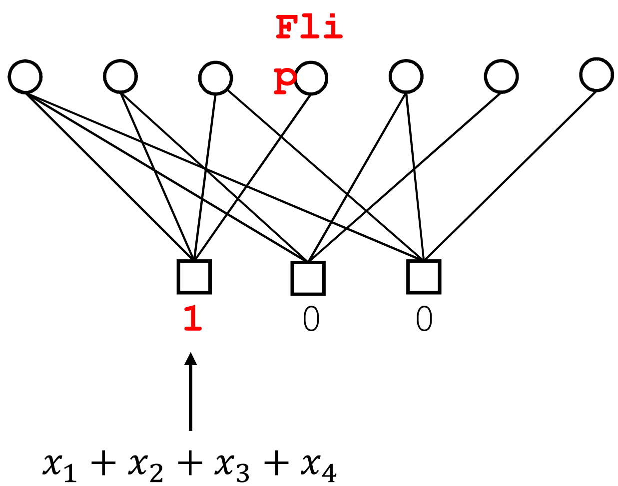
Overview of lecture 2

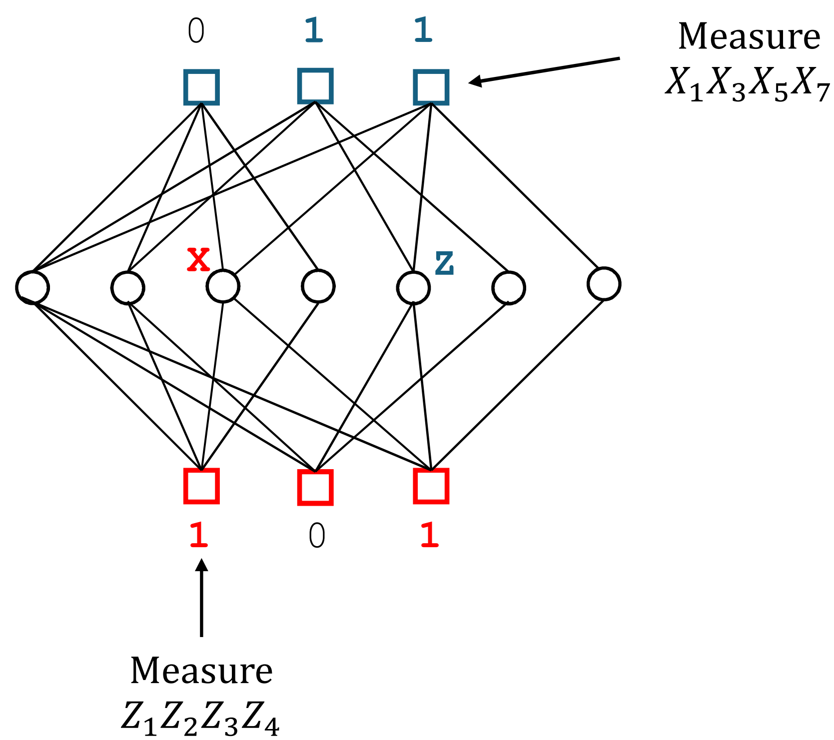
- Examples of quantum LDPC codes
- Syndrome extraction circuits for surface codes
- Syndrome extraction circuits for LDPC codes
- Performance of QLDPC codes

Classical and quantum Hamming codes

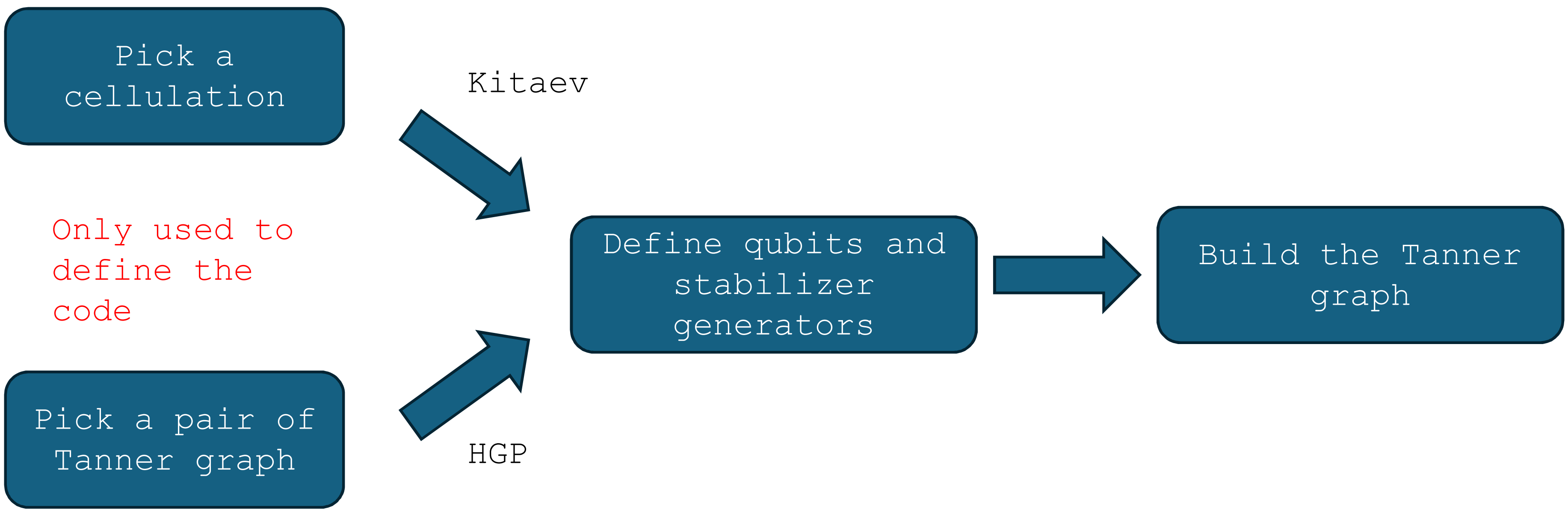
Quantum codes detect X errors and Z errors:

Classical code detect bit flips:





Design flow

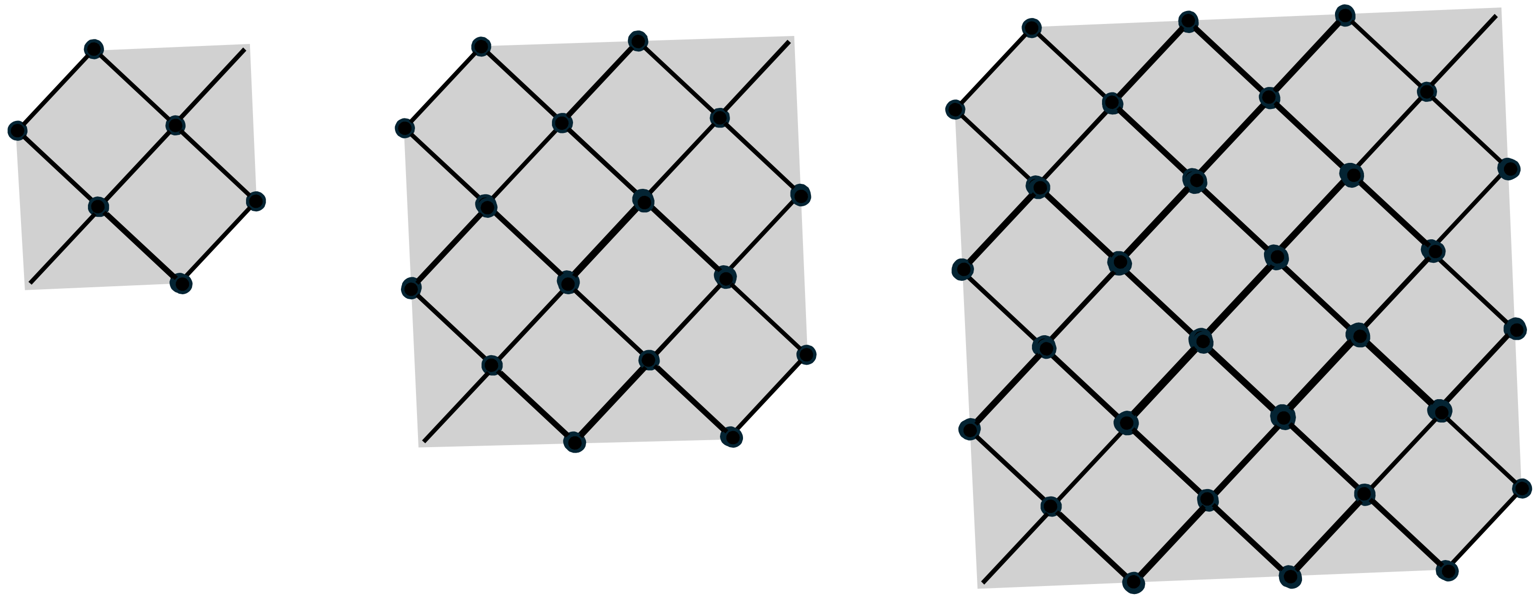

Pick a cellulation
Kitaev
Only used to define the code
Pick a pair of Tanner graph
HGP
Define qubits and stabilizer generators
Build the Tanner graph

The rotated surface code

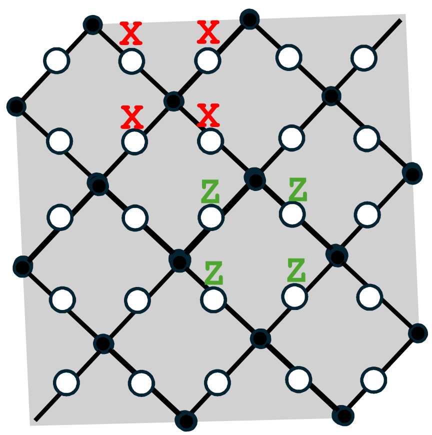
Planar cellulation

Instead of a torus, we consider a planar region:

- No identification of the opposite side.
- Rotate the square grid.
- Cute the corners faces and vertices.



Definition of the rotated surface code



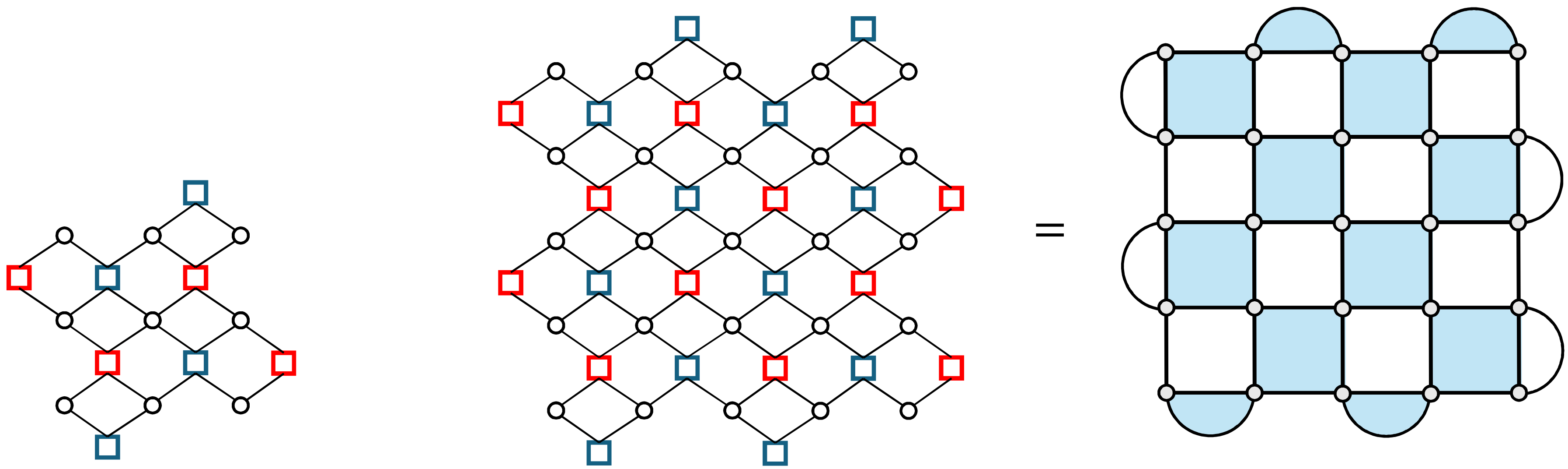
We use Kitaev's construction for our rotated planar cellulation:

- Place a qubit on each edge.
- Define a X stabilizer generator for each vertex.
- Define a Z stabilize generator for each face.

Remark.

- The boundary vertices and faces define weight-2 generators.

Tanner graphs of the rotated surface code



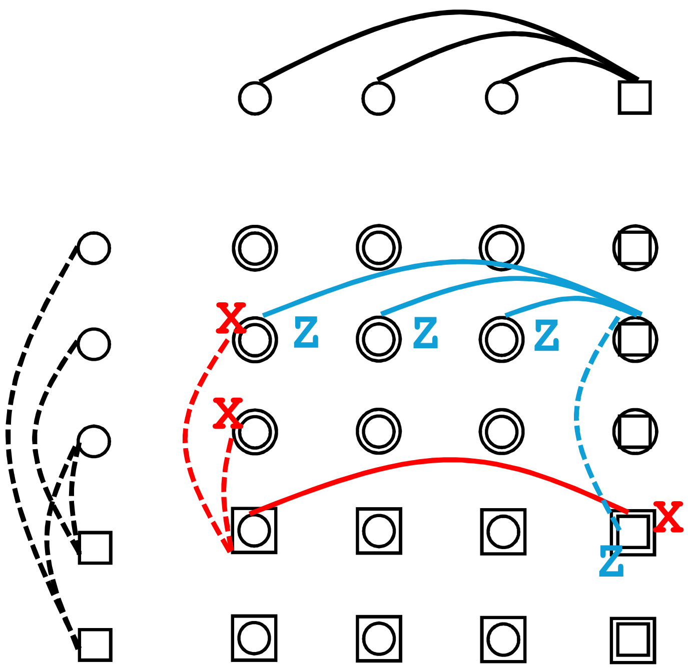
3 x 3 surface code
Corrects 1 error

5 x 5 surface code
Corrects 2 errors

The problem with surface codes: The encode only 1 logical qubit.

Hypergraph Product (HGP) Codes

Example – Hypergraph product code



Consider two bipartite graph

- Place a qubit on each circle-circle.
- Place a qubit on each square-square.
- Define a X generator for each square-circle.
- Define a Z generator for each circle-square.

What is n?

- n=11

What is the first stabilizer generator?

- III XII XII XI

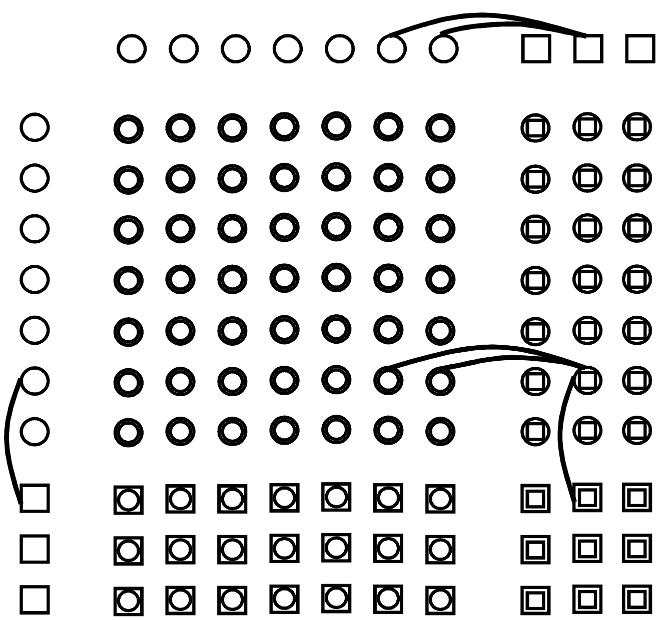
Number of logical qubits of HGP codes

Theorem. For HGP(C_1, C_2) codes, we have

- $n = n_1 n_2 + r_1 r_2$
- $k = k_1 k_2 + k_1^T k_2^T$
- $d \geq \min(d_1, d_2, d_1^T d_2^T)$
- If $k_1 > 0$, then $d \leq d_2$
- If $k_1^T > 0$, then $d \leq d_2^T$

By selected random sparse graph we can achieve $k \propto n$ and $d \propto \sqrt{n}$.

A family of LDPC codes with constant rate



Select two random Tanner graphs with

- 4s bits with degree 3.
- 3s checks with degree 4.

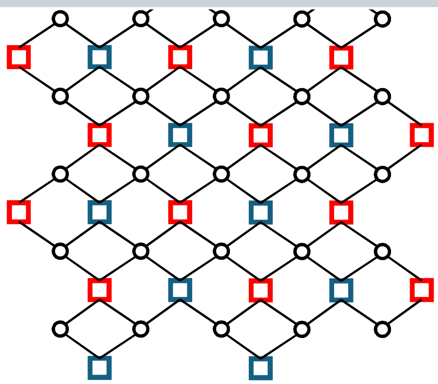
We get HGP codes with

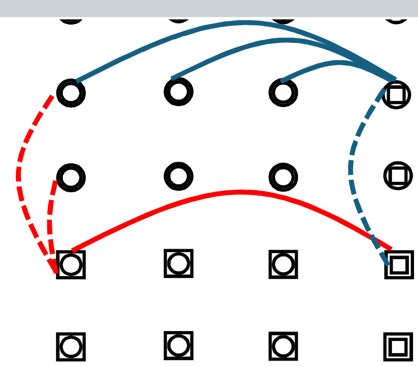
- $n = 25s^2$
- $k = s^2$
- Generator degree $= 7$
- Qubit degree = 6 or 8

We also impose girth ≥ 8 and we select the best code out of 50 – 1000

Surface codes vs LDPC codes

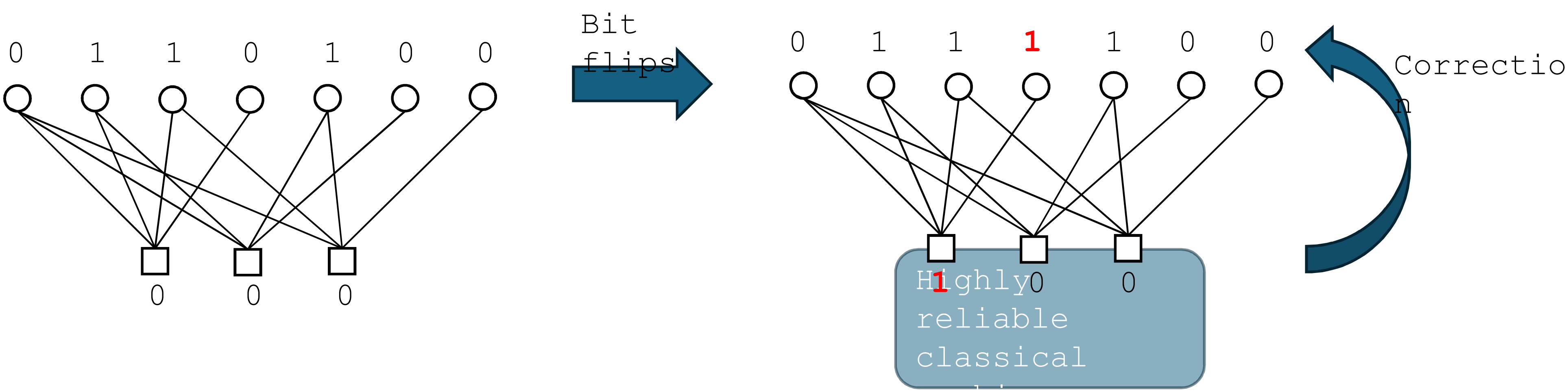
Code	Surface codes	LDPC codes
# logical qubits k	1	$\frac{n}{25}$
Measurement weight	4	7
# ancilla qubits	n	?



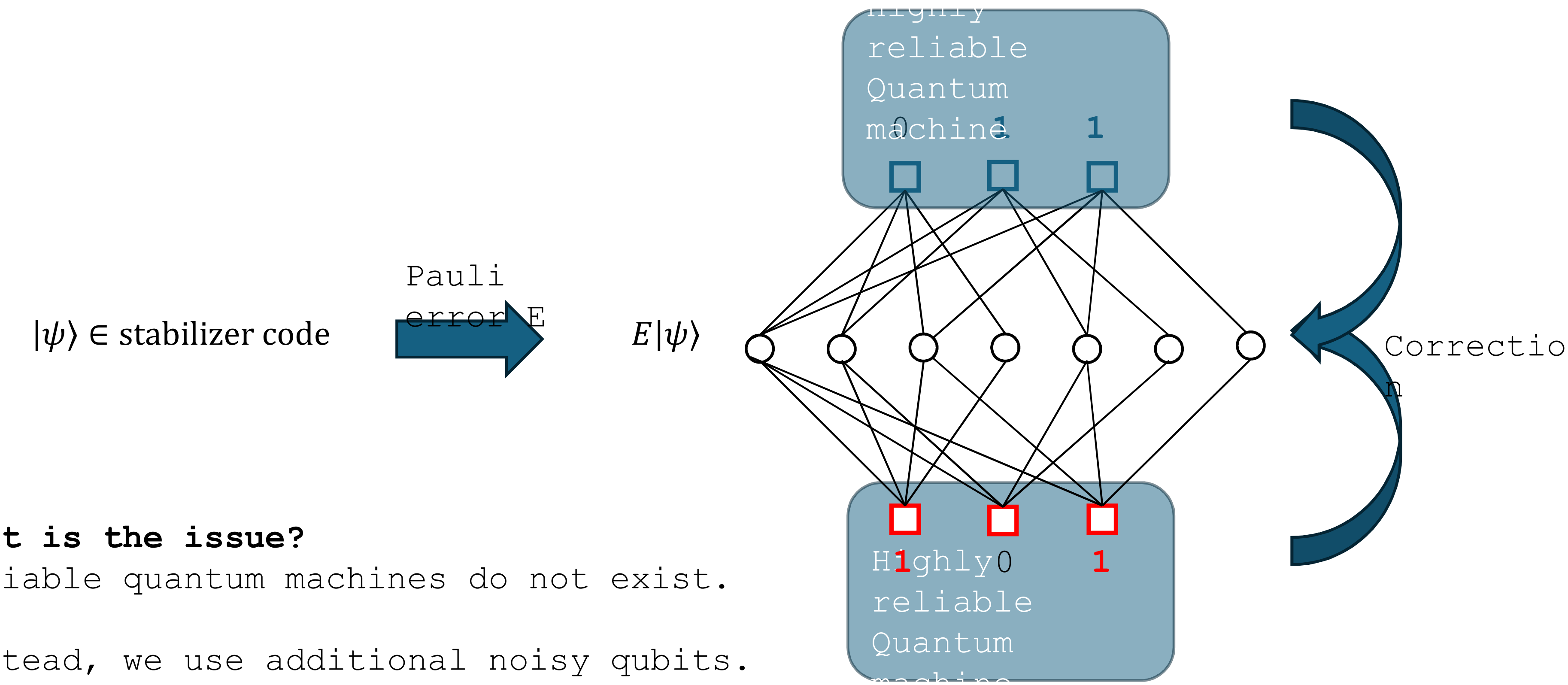


Syndrome extraction circuits

Classical vs Quantum error correction



Classical vs Quantum error correction

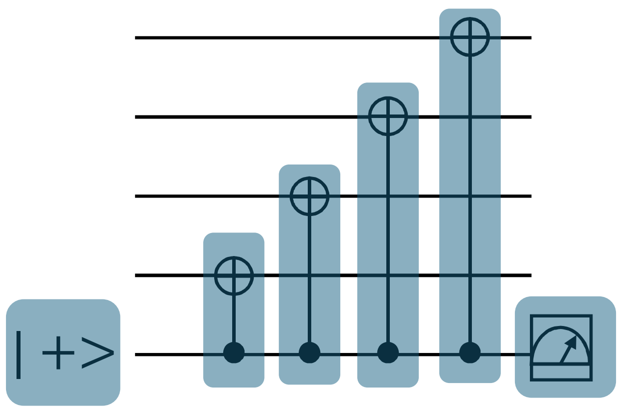


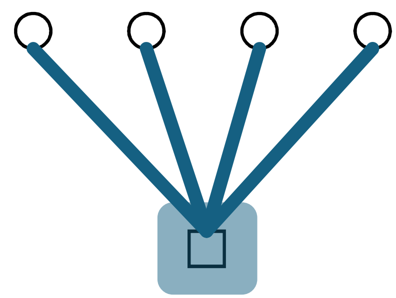
What is the issue?

Reliable quantum machines do not exist.

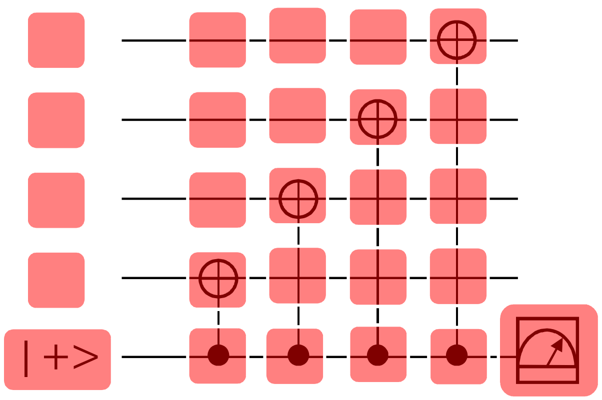
Instead, we use additional noisy qubits.

Measurement of a X check





Standard Pauli noise models



Perfect measurement model:

- Noise on data qubits

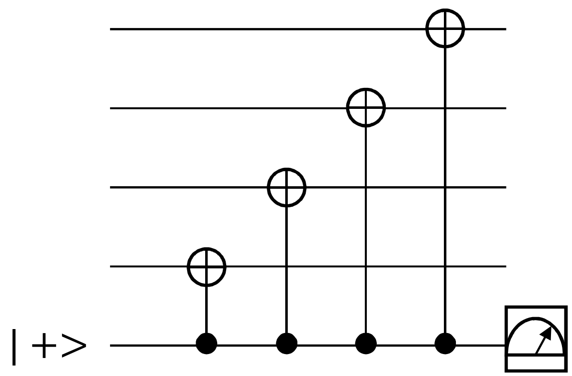
Phenomenological model:

- Noise on data qubits
- Noise on measurements

Circuit noise:

- Noise on data qubits
- Noise on measurements
- Noise on ancilla qubits
- Noise on gates
- Noise on waiting qubits

The problem with high random stabilizer codes



Question.

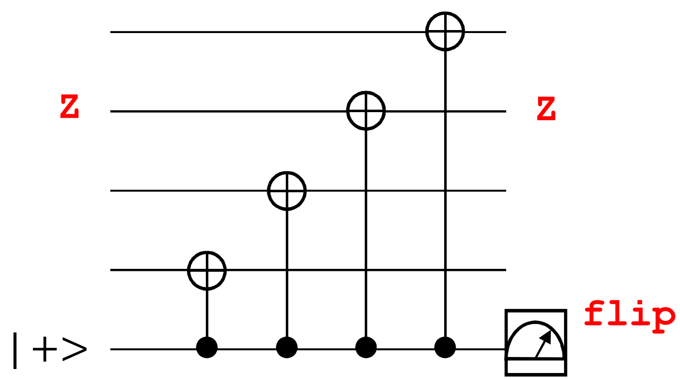
Can we use this *noisy* circuit with random stabilizer codes?

Strategy:

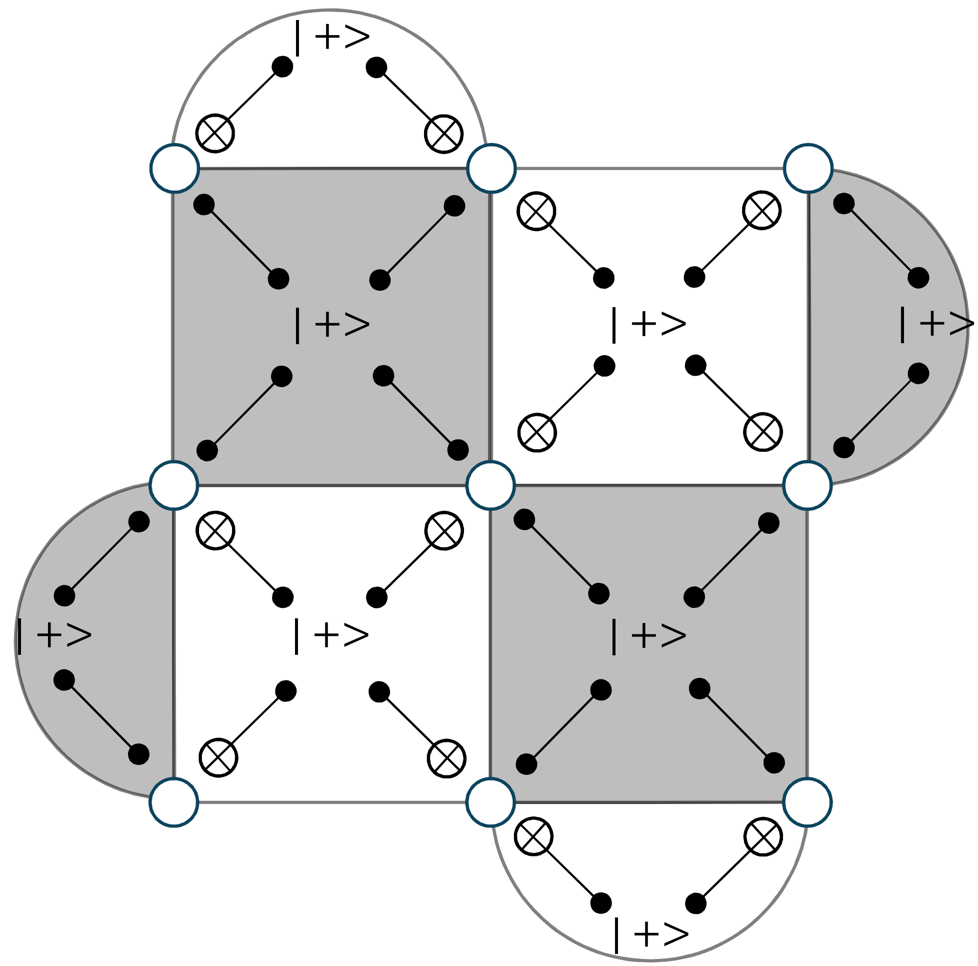
- Suppose that we have 1000 qubits.
- Pick a random [[1000, 500]] stabilizer code.
- Measure the 500 stabilizers generators.

Syndrome extraction circuits for surface codes

Syndrome extraction circuit



X plaquette circuit

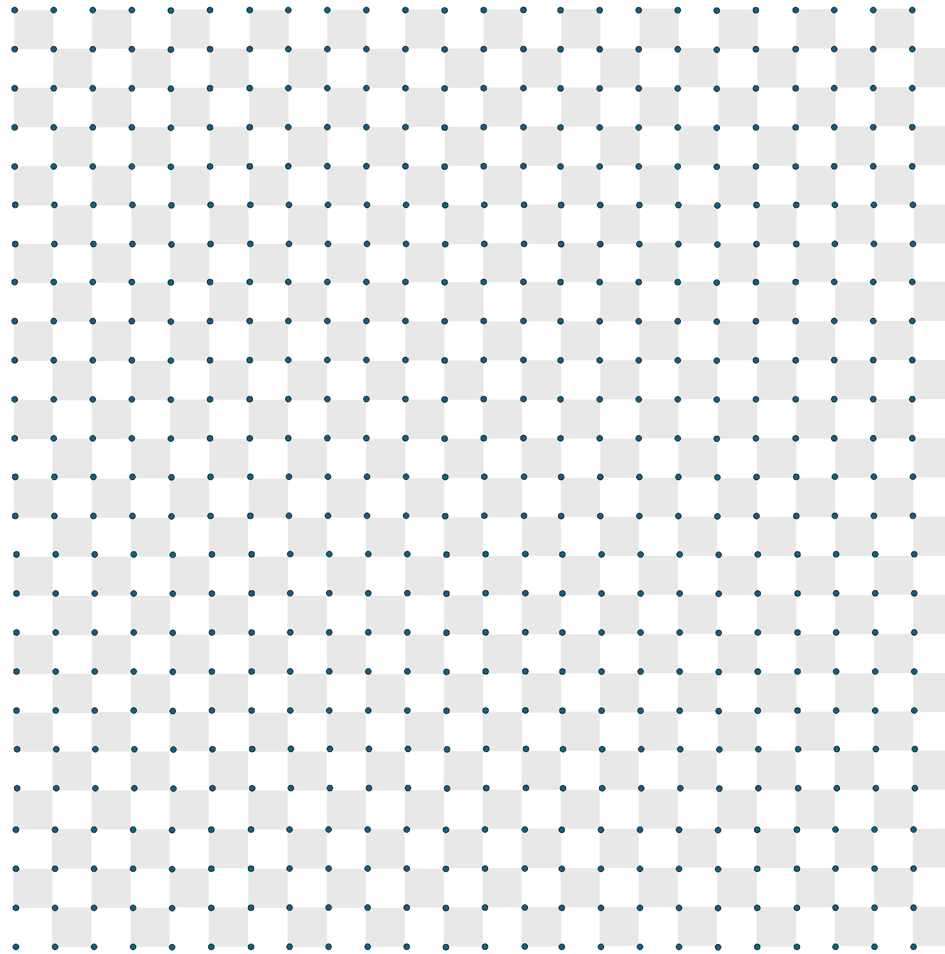


Surface codes

Why surface codes?
- High noise threshold (about 1%).
- Implemented with 2D local gates.

Main issue:
- Thousands of physical qubits per logical qubit.

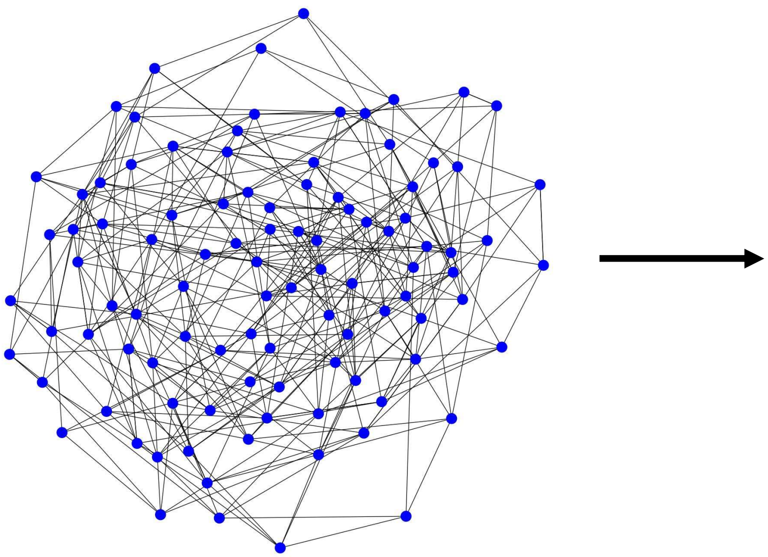
Here: 1250 physical qubits



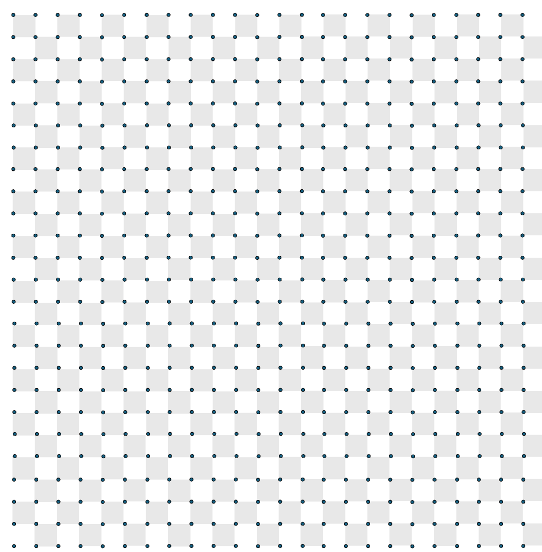
25x25 surface code:
Physical error rate: $10^{-3} \Rightarrow$ Logical error rate: 10^{-12}

Syndrome extraction circuits for LDPC codes

Can we implement quantum LDPC codes?



Typical LDPC code



Typical quantum computer

Main results

With 2D local gates: arxiv:2109.14599

- Need large depth circuits or many ancilla qubits.
- Numerically: poor performance.

With long-range connections: arxiv:2109.14609

- Layout based on only a few planar layers.
- Numerically outperform surface codes.

Bounds on syndrome extraction circuits

For surface codes: depth = 6.

Theorem. (informal) For quantum LDPC codes implemented with 2D local gates:

$$\text{depth} \geq \text{constant} \times \frac{n}{\sqrt{q}}$$

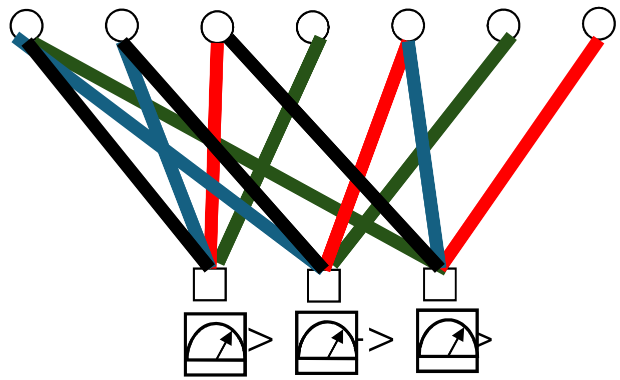
where q = total number of qubit used.

	Constant depth	**Constant qubit overhead**
Bound	# ancilla $\geq$ constant $\times\ n^2$	Depth $\geq$ constant $\times \sqrt{n}$
Saturating circuit	Switch-based circuit (next slide)	HGP code circuit

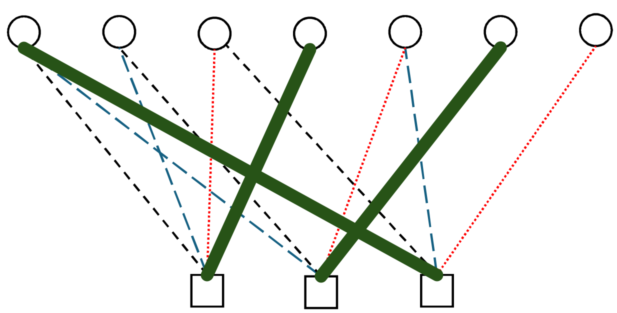
2D local circuits

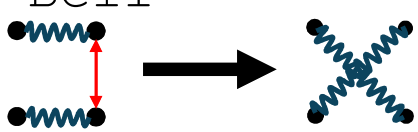
Simultaneous measurement of all the X checks

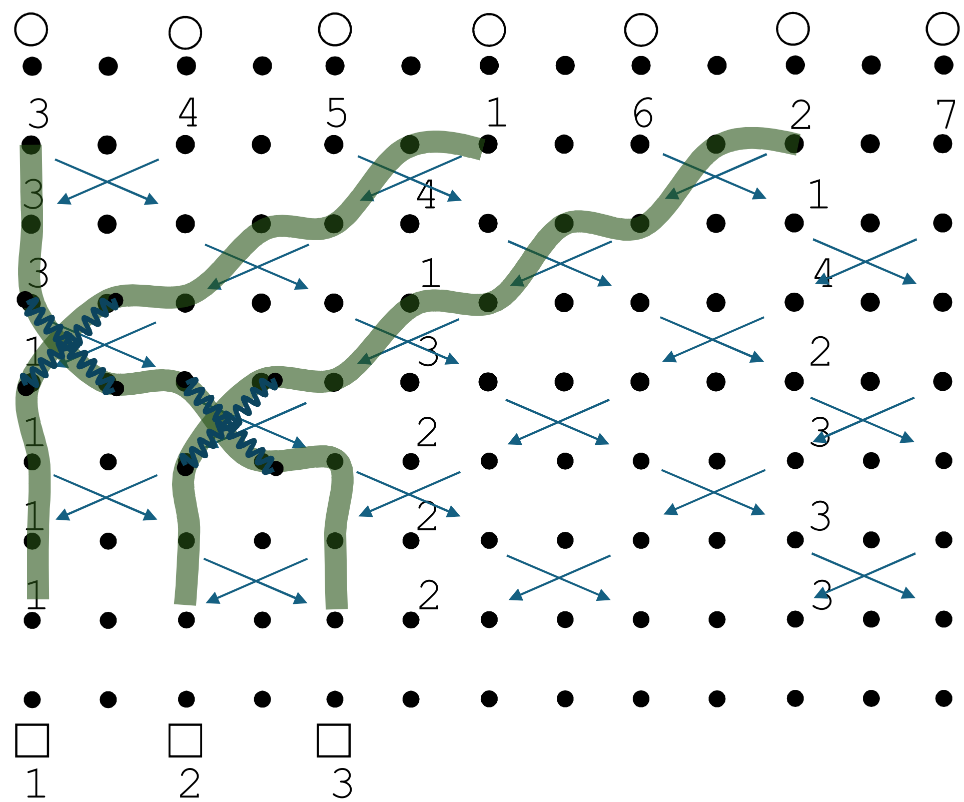
1. Construct an edge coloration
2. Prepare a readout qubits in $|+\rangle$ for each check.
3. For each color c do:
4. Apply a CNOT on each edge with color c
5. Measure readout qubits in the X basis.



Switch-based circuit

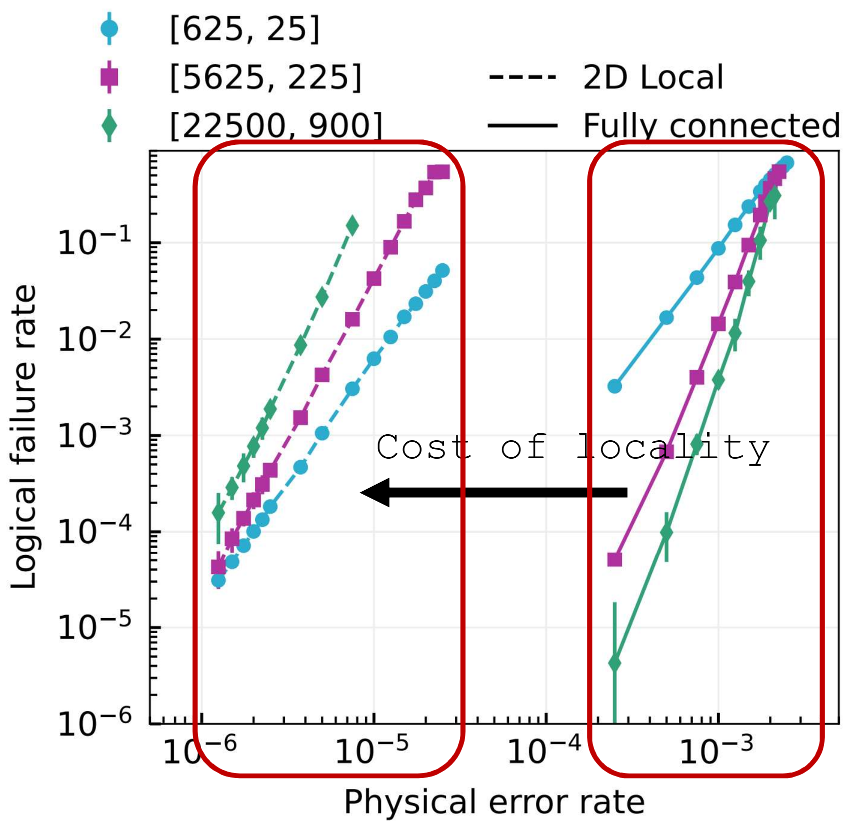






2D local vs fully connected implementation

- With fully connected qubits:
  - # ancillas: $O(n)$
  - depth: $O(1)$
- With 2D local gates:
  - # ancilla qubits: $O(n)$
  - Depth $O(\sqrt{n})$



With long-range gates

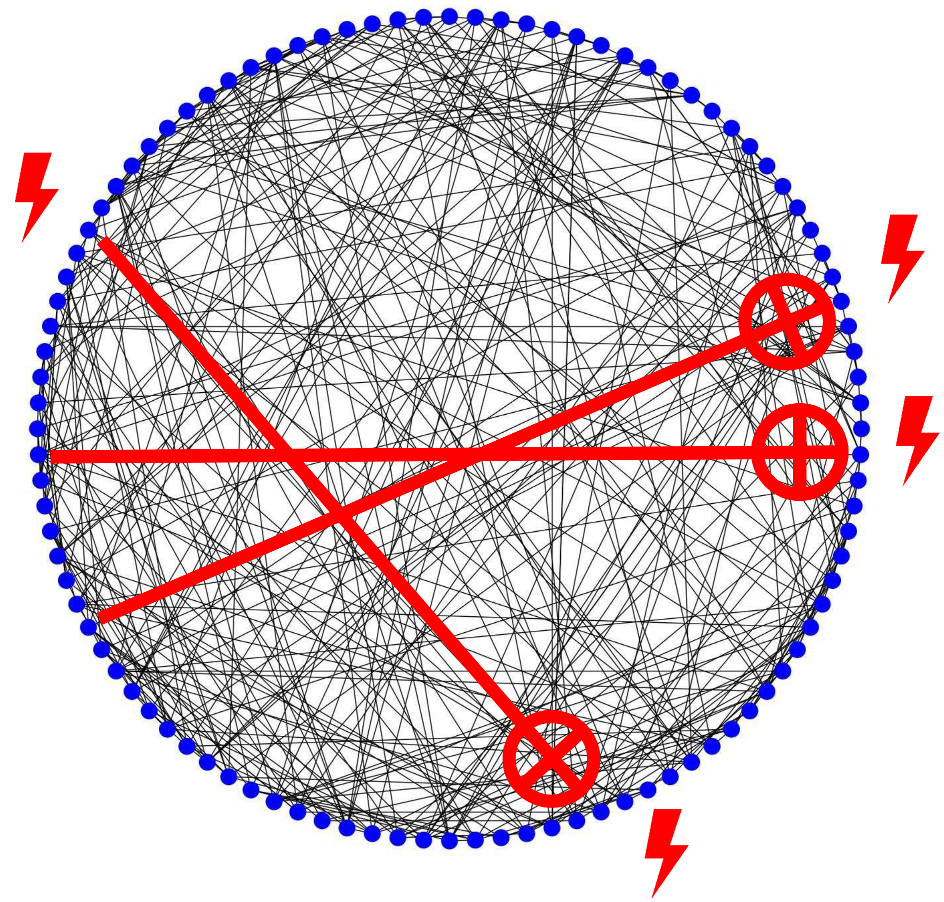
Naïve layout with long-range connectivity

Issue:

- Crossing gates may induce correlated errors.

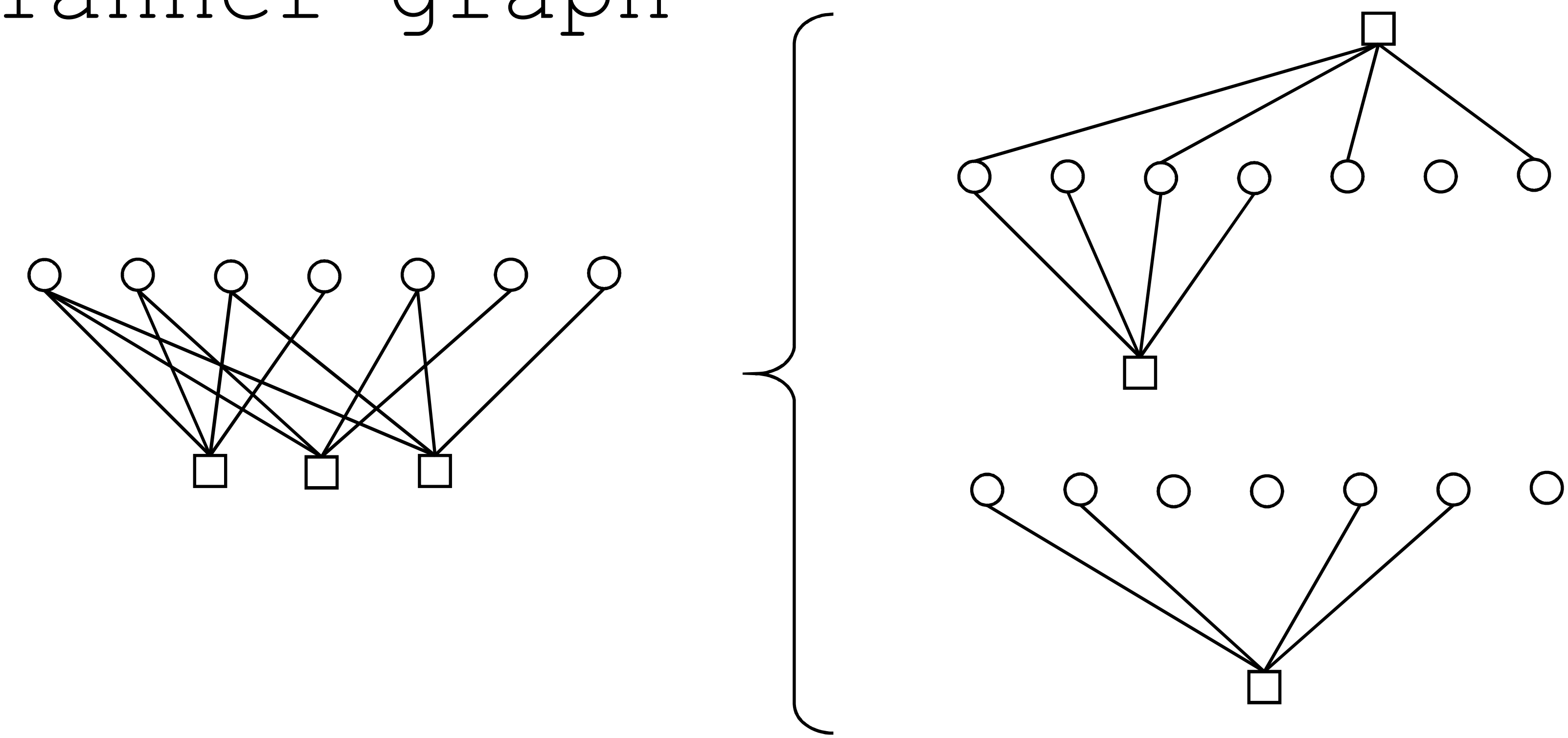
Goal:

- A small number of crossing gates.
- Short depth.



Graph with 100 vertices with degree 8

Planar decomposition of the Tanner graph



Degree = 4

planar layers = 2

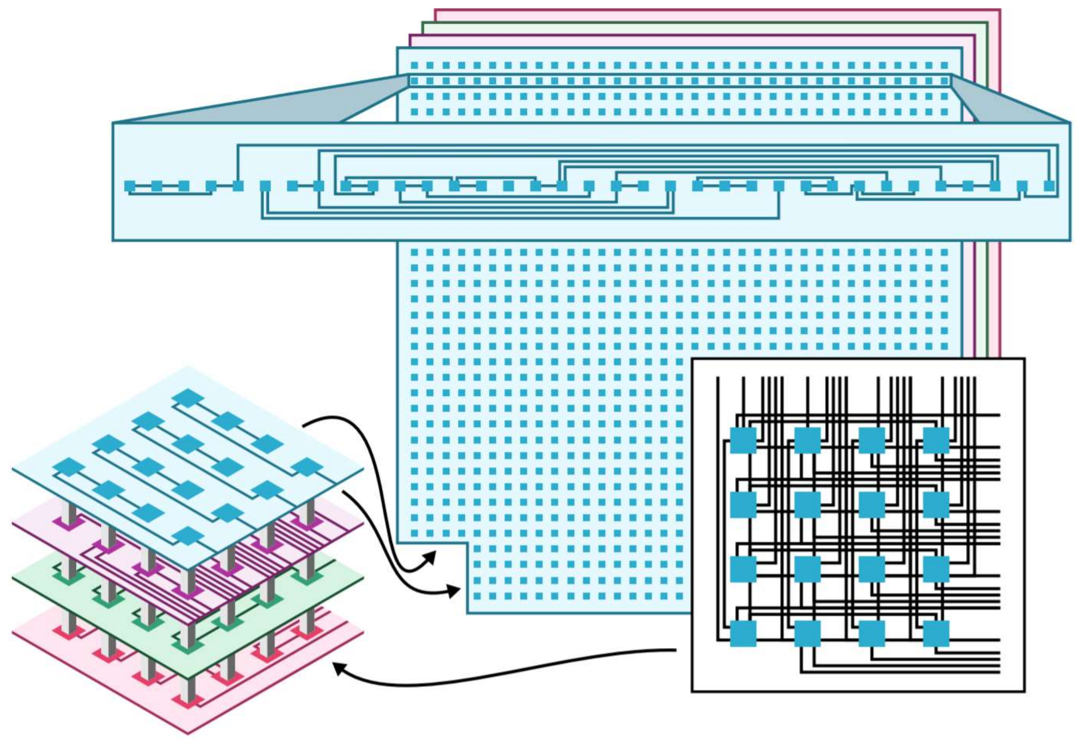
ℓ-planar layout

Assumptions:

- CSS code.
- Degree(Tanner graph) = δ.

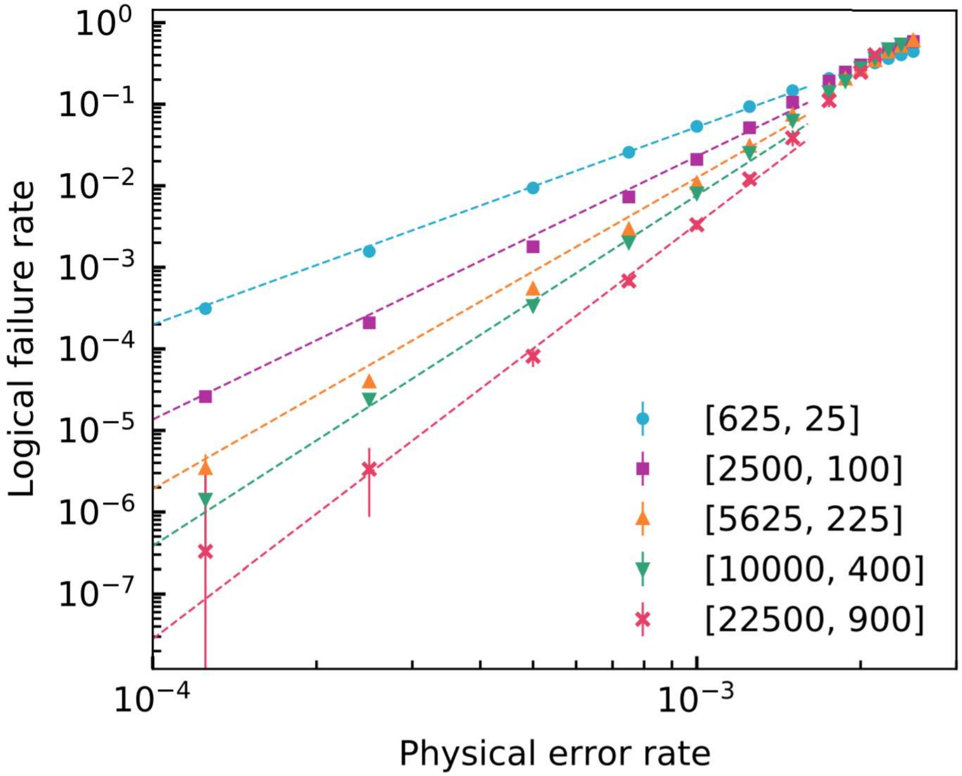
We constructed a syndrome extraction circuit with:

- $\left\lceil \frac{\delta}{2} \right\rceil$ planar layers of gates.
- Depth = $2\delta + 2$.



Improved depth for HGP codes.

Numerical results



Noise threshold:

0.28% (instead of 0.7% for surface codes)

physical qubits per logical qubit:

49 (instead of thousands for surface codes)

Logical failure rate	10^{-9}	10^{-12}	10^{-15}
Logical qubits	1600	6400	18 496
Surface code physical qubits	387 200	2 880 000	13 354 112
HGP code physical qubits	78 400	313 600	906 304
Improvement using HGP codes	4.94×	9.18×	14.73×