Soft Collinear Effective Theory & Collider Physics

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Amplitudes
IAS
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Soft Collinear Effective Theory (SCET)

EFT for hard interactions which produce energetic (collinear) and soft particles.

Higgs production, DY, ...
Jet Physics
Jet Substructure
B-Decays and CP violation
Quarkonia Production
TMDs / Nuclear Physics
(Heavy Ion collisions)

builds on extensive past literature (CSS factorization, exclusive fact, …)

Infrared Structure of Gauge Theory
Factorization for Collider Processes
Higher order Resummation
Gauge theory at Subleading Power
Subtractions for Fixed Order QCD
High Energy Limit / Regge phenomena

“EFT for Collider Physics”

Bauer, Fleming, Luke, Pirjol, IS ’00, ’01

builds on extensive past literature (CSS factorization, exclusive fact, …)
Outline

SCET Formalism:
- Introduction to SCET & Factorization
- Wilson Lines, Large Logs and Renormalization Group
- Forward Scattering & Factorization Violation

Collider Physics Applications:
- High Precision Resummation $e^+e^-$
- High Precision Resummation $pp$
- Power Corrections
- Amplitudes in the Regge Limit
Non-perturbative Factorization:

parton distributions
\[ d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F \]

hadronization: fragmentation fns., soft hadronization, ...
(QFT operators)

universal hadronic dynamics
via
universal hadronic functions

perturbative cross section

hadronization:

\[ f_a f_b \otimes \hat{\sigma} \otimes F \]

Non-perturbative Factorization:
Non-perturbative Factorization:

parton distributions

\[ d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F \]

hadronization:
fragmentation fns.,
soft hadronization, ... (QFT operators)

universal hadronic dynamics
via
universal hadronic functions

time
Perturbative Factorization: for multi-scale problems with $N$ jets

$$\hat{\sigma}_{\text{fact}} = I_a I_b \otimes H \otimes \prod_i J_i \otimes S$$

- $\mu_B$
- $\mu_H$
- $\mu_J$
- $\mu_S$

$\mu_p \approx \Lambda_{\text{QCD}}$

$\mu_S \approx p_{\text{soft}}$

$\mu_H \approx Q$

$J_1$

$J_2$

$J_3$

$E$

$\mu_H$

$\mu_B$

$\mu_J$, $\mu_B$

$\mu_S$

$\mu_p$
Perturbative Factorization: for multi-scale problems with fixed # jets

\[ \hat{\sigma}_{\text{fact}} = I_a I_b \otimes H \otimes \prod_i J_i \otimes S \]

\[ \mu_B \quad \mu_H \quad \mu_J \quad \mu_S \]

Perturbative Universality

- \( H \) determined by hard process, independent of jet radius, etc.
- \( J_i, I_{a,b} \) splitting and virtual effects for parton \( i \), encode jet dynamics, independent of \( H \)
- \( S \) soft radiation, all partons contribute, eikonal Feynman rules universal soft dynamics

Scale dependence \( \leftrightarrow \) RGE sums up logarithms \( \log \left( \frac{\mu_H}{\mu_S} \right) \),…
Perturbative QCD Results:

fixed order:

\[ \hat{\sigma} = \sigma_0 \left[ 1 + \alpha_s + \alpha_s^2 + \ldots \right] \]

\[ = \text{LO} + \text{NLO} + \text{NNLO} + \ldots \]

SCET anomalous dimensions:

resummation of large (double) logs

\[ L = \log(\ldots) \]

\[ \log \left( \frac{\Lambda_{\text{QCD}}}{Q} \right), \]

\[ \log \left( \frac{p_T}{Q} \right), \ldots \]

\[ \ln \hat{\sigma}(y) = \sum_k L(\alpha_s L)^k + \sum_k (\alpha_s L)^k + \sum_k \alpha_s (\alpha_s L)^k + \sum_k \alpha_s^2 (\alpha_s L)^k + \ldots \]

\[ = \text{LL} + \text{NLL} + \text{NNLL} + \text{N}^3\text{LL} + \ldots \]
Soft Collinear Effective Theory

Fields for various Modes:

\[ \xi_{n_1}, A_{n_1}^\mu \]
\[ \xi_{n_2}, A_{n_2}^\mu \]
\[ \xi_{n_3}, A_{n_3}^\mu \]

\[ \mu_p \]
\[ \mu_B \]
\[ \mu_S \]
\[ \mu_H \]
\[ J_1 \]
\[ J_2 \]
\[ J_3 \]
\[ \ell^+ \]
\[ \ell^- \]

Dominant contributions from isolated regions of momentum space

n-collinear
\[ (n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda) \]

soft
\[ (n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q\lambda^k \]

\[ k \geq 1 \]

Power counting \( \lambda \ll 1 \)
Key Simplifying Principle is to Exploit the Hierarchy of Scales

\[ \mathcal{L} = \sum_i C_i O_i \]

\[ d\sigma = \int \text{(phase space)} \left| \sum_i C_i \langle O_i \rangle \right|^2 = \sum_j H_j \otimes \text{(longer distance dynamics)}_j \]

Wilson coefficients + operators at \( \mu_H \)

Amplitudes!
Hard-collinear factorization

Hard scale operators from building block fields:

\[ \mathcal{O} = (B_{n_a \perp})(B_{n_b \perp})(B_{n_1 \perp})(\bar{\chi}_{n_2})(\chi_{n_3}) \]

“quark jet” \( \chi_n = (W_n^\dagger \xi_n) \)

“gluon jet” \( B_{n \perp}^\mu = \frac{1}{g} [W_n^\dagger i D_\perp^\mu W_n] \) or \( B_{n \perp}^{A\mu} = \frac{1}{g} \frac{1}{\bar{n} \cdot \partial_n} \bar{n}_\nu G_n^{B\nu \mu} \mathcal{W}_n^{BA} \)
Often convenient to use helicity basis for building blocks to make it easier to match to amplitude calculations

\[ \mathcal{B}_{n\perp}^{\pm} \quad J_{n\bar{n}}^{\pm} \]

see 1508.02397
\[ d\sigma = B_{a,b} \otimes H_j \otimes \prod_i J_i \otimes \text{(longer distance dynamics)} \]
Soft-collinear factorization

Soft radiation knows only about bulk properties of radiation in the jets (color & direction)

Soft Wilson lines: \( (S_{n_a} S_{n_b} S_{n_1} S_{n_2} S_{n_3}) \)

Soft function \( S = \) Matrix Elements of Soft Wilson Lines
Leading Power Glauber Lagrangian:

\[ \mathcal{L}_{G}^{(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} O_{iB}^{B} \frac{1}{p_{\perp}^{2}} O_{BC}^{n} \frac{1}{p_{\perp}^{2}} O_{jC}^{\bar{n}} + \sum_{n} \sum_{i,j=q,g} O_{n}^{iB} \frac{1}{p_{\perp}^{2}} O_{j}^{nB} \]

(3 rapidity sectors)

\( n-\bar{n} \) fwd. scattering

Glauber potential

\[ \frac{1}{q_{\perp}^{2}} \]

(2 rapidity sectors)

\( n-S \) fwd. scattering

Lipatov vertex

constructed from top-down matching: QCD → SCET
Leading Power Glauber Lagrangian:

\[
\mathcal{L}_{G}^{(0)} = \sum_{n, \bar{n}} \sum_{i, j=q, g} \mathcal{O}_{n}^{iB} \frac{1}{P_{\perp}^{2}} \mathcal{O}_{s}^{BC} \frac{1}{P_{\perp}^{2}} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i, j=q, g} \mathcal{O}_{n}^{iB} \frac{1}{P_{\perp}^{2}} \mathcal{O}_{s}^{jB} \]

In Hard Scattering

- Glauber Lagrangian can spoil factorization by coupling sectors in a non-factorizable manner. (Describes ONLY non-trivial fact. violation.)

- Its effects often cancel due to unitarity (summing over inclusive enough final states) or by exponentiating into an unobservable phase.

- Lagrangian can be used to systematically study non-factorizable Collider physics phenomena. (eg. super leading logs, “underlying event”)

In Forward Scattering \( s \gg |t| \)

- Describes the leading scattering process. Old and well studied limit.

- SCET provides top-down EFT description, new tools
SCET Lagrangian at leading power

\[ \mathcal{L} = \mathcal{L}^{(0)}_{\text{dyn}} + \mathcal{L}^{(0)}_{\text{hard}} + \mathcal{L}^{(0)}_G \]

- Dynamics of infrared modes
- Hard Scattering operators (typically once)
- Glauber gluon exchange (only factorization violating term)

- \( \mathcal{L}^{(0)}_{\text{hard}} = \sum_i C_i^{(0)} \mathcal{O}_i^{(0)} \) Leading operators for a given process
- \( \mathcal{L}^{(0)}_{\text{dyn}} = \sum_n \mathcal{L}_n^{(0)} + \mathcal{L}^{(0)}_{\text{soft}} \) Collinear and Soft dynamics (Factorizes after soft-collinear decoupling)

Copies of QCD* give dynamics in different sectors, with hard operators providing the only connection between sectors
SCET Lagrangian at leading power

\[ \mathcal{L} = \mathcal{L}^{(0)}_{\text{dyn}} + \mathcal{L}^{(0)}_{\text{hard}} + \mathcal{L}^{(0)}_{\text{G}} \]

- Dynamics of infrared modes
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- \( \mathcal{L}^{(0)}_{\text{hard}} = \sum_i C_i^{(0)} \mathcal{O}_i^{(0)} \)
- \( \mathcal{L}^{(0)}_{\text{dyn}} = \sum_n \mathcal{L}_n^{(0)} + \mathcal{L}^{(0)}_{\text{soft}} \)

Leading operators for a given process

Collinear and Soft dynamics (Factorizes after soft-collinear decoupling)

Factorization

\[ d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F \]
\[ \hat{\sigma}_{\text{fact}} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S \]
Applications
Dijet production \( e^+ e^- \rightarrow 2 \) jets

\[
\tau = 1 - T
\]

\( \tau \ll 1 \)

\[
\frac{d\sigma}{d\tau} = \sigma_0 H(Q, \mu) Q \int d\ell \, d\ell' \, J_T(Q^2 \tau - Q\ell, \mu) S_T(\ell - \ell', \mu) F(\ell')
\]

- hard function
- jet functions (combined)
- perturbative soft function
- non-perturbative soft function

\[
\frac{d\sigma^{\text{nonsingular}}}{d\tau} + \frac{d\sigma^{\text{singular}}}{d\tau}
\]

\begin{align*}
&\text{nonsingular} \\
&\text{singular} \\
&\text{total} \\
&\text{singular subt.} \\
&\text{nonsingular subt.}
\end{align*}
\[ \alpha_s(m_Z) \text{ from Thrust} \quad e^+e^- \rightarrow \text{jets} \quad \text{Aim at 1\% precision} \]

- \[ \mathcal{O}(\alpha_s^3) + N^3\text{LL} + \frac{\Omega_1}{Q\tau} \text{ power correction} + \text{renormalon subtractions, R-RGE} \]

  + full treatment of \{peak, tail, multijet\} + QED effects + b-mass effects + global fit, various Q’s

\[
\frac{1}{\sigma} \frac{d\sigma}{d\tau} = S = S^{\text{pert}} \otimes S^{\text{mod}}
\]
\( \alpha_s(m_Z) \) from Thrust \( e^+e^- \rightarrow \text{jets} \) [Aim at 1% precision]

- \( \mathcal{O}(\alpha_s^3) + N^3\text{LL} + \frac{\Omega_1}{Q_\tau} \) power correction + renormalon subtractions, R-RGE

  + full treatment of \{peak, tail, multijet\} + QED effects + b-mass effects + global fit, various Q’s

factorize pert. & nonperturbative soft effects:

\[ S = S^{\text{pert}} \otimes S^{\text{mod}} \]
$\alpha_s(m_Z)$ from Thrust  
\[ e^+e^- \rightarrow \text{jets} \]  
Aim at 1% precision  
Becher, Schwartz `09  
Abbate, Fickinger,  
Hoang, Mateu, I.S. `10  
with $O(\alpha_s^3)$ from  
Gehrmann-De Ridder et al.  
& Weinzierl (`07–`09)  

\[ O(\alpha_s^3) + N^3\text{LL} + \frac{\Omega_1}{Q \tau} \text{power correction} + \text{renormalon subtractions, R-RGE} \]  
full treatment of \{peak, tail, multijet\} + QED effects + b-mass effects + global fit, various Q’s  

factorize pert. & nonperturbative soft effects:  
\[ S = S^{\text{pert}} \otimes S^{\text{mod}} \]
\[ \alpha_s(m_Z) \text{ from Thrust} \]

\[ e^+e^- \rightarrow \text{jets} \quad \text{Aim at 1\% precision} \]

- \( \mathcal{O}(\alpha_s^3) + N^3\text{LL} + \frac{\Omega_1}{\mathcal{Q}\mathcal{T}} \text{ power correction} + \text{renormalon subtractions, R-RGE} \)
- \text{full treatment of \{peak, tail, multijet\} + QED effects + b-mass effects + global fit, various Q's} \]

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \]

factorize pert. & nonperturbative soft effects:

\[ S = S^{\text{pert}} \otimes S^{\text{mod}} \]

Becher, Schwartz `09
Abbate, Fickinger, Hoang, Mateu, I.S. `10
with \( \mathcal{O}(\alpha_s^3) \) from Gehrmann-De Ridder et al. & Weinzierl (`07~`09)
\( \alpha_s(m_Z) \) from Thrust\hspace{1cm} e^+e^- \rightarrow \text{jets}\hspace{1cm} \text{Aim at 1\% precision}\hspace{1cm} \text{Becher, Schwartz '09}\hspace{1cm} \text{Abbate, Fickinger, Hoang, Mateu, I.S. '10}\hspace{1cm} \text{with } \mathcal{O}(\alpha_s^3) \text{ from Gehrmann-De Ridder et al. & Weinzierl ('07-'09)}

\( \mathcal{O}(\alpha_s^3) + N^3LL + \frac{\Omega_1}{Q_T} \) power correction + renormalon subtractions, R-RGE

+ full treatment of \{peak, tail, multijet\} + QED effects + b-mass effects + global fit, various Q's

\[ \frac{\tau}{\sigma} \frac{d\sigma}{d\tau} \]

Fit at \( N^3LL' \) for \( \alpha_s(m_Z) \) & \( \Omega_1 \)

theory scan error

[Graph showing data points and fitted curves for \( \tau/\sigma \) vs. \( \tau \) with various experimental collaborations represented.]
Consistency checks

QED & b-mass effects small

\[ \Delta \alpha_s(m_Z) = -0.0005 \]

Thrust vs. thrust moments

Agreement beyond the fit region

Thrust vs. C-parameter
**Small $\alpha_s(m_Z)$?**

**thrust 2010:** $\alpha_s(m_Z) = 0.1135 \pm 0.0011$

**PDG 2023:** $\alpha_s(m_Z) = 0.1180 \pm 0.0009$

**thrust 2023 reanalysis:** Bell, Lee, Makris, Talbert, Yan (2023), also small $\alpha_s$

? Power corrections for 2-jets ($\Omega_1$) versus 3-jets ($\neq \Omega_1$)

Luisoni, Monni, Salam (2021)

Caola, Ravasio, Limatola, Melnikov, Nason, Ozcelik (’21-’22)

Nason, Zanderighi (2023)

Benitez-Rathgeb, Hoang, Mateu, IS, Vita (2024)

![Graph showing models for 3-jet show change to $\Omega_1$](image1)

![Graph showing fits in restricted dijet region give small $\alpha_s$](image2)
Energy Energy Correlators and power corrections

Exciting class of observables for collider physics (both theoretically and experimentally)

$$\frac{d\Sigma}{d\chi} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta(\chi - \theta_{ij})$$

perturbative QCD

$$\frac{1}{\sigma_0} \frac{d\Sigma}{d\chi} = \frac{1}{\sigma_0} \left( \frac{d\hat{\Sigma}}{d\chi} \right) + \frac{2}{\sin^3 \chi} \frac{\Omega_1}{Q}$$

universal power correction describing hadronization

Korchemsky, Sterman (1999)

$$\Omega_1 \equiv \frac{1}{N_c} \langle 0 | \text{tr} \bar{Y}_n Y_n^\dagger \mathcal{E}_T(0) Y_n \bar{Y}_n | 0 \rangle$$

modified perturbative QCD

$$\sum_n c_n(\chi, \mu/Q) \alpha^n_s(\mu) + d_n\text{-series}$$

scheme change to remove leading renormalon

$$\Omega_1(R) = \bar{\Omega}_1 - R \sum_n d_n(\mu/R) \alpha^n_s(\mu)$$

MS scheme $\implies$ $R$ scheme

Hoang, I.S.(2007); Hoang Kluth(2008); Schindler, Sun, I.S. (2023)
Better convergence
• Agrees with data!
• NOT a fit

Conﬁrms universality
\(\Omega_1\), \(\alpha_s\) from thrust ﬁt

• Better convergence
• Agrees with data!
• Confirms \(\Omega_1\) universality

EEC With Power Corrections

Schindler, Sun, I.S. (2023)

\(R\) scheme: \(\Omega_1\), \(\alpha_s\) from thrust ﬁt

\[
\frac{1}{\sigma} \frac{d\Sigma}{d\chi} = \frac{1}{\sigma} \frac{d\sigma}{d\chi}
\]

Schindler, Sun, I.S. (2023)

\(R\) scheme: \(\Omega_1\), \(\alpha_s\) from thrust ﬁt

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\frac{1}{\sigma} \frac{d\Sigma}{d\chi} = \frac{1}{\sigma} \frac{d\sigma}{d\chi}
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\(\Omega_1\), \(\alpha_s\) from thrust ﬁt

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\frac{1}{\sigma} \frac{d\Sigma}{d\chi} = \frac{1}{\sigma} \frac{d\sigma}{d\chi}
\]

\(\Omega_1\), \(\alpha_s\) from thrust ﬁt

\[
\frac{1}{\sigma} \frac{d\Sigma}{d\chi} = \frac{1}{\sigma} \frac{d\sigma}{d\chi}
\]
Projected N-point Energy Correlators

\[ \langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_N \rangle \]

\[ \theta_L = \max(\theta_{ij}) \]

Power Corrections

Lee, Pathak, I.S., Sun (2024)

\[ \frac{1}{\sigma} \frac{d\sigma^{[N]}}{d\theta_L} = \frac{1}{\sigma} \frac{d\hat{\sigma}^{[N]}}{d\theta_L} + \frac{4N}{2^N} \frac{\Omega_1}{Q \sin^3 \theta_L} \]

\[ x_L = (1 - \cos \theta_L)/2 \]
Projected N-point Energy Correlators \( e^+e^- \)

Chen, Moult, Zhang, Zhu (2020)

\[
\langle \mathcal{E}_1 \mathcal{E}_2 \cdots \mathcal{E}_N \rangle
\]
\[
\theta_L = \max(\theta_{ij})
\]

Power Corrections

Lee, Pathak, I.S., Sun (2024)

\[
\frac{1}{\sigma} \frac{d\sigma^{[N]}}{d\theta_L} = \frac{1}{\sigma} \frac{d\hat{\sigma}^{[N]}}{d\theta_L} + \frac{4N}{2^N} \frac{\overline{\Omega}_1}{Q \sin^3 \theta_L}
\]

Resummation \( \theta_L \ll 1 \)

Dixon, Moult, Zhu (2019)

Chen, Moult, Zhang, Zhu (2020)

\[
\frac{1}{\sigma} \frac{d\sigma^{[N]}}{d\theta_L} \sim \int dx \, x^N \hat{J}^{[N]} \cdot \hat{H}
\]
**EEC in back-to-back limit**

\[ N^4 \text{LL} \quad \text{Duhr, Mistlberger, Vita (2022)} \]


Key new ingredients:

- OPE for TMD PDFs and FFs to 3-loops (all channels)  
  Ebert, Mistlberger, Vita (2020)  
  Luo, Yang, Zhu, Zhu (2020)

\[
\begin{align*}
  f_{i/h}^{pert}(x, b_T, \mu, \zeta) &= \sum_j \int \frac{dy}{y} C_{ij}(x/y, b_T, \mu, \zeta) f_{j/h}(y, \mu) \\
  \gamma_{\zeta}^q[\alpha_s] &= \alpha_s \gamma_{\zeta}^{(1)} + \alpha_s^2 \gamma_{\zeta}^{(2)} \\
  &+ \alpha_s^3 \gamma_{\zeta}^{(3)} + \alpha_s^4 \gamma_{\zeta}^{(4)} + \ldots 
\end{align*}
\]

(3-loop result: Li, Zhu 2016; Vladimirov 2016)

- CS kernel to 4-loops  
  Duhr, Mistlberger, Vita (2022)  
  Moult, Zhu, Zhu (2022)

\[
\begin{array}{c|c|c|c|c|c}
\text{Accuracy} & \mathcal{H}, \mathcal{J} & \Gamma_{\text{cusp}}(\alpha_s) & \gamma_{H}^q(\alpha_s) & \gamma_{T}^q(\alpha_s) & \beta(\alpha_s) \\
\hline
\text{LL} & \text{Tree level} & 1\text{-loop} & - & - & 1\text{-loop} \\
\text{NLL} & \text{Tree level} & 2\text{-loop} & 1\text{-loop} & 1\text{-loop} & 2\text{-loop} \\
\text{NLL}' & 1\text{-loop} & 2\text{-loop} & 1\text{-loop} & 1\text{-loop} & 2\text{-loop} \\
\text{NNLL} & 1\text{-loop} & 3\text{-loop} & 2\text{-loop} & 2\text{-loop} & 3\text{-loop} \\
\text{NNLL}' & 2\text{-loop} & 3\text{-loop} & 2\text{-loop} & 2\text{-loop} & 3\text{-loop} \\
\text{N^3LL} & 2\text{-loop} & 4\text{-loop} & 3\text{-loop} & 3\text{-loop} & 4\text{-loop} \\
\text{N^3LL}' & 3\text{-loop} & 4\text{-loop} & 3\text{-loop} & 3\text{-loop} & 4\text{-loop} \\
\text{N^4LL} & 3\text{-loop} & 5\text{-loop} & 4\text{-loop} & 4\text{-loop} & 5\text{-loop} \\
\text{N^4LL}' & 4\text{-loop} & 5\text{-loop} & 4\text{-loop} & 4\text{-loop} & 5\text{-loop} \\
\end{array}
\]

**Resummation \( \chi \to \pi \)**

\[
\frac{e^+e^- \to \gamma^* \to \text{hadrons}}{\alpha_s(m_Z) = 0.118}
\]

\[\chi^\circ \quad \text{v} \quad \text{x} \quad 162 \quad 164 \quad 166 \quad 168 \quad 170 \quad 172 \quad 174 \quad 176 \quad 178 \quad 180\]

\[\begin{array}{c}
\text{NNLL} \\
\text{N^3LL} \\
\text{N^3LL}' \\
\text{N^4LL} \\
\text{N^4LL}'
\end{array}\]
High Precision Resummation $pp$
Higgs $q_T$ spectrum

- **Gluon Fusion**
- **Higgs recoils against Jets**

**Resummation**

$$q_T \frac{d\sigma}{dq_T} = \sum_{i=1}^{\infty} \alpha_i^i \sum_{j=0}^{2i-1} \ln^j \frac{q_T}{m_H} + \mathcal{O}(q_T^2)$$

**Non-perturbative**

**Transition Region**

**Fixed Order**

$$q_T \frac{d\sigma}{dq_T} = \alpha_s h_1 + \alpha_s^2 h_2 + \alpha_s^3 h_3 + \ldots$$

**LO**, **NLO**, **NNLO**

**nonsingular**
Small $q_T$ factorization

\[
\frac{d^2\sigma}{dq_TdY} = W^{(0)}(q_T,Y) + W^{\text{non.sing.}}(q_T,Y)
\]

Collins, Soper, Sterman SCET

\[
W^{(0)}(q_T,Y) = \int \frac{d^2\tilde{b}}{(2\pi)^2} e^{i\tilde{b} \cdot \vec{q}_T} W(x_a, x_b, m_H, \tilde{b})
\]

$\mu \approx Q$

$\mu \approx 1/b$

$\mu \approx \Lambda_{\text{QCD}}$

$\nu$ = “rapidity” RGE scale

$\mu =$ invariant mass scale

\[
W(x_a, x_b, m_H, \tilde{b}) = \left[ C_V(m_t, m_H, \mu) \right]^2 S(\tilde{b}, \mu, \nu) B_{g/N_1}^{\alpha\beta}(x_a, Q, \tilde{b}, \mu, \nu) B_{g/N_2}^{\alpha\beta}(x_b, Q, \tilde{b}, \mu, \nu)
\]

\[
B_{g/N}^{\alpha\beta}(x, Q, \tilde{b}, \mu, \nu) = \sum_k \int \frac{d\xi}{\xi} \mathcal{I}_{gk}^{\alpha\beta}(\frac{x}{\xi}, \tilde{b}, \mu, \nu) f_{k/N}(\xi, \mu) + \mathcal{O}(\Lambda_{\text{QCD}}^2 \tilde{b}^2)
\]
Small $q_T$ factorization

\[ \frac{d^2 \sigma}{dq_T dY} = W^{(0)}(q_T, Y) + W^{\text{non.sing.}}(q_T, Y) \]

Single Scale Functions:

- Hard function: $\ln \frac{Q^2}{\mu^2}$
- Beam function: $\ln(b^2 \mu^2)$, $\ln \frac{Q^2}{\nu^2}$
- Soft function: $\ln(b^2 \mu^2)$, $\ln(b^2 \nu^2)$

\[ W(x_a, x_b, m_H, \vec{b}) = \left[ C_V(m_t, m_H, \mu) \right]^2 \left( S(\vec{b}, \mu, \nu) B^\alpha_{g/N_1} (x_a, Q, \vec{b}, \mu, \nu) B^\beta_{g/N_2} (x_b, Q, \vec{b}, \mu, \nu) \right) \]

Resummation:

\[ \ln W = L \sum_k (\alpha_s L)^k + \sum_k (\alpha_s L)^k + \alpha_s \sum_k (\alpha_s L)^k + \alpha_s^2 \sum_k (\alpha_s L)^k \]

\[ L = \ln(m_H b) \]

LL \quad \text{NLL} \quad \text{NNLL} \quad \text{N3LL} \]
The Higgs $p_T$ Spectrum and Total Cross Section with Fiducial Cuts at $N^3LL'+N^3LO$

Consider $gg \to H \to \gamma\gamma$ with ATLAS fiducial cuts:

$$p_T^{\gamma_1} \geq 0.35 \, m_H , \quad p_T^{\gamma_2} \geq 0.25 \, m_H , \quad |\eta^\gamma| \leq 2.37 , \quad |\eta^\gamma| \notin [1.37, 1.52]$$

$$\sigma_{\text{fid}} = \int dq_T dY \, A(q_T, Y; \Theta) \, W(q_T, Y)$$

Fiducial cross section measures deviation from SM gluon-fusion:

$$A = \text{acceptance}$$

Acceptance causes a need for resummation to obtain Fiducial cross section

Cutting on photon $p_T$ induces large logs
Resummation Inputs

- **Three-loop soft and hard function** … includes in particular the three-loop virtual form factor
  [Li, Zhu, ’16] [Baikov et al. ’09; Lee et al. ’10; Gehrmann et al. ’10]

- **Three-loop unpolarized and two-loop polarized beam functions**
  [Ebert, Mistlberger, Vita ’20; Luo, Yang, Zhu, Zhu ’20]
  [Luo, Yang, Zhu, Zhu ’19; Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov ’19]

- **Four-loop cusp, three-loop noncusp anomalous dimensions**
  [Brüser, Grozin, Henn, Stahlhofen ’19; Henn, Korchemsky, Mistlberger ’20; v. Manteuffel, Panzer, Schabinger ’20] [Li, Zhu, ’16; Moch, Vermaseren, Vogt ’05; Idilbi, Ma, Yuan ’06; Vladimirov ’16]

- **Four-loop CS kernel, from conformal relation between UV & rapidity anom. dims**
  [Vladimirov, 1610.05791 → Duhr, Mistlberger, Vita, 2205.02242; Moult, Zhu, Zhu, 2205.02249]

Fixed Order Inputs

- **At NNLO**, renormalize & implement bare analytic results for \( W(q_T,Y) \)
  [Dulat, Lionetti, Mistlberger, Pelloni, Specchia ’17]

- **At N^3 LO**, use existing binned NNLO_{1} results from NNLOjet
  [Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier ’15-16; as used in Chen et al. ’18; Bizoń et al. ’18]

- **Use N^3 LO total inclusive cross section as additional fit constraint on underflow**
  [Mistlberger ’18]

**Implemented in C++ Library “SCETlib”**
Higgs Results

The fiducial $q_T$ spectrum at $N^3LL'+N^3LO$

![Graph showing the fiducial $q_T$ spectrum at $N^3LL'+N^3LO$.](image)

The total fiducial cross section at $N^3LO$ and $N^3LL'+N^3LO$

![Graph showing the total fiducial cross section at $N^3LO$ and $N^3LL'+N^3LO$.](image)

Precision and convergence improved

Billis, Dehnadi, Ebert, Michel, Tackmann

(2021)
**Drell-Yan Results**

**Fixed Order Inputs**

- **Fiducial** $Z$+jet MC data at $\mathcal{O}(\alpha_s^2)$ from MCFM
  

- **Very recently:** Precise fiducial $Z$+jet MC data at $\mathcal{O}(\alpha_s^3)$ from NNLOjet
  
  [Chen et al., 2203.01565 – many thanks to the NNLOjet collaboration for providing the raw data.]
Power Corrections
SCET beyond leading power

\[ \mathcal{L} = \sum_{p \geq 0} \mathcal{L}_{\text{dyn}}^{(p)} + \sum_p \mathcal{L}_{\text{hard}}^{(p)} + \mathcal{L}_{\text{G}}^{(0)} \]

\( \mathcal{O}(\lambda^p) \)

Dynamics of infrared modes

Hard Scattering operators (typically once)

Only leading term can spoil factorization

Subleading Lagrangians

Subleading Hard Scattering Operators
Sudakov suppression at subleading power?

\[ \frac{1}{\sigma_0} \frac{d\sigma^{(2), e^+ e^-}_{LL}}{d\tau} = \left( \frac{\alpha_s}{4\pi} \right) 8C_F \log(\tau) e^{-4C_F \left( \frac{\alpha_s}{4\pi} \right) \log^2(\tau)} + \frac{C_F}{(C_F - C_A) \log(\tau)} \left( e^{-4C_F \left( \frac{\alpha_s}{4\pi} \right) \log^2(\tau)} - e^{-4C_A \left( \frac{\alpha_s}{4\pi} \right) \log^2(\tau)} \right) \]

\begin{align*}
\text{Soft Quark Sudakov} & \\
\text{Collinear Quark Correction} & \\
\text{Soft Quark Correction} & \end{align*}

Proof (refactorization)

Beneke, Garny, Jaskiewicz, Strohm, Szafron, Vernazza, Wang '22

Conjecture

Moult, IS, Vita, Zhu '19

Endpoint singularities!

\[ \int_0^1 \frac{dx}{x} \]
Regge Amplitudes
Regge Amplitudes in SCET

\[ \mathcal{O}_n^A \frac{1}{P^2} \mathcal{O}_S^{AB} \frac{1}{P^2} \mathcal{O}_{\bar{n}}^B, \quad \mathcal{O}_n^A \frac{1}{P^2} \mathcal{O}_s^A \]

- \( \mathcal{L}^{(0)}_G \) Lagrangian gives forward scattering amplitudes \( s \gg |t| \) any loop order both planar and non-planar graphs any color channel large (Regge) logs from rapidity RGE

\[ \ln \left( \frac{s}{-t} \right) = \ln \left( \frac{s}{\nu^2} \right) + \ln \left( \frac{\nu^2}{-t} \right) \]

collinear loop soft loop

\[ \mathcal{A}(i, j) \]
- Glauber loops (simple)

\[ \propto (i\pi) \int \frac{d^{d-2}k_\perp}{(k_\perp^2)(k_\perp + q_\perp)^2} \]

- Same color as QCD box graph (includes \(8_A\))

Glauber \(\neq\) Reggeon
eg. Gluon Reggeization $\mathcal{A}(1,1)$

single Glauber exchange ($\mathcal{G}_A$)

rapidity divergent due to Wilson lines in collinear (soft) operators

$$\nu \frac{d}{d\nu} J_{(1)} = -\alpha(t) J_{(1)}$$
evolve: $\nu^2 = s \rightarrow \nu^2 = -t$
gives: $$\left( \frac{s}{-t} \right)^\alpha(t)$$

Turns out that there are no $1 \rightarrow (j \geq 2)$ transitions: $J_{(1)}^{\text{bare}} = Z_{(1,1)} J_{(1)}^{\text{ren}}$

provides natural definition for Gluon Regge trajectory at any loop order
General rapidity renormalization

\[ A = \sum_{ij} A(i,j) = \sum_{ij} J_\kappa(i) \otimes i S_\alpha^{\beta(i,j)} \otimes j J_\kappa'(j) = J_\kappa \cdot S \cdot J_\kappa' \]

\[ J^{\text{bare}} = J^{\text{ren}} \cdot Z_J \quad S^{\text{bare}} = Z_S \cdot S^{\text{ren}} \cdot Z_S \quad Z_S = Z_J^{-1} \]

2 Glauber exchange reproduces \( 1_S \) (pomeron), \( 8_S \), 27 BFKL equations

Again 2 possible ways to do calculation

\[ = 0 \]

collapse rule
3 Glauber exchange

\[ 8 \otimes 8 \otimes 8 = 1^2 \oplus 8^8 \oplus 10^4 \oplus \overline{10}^4 \oplus 27^6 \oplus 35^2 \oplus \overline{35}^2 \oplus 64 \]

Regge cuts at this order

Meaning: \( 27 \oplus \cdots \oplus 27, \) 6 copies of 27’s
Interesting complementarity to Reggeon EFTs

QCD $\rightarrow$ SCET $\rightarrow$ Wilson Line Regge EFT

$\mathcal{L}^G_0$

Caron-Huot, Gardi, Vernazza, ...

- Operator definition for impact factors
  $$\langle p | O_{n}^{A_1} \cdots O_{n}^{A_N} | p' \rangle$$
- Collinear loop calculations for rapidity logs
- Different structure for vanishing transitions $1 \rightarrow j$ vs. eg. $(j - 1) \rightarrow j$
- Signature and crossing symmetry not manifest from start

- Glauber operators can also be used to study factorization violation in hard scattering
Other Areas (no time to discuss)

- SCET for B-physics (SCET+HQET)
- SCET for quarkonia (SCET+NRQCD)
- SCET for jet substructure, often called SCET$_+$
- SCET for heavy-ions (SCET coupled to medium)
- SCET for electroweak logarithms
- SCET for Dark Matter annihilation
- SCET for gravitational scattering amplitudes

For further references see my SCET review in 50 yrs of QCD, 2212.11107
Summary:

- Precision Resummation

- Regge Amplitudes

- Nonperturbative corrections

- Power Corrections

\[ J_{P}^{(1)} \sim \frac{C_{f}^{(0)}}{2\omega_{a}} \bar{n}_{\bar{n},\omega_{b}} [S_{n}^\dagger S_{n}] \gamma^{\mu} \not{P}_{\perp} \not{n}_{\bar{n},\omega_{a}} \]

\[ J_{B}^{(1)} \sim (n^{\mu} + \bar{n}^{\mu}) \int d\omega_{c} C_{f}^{(1)}(Q, \omega_{c}) \bar{n}_{\bar{n},\omega_{b}} [S_{n}^\dagger S_{n}] \not{B}_{\perp n_{-} - \omega_{c} n_{+}} \]

SCET is a powerful tool for Collider Physics