Three Cheers For

"Shut Up And Calculate!"

(1951) Withheld 3rd Cheer because Democracy encourages Mediocrity.....

"Shut Up + Calculate!" is the best innoculation against mediocrity we have in fundamental physics!

"But what does Quantum Mechanics Mean?"

vs

"Shut Up And Calculate!"

Brave Asker of Big Questions

Problem-set addicted, Calculation Monkey-Nerd

"Shut Up + Calculate" $\sim$ "Actions speak louder than words"

"Shut Up + Calculate" $\sim$ "Where's the Beef?"

"Shut Up
+ Calculate"

$\sim$

A short-hand for the
most effective strategy
we have found to
attack + make real progress
on the biggest, hardest + most
"conceptual" mysteries of physics

Structure of Physical Laws

Mathematical Structure	+	Physical Interpretation: Dictionary + Grammar for relating Math. Struct. to real world
"Equations"		"Words"

The Creative Tension of Theoretical Physics

Grand Physical / Philosophical Principles ⟷ Insightfully Chosen Calculations (generating "theorists data")

Romantic Dreamer ⟷ Careful Pragmatist

Structure of Physical Laws

Mathematical Structure + Physical Interpretation: Dictionary + Grammar for relating Math. Struct. to real world

~ "Equations" ~ "Words"

Ultimately More Important!

* Imprecise + Slippery
* Lead to Complacency
* Words can change radically, while eqns are unaltered!

... Final Form of Laws Are So Simple ...

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

$$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

Surely we can find them with the right philosophical mindset — what is all this "theorists data/shut up calculate" crap?

Grand Physical / Philosophical Principles ⟷ Insightfully Chosen Calculations (generating "theorists data")

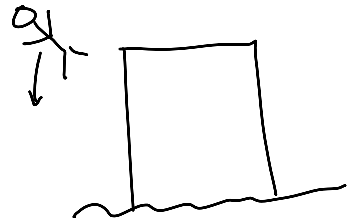
INDISPENSABLE!

But also, the easier part...

Where the heart of the action is

Why? For mysterious reasons, laws are Mathematical, Simple + Rigid.

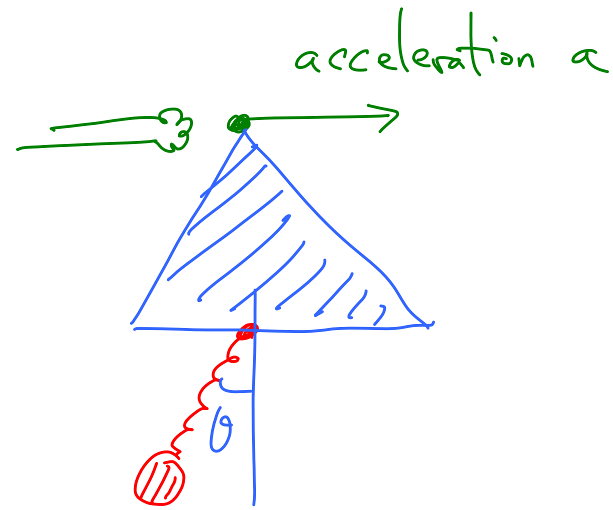
Much easier (+ more forgiving of little errors!) to guess the correct equations when you're in basin of attraction of the right ideas.

Einstein :  → General Relativity

Heisenberg : "Only Speak of Observables!" → Quantum Mechanics

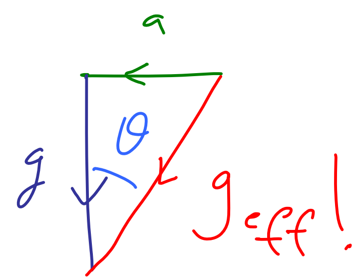
How I discovered eq. principle in 1986

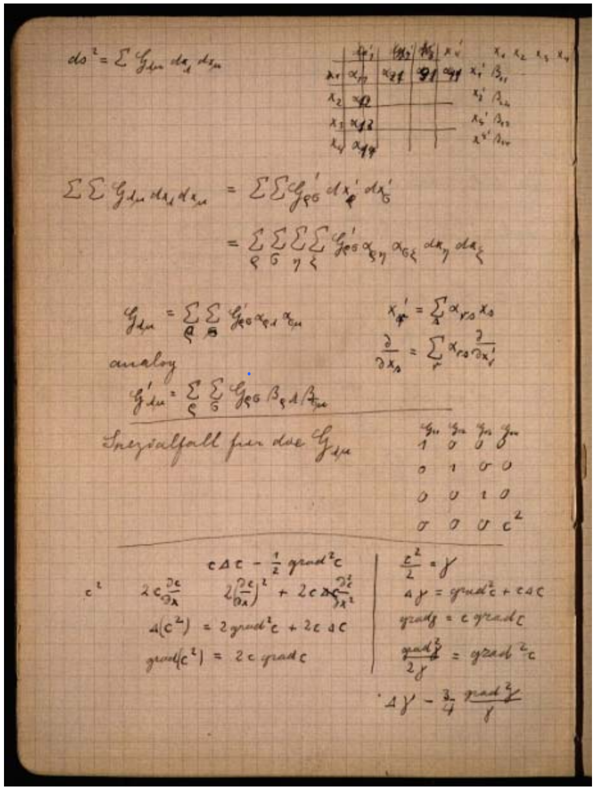
$g \downarrow$



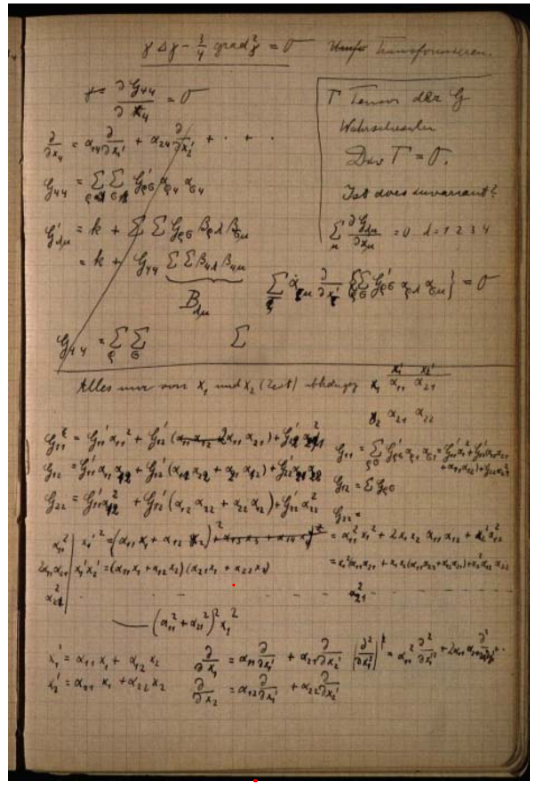
... After Lots of Hard Work ....

$$\tan\theta = \frac{a}{g} \;!$$

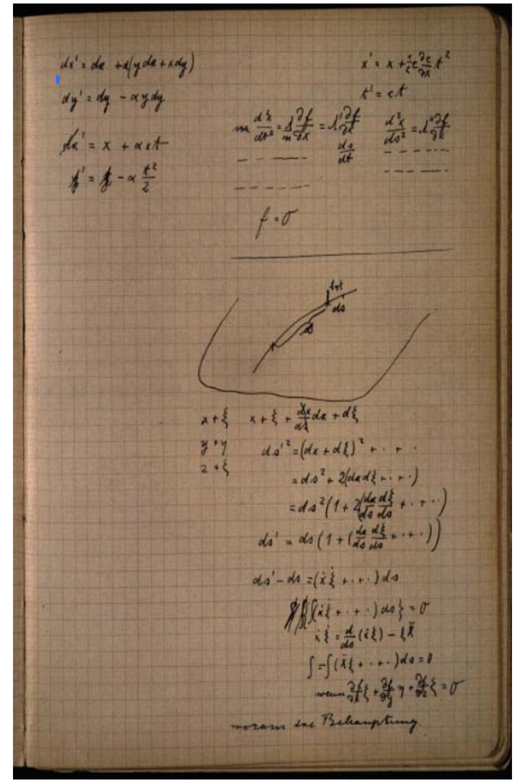




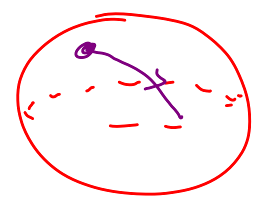
The "curved spacetime metric" appears

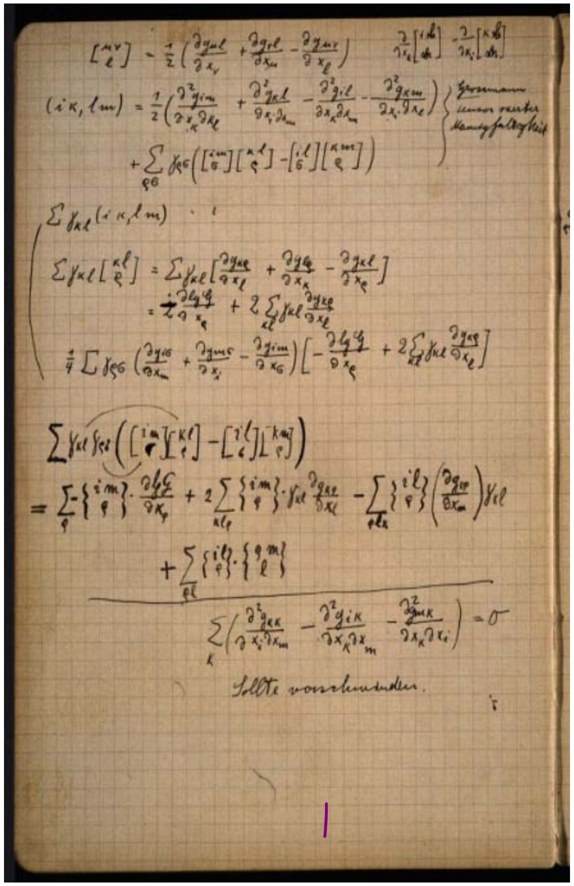


... practicing ...

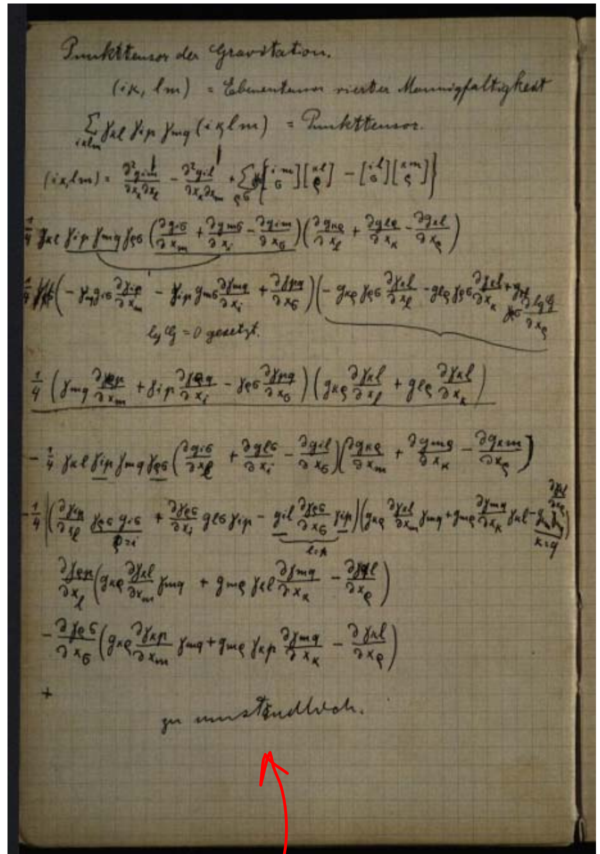


curved motion in space




Sollte verschwinden.

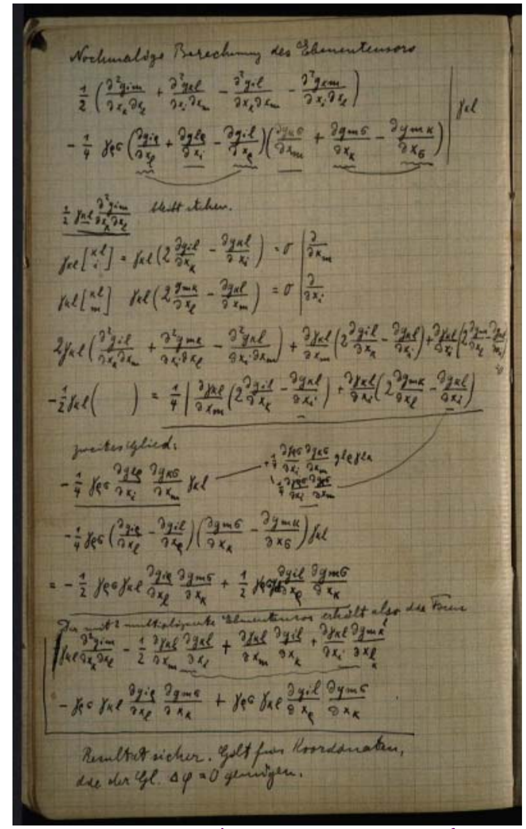
Riemann Tensor Appears!


Punkttensor der Gravitation.

(ix, lm) = Elementartensor vierter Mannigfaltigkeit

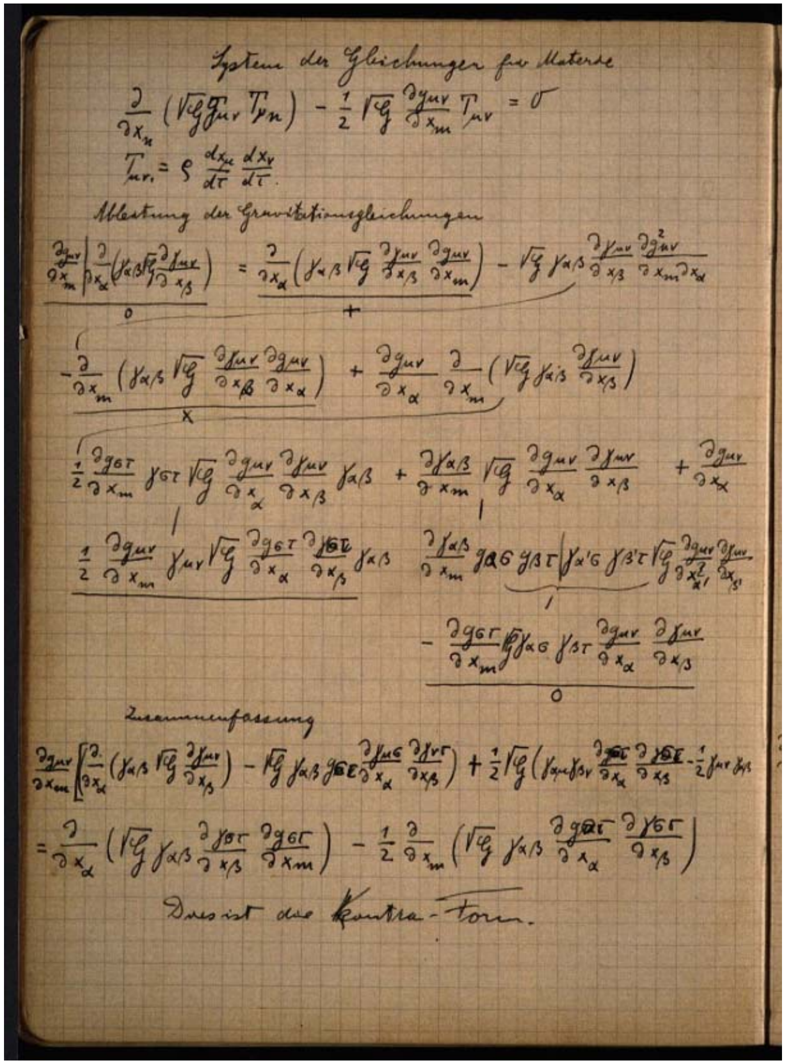
zu umständlich.

... "Too involved" ...


Nochmalige Berechnung des Elemententensors

Resultat sicher. Gilt für Koordinaten, die der Gl. $\Delta\varphi = 0$ genügen.

Achingly Close!


System der Gleichungen für Materie

$$\frac{\partial}{\partial x_n}\left(\sqrt{g}\, g_{\mu\nu} T_{\nu n}\right) - \frac{1}{2}\sqrt{g}\,\frac{\partial \gamma_{\mu\nu}}{\partial x_m} T_{\mu\nu} = 0$$

$$T_{\mu\nu} = \rho \frac{dx_\mu}{d\tau}\frac{dx_\nu}{d\tau}$$

Ableitung der Gravitationsgleichungen

Zusammenfassung

Dies ist die Kontra-Form.

"Entwurf Theory":

Huge + Confusing Mistake

← But so neat!

+ Fully justified by "philosophy" (irrelevant)

+ "physical arguments" (wrong)

Heisenberg's Journey

* Struggle to extend Bohr's model for hydrogen to Helium! (+ other realistic systems)
* Too hard: switch to toy model

$$\ddot{x} + \omega_0^2 x + \lambda x^2 = 0$$

"Anharmonic Oscillator"

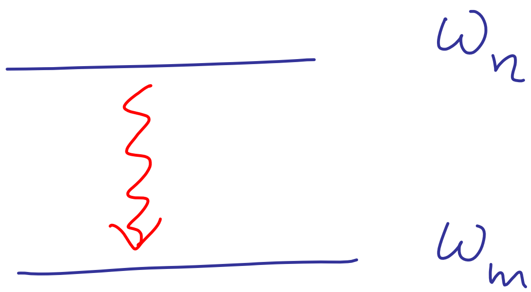
* Useful to expand in harmonics:

$$\boxed{x(t) \longleftrightarrow x_n}$$

$$x(t) = \sum_n x_n e^{in\omega_0 t}$$

$$x^2(t) = \sum_n (x^2)_n e^{in\omega_0 t} \quad \text{with} \quad (x^2)_n = \sum x_m x_{n-m}$$

Now the Magic!



What we observe is $\omega_{nm} = \omega_n - \omega_m$

So we shouldn't have "X_n", we should also have "X_{nm}"!

+

$$(X^2)_{nm} = \sum_k X_{nk} X_{km}$$

A GUESS
A LEAP

in x. A significant difficulty arises, however, if we consider two quantities $x(t)$, $y(t)$, and ask after their product $x(t)y(t)$. If $x(t)$ is characterized by $\mathfrak{A}$, and $y(t)$ by $\mathfrak{B}$, we obtain the following representations for $x(t)y(t)$:

Classical:

$$\mathfrak{C}_\beta(n) = \sum_{-\infty}^{+\infty}{}_\alpha \mathfrak{A}_\alpha(n)\mathfrak{B}_{\beta-\alpha}(n).$$

Quantum-theoretical:

$$\mathfrak{C}(n, n-\beta) = \sum_{-\infty}^{+\infty}{}_\alpha \mathfrak{A}(n, n-\alpha)\mathfrak{B}(n-\alpha, n-\beta).$$

Whereas in classical theory $x(t)y(t)$ is always equal to $y(t)x(t)$, this is not necessarily the case in quantum theory. In special instances,

Classical:

$$\begin{aligned} \omega_0^2 a_0(n) + \tfrac{1}{2}a_1^2(n) &= 0; \\ -\omega^2 + \omega_0^2 &= 0; \\ (-4\omega^2 + \omega_0^2)a_2(n) + \tfrac{1}{2}a_1^2 &= 0; \\ (-9\omega^2 + \omega_0^2)a_3(n) + a_1a_2 &= 0; \\ \cdots\cdots\cdots\cdots\cdots & \end{aligned} \tag{18}$$

Quantum-theoretical:

$$\begin{aligned} \omega_0^2 a_0(n) + \tfrac{1}{4}[a^2(n+1, n) + a^2(n, n-1)] &= 0; \\ -\omega^2(n, n-1) + \omega_0^2 &= 0; \\ [-\omega^2(n, n-2)+\omega_0^2]a(n, n-2)+\tfrac{1}{2}[a(n, n-1)a(n-1, n-2)] &= 0; \\ [-\omega^2(n, n-3) + \omega_0^2]a(n, n-3) & \\ +\tfrac{1}{2}[a(n, n-1)a(n-1, n-3)]+\tfrac{1}{2}[a(n, n-2)a(n-2, n-3)] &= 0; \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \end{aligned} \tag{19}$$

3. As a **simple example**, the anharmonic oscillator **will** now be treated:

$$\ddot{x} + \omega_0^2 x + \lambda x^2 = 0. \tag{17}$$

Classically, this equation is satisfied by a solution of the form

$$x = \lambda a_0 + a_1 \cos \omega t + \lambda a_2 \cos 2\omega t + \lambda^2 a_3 \cos 3\omega t + \ldots \lambda^{\tau-1} a_\tau \cos \tau\omega t,$$

where the a are power series in λ, the first terms of which are independent of λ. Quantum-theoretically we attempt to find an analogous expression, representing x by terms of the form

$$\lambda a(n, n); \quad a(n, n-1) \cos \omega(n, n-1)t;$$
$$\lambda a(n, n-2) \cos \omega(n, n-2)t;$$
$$\ldots \lambda^{\tau-1} a(n, n-\tau) \cos \omega(n, n-\tau)t \ldots.$$

Precision of Equations Brings Deep Truths out of the dark - rigidity of equations makes guessing + analogizing very powerful but this was the case for all terms evaluated) turns out to be

$$W = \frac{(n+\frac{1}{2})h\omega_0}{2\pi} + \lambda\,\frac{3(n^2+n+\frac{1}{2})h^2}{8\cdot 4\pi^2\omega_0^2 m}$$

$$- \lambda^2\,\frac{h^3}{512\pi^3\omega_0^5 m^2}\left(17n^3 + \tfrac{51}{2}n^2 + \tfrac{59}{2}n + \tfrac{21}{2}\right). \qquad (27)$$

This energy can also be determined using the *Kramers–Born* approach by treating the term $\frac{1}{4}m\lambda x^4$ as a perturbation to the harmonic oscillator. The fact that one obtains exactly the same result (27) seems to me to furnish remarkable support for the quantum-mechanical equations which have here been taken as basis. Furthermore, the

When equations work magically + non-trivially, you know you're on the right track!

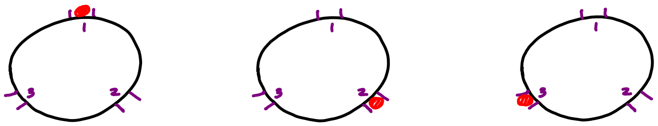
Why so much torture when "answer is so simple!"? Because in the trenches of discovery it is crucial to find both why "what is right is right" and why "everything else is wrong!"

"Words + Philosophy" are especially useless for this purpose — since even when you're right they can change + adjust! It's equations + calculations that are sure things in totally uncharted territory.

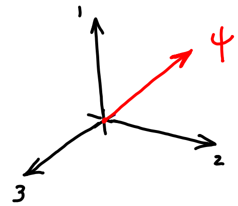
QM is most mind blowing discovery of humanity. Way too big a leap to make "methodically, carefully" — need to see magic! Correct equations came first, correct words much later. A real triumph of "Shut Up + Calculate" — and template for attacking similarly huge questions today.

Quantum Mechanics

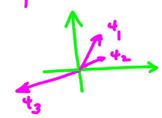
* State of a system is given by "state vector"



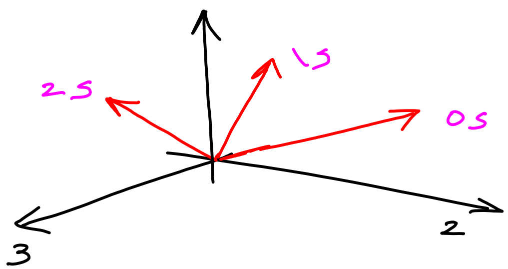
$\Psi_{\text{ball in 1}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\quad \Psi_{\text{ball in 2}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\quad \Psi_{\text{ball in 3}} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$



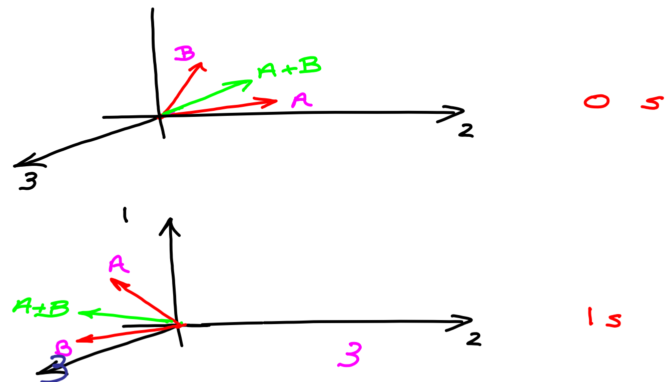
$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$ ⟸ "Complex" numbers



Probability to measure $\frac{1}{\substack{2\\3}} = (\text{length of } \psi_{\substack{1\\2\\3}})^2$



length = 1 ~~~~
at all times
(all prob add to 1!)



"Linear Evolution"

* Physical Observables are operations ("hermitian operators") that in general transform states to other states.

If $A\vec{\psi} = a\vec{\psi}$, A has def. value a

In general $AB\vec{\psi} \neq BA\vec{\psi}$

e.g. $(XP-PX)\vec{\psi} = i\hbar\vec{\psi}$

or $(S_xS_y-S_yS_x)\vec{\psi} = i\hbar S_z\vec{\psi}$

{...And are a necessary feature of Quantum treatment of translations + rotations....}

Central Novelty is "$AB \neq BA$"

If $\vec{\psi}_a$ have definite "A"

$\vec{\phi}_b$ have definite "B"

$\vec{\psi}_a$ can have value b w/ probability $|\vec{\psi}_a^* \cdot \vec{\phi}_b|^2$

This quantum novelty is suppressed for "large" systems made of N copies of "small" systems, as $N \longrightarrow \infty$.

Say $AB - BA = C$ . . . Let $\overline{A} = \frac{1}{N} \sum_i A_i$ etc.

Then $(\overline{A}\,\overline{B} - \overline{B}\,\overline{A}) = \frac{1}{N} \overline{C}$ . . .

Related, say $\vec{\psi} = \sqrt{\frac{2}{3}}\, \psi_{up} + \sqrt{\frac{1}{3}}\, \psi_{down}$

Does not have definite value of "up" or "down"

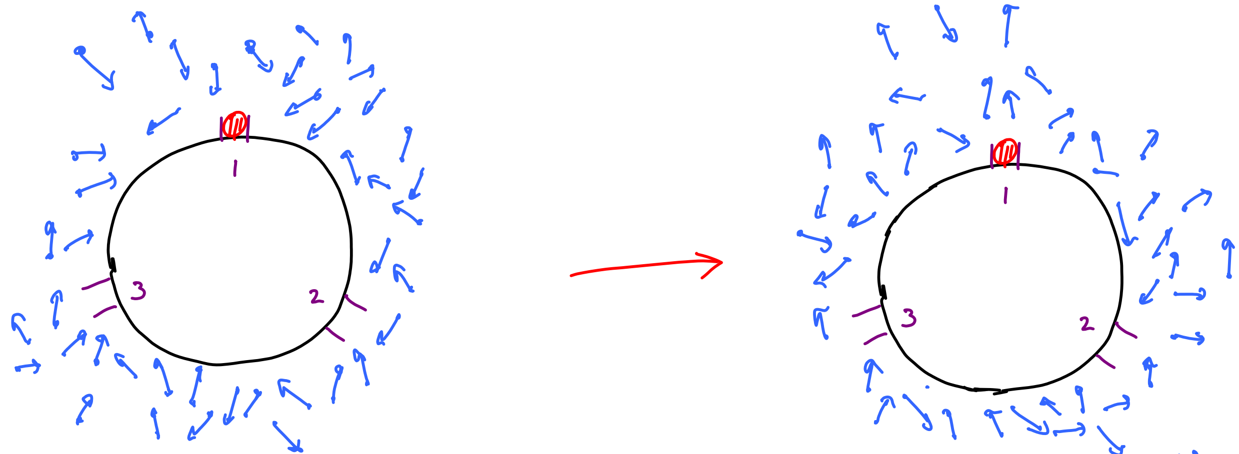
$\underbrace{\vec{\psi} \times \cdots \times \vec{\psi}}_{N \text{ copies of state}}$

"average up" $= \frac{2}{3}$

"average down" $= \frac{1}{3}$

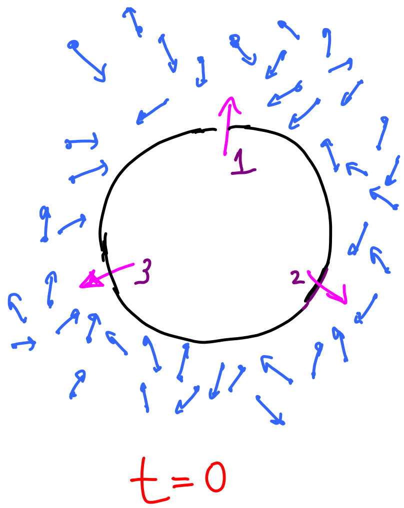
Completely Obvious fact about probability

"Measurement" just another Physical Process



$t=0: \quad \Psi = [\psi_1, \psi_{Air}]$

$t = 1s \quad \Psi = [\psi_1, \psi_{Air}^{(1)}]$



$$\Psi = \left[\psi_1 + \psi_2 + \psi_3 ,\ \psi_{Air} \right]$$

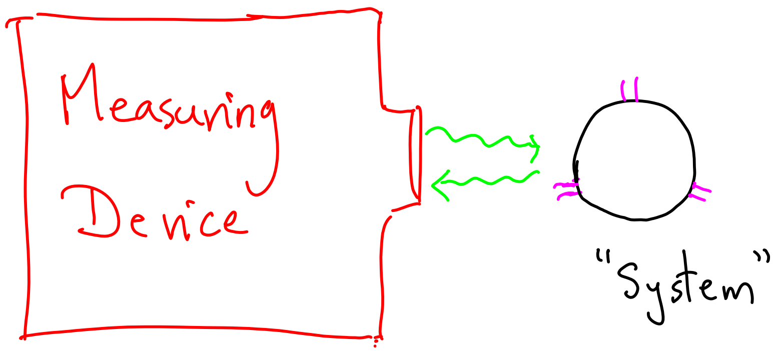
$$\downarrow$$

$$\psi_1 \psi_{Air}^{(1)} + \psi_2 \psi_{Air}^{(2)} + \psi_3 \psi_{Air}^{(3)}$$

But crucially, $\vec{\psi}_{Air}^{+(1)} \cdot \psi_{Air}^{(2)} \sim (0.999)^{10^{30}}$ (!) $\longrightarrow 0$

Quantum interference lost, "decoherence" + emergence of classical world {99.9% understood by founders of QM}

Exact Quantum Predictions



Infinitely many measurements with an Infinitely large measuring apparatus!

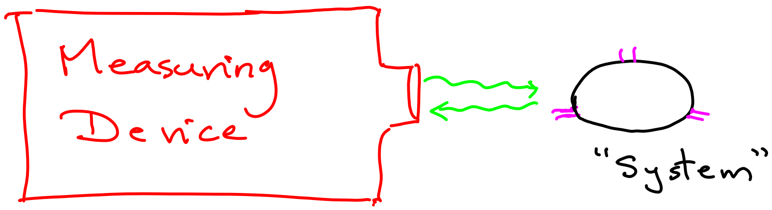
When these conditions are met ....

THAT'S IT. NO FOG, NO CONFUSION, NO "CRISIS IN INTEPRETING QM" — EVEN CONCEPTUALLY

{And again, apart from minor clarifications of decoherence in 70's + 80's, all known for ~80 yrs!}

[See Sidney Coleman Video: "QM in Your Face!" for more]

Exact Quantum Predictions

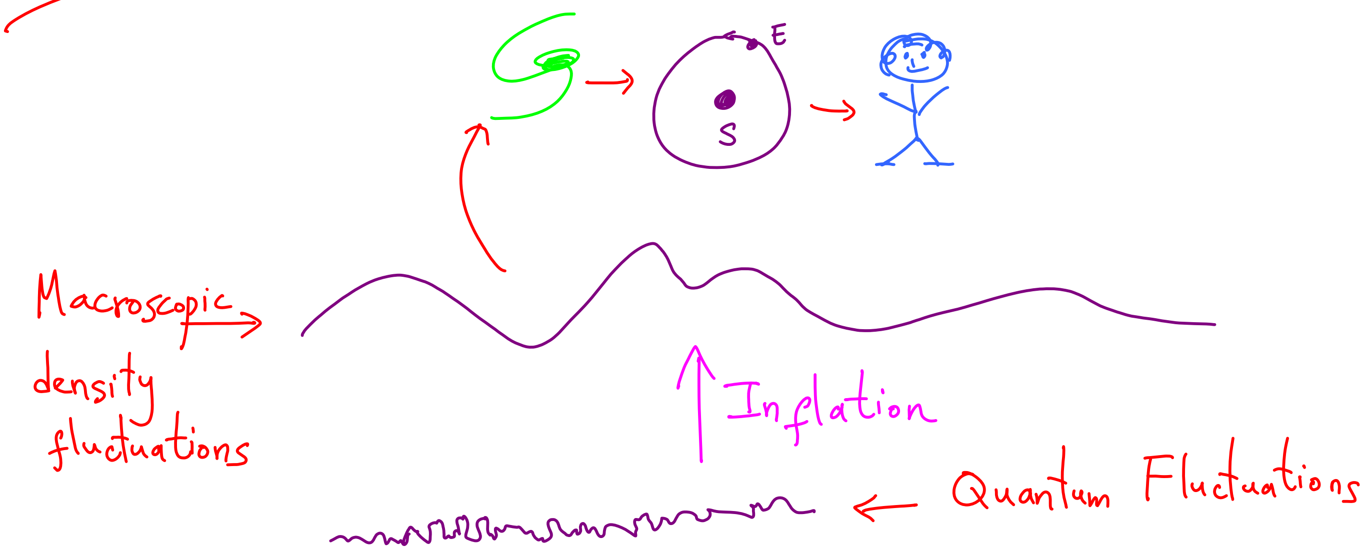


Infinitely many measurements with an Infinitely large measuring apparatus!

GRAVITY + esp COSMOLOGY give the "deep questions about QM" real teeth...

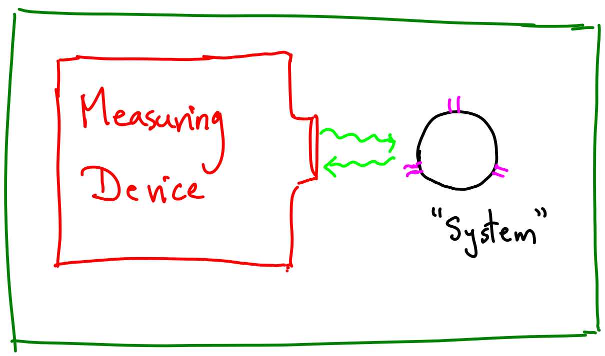
but it has always been the "shut up and calculate" folks who exposed these teeth!

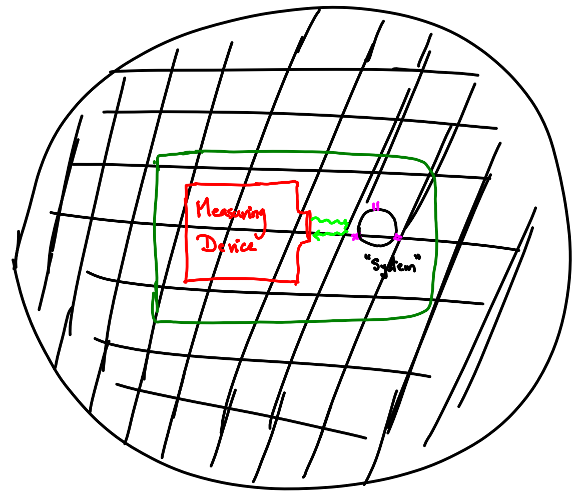
Simple Example: Inflation + Ψ_U



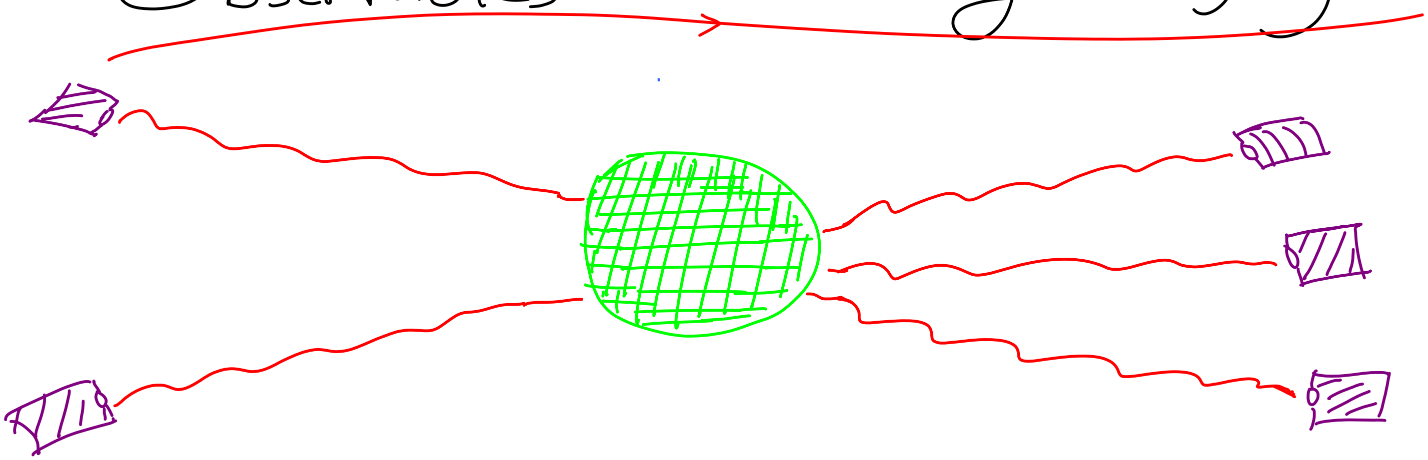
["Inf. many experiments" done by universe itself, in different regions of space!]

No Local Observables!





Observables on "Boundary at Infinity"

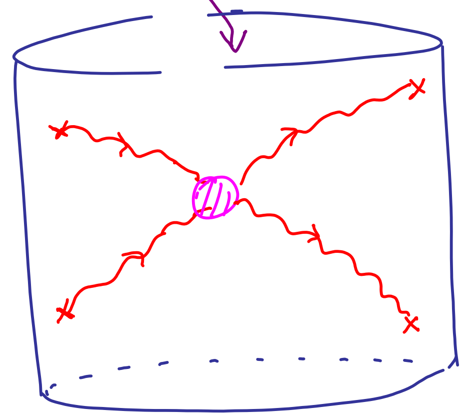


In flat space: "Scattering Amplitudes"
Crucial insight from 60's [de Witt, Penrose..]
Central focus of early work in String Theory

$$(\text{Quantum Gravity})_{D+1} = (\text{Quantum Field Theory})_{D}$$

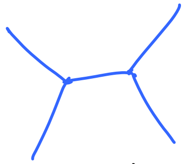
Emergent Space, Gravity, Strings ...

time

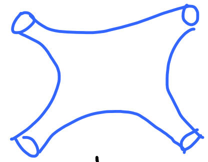


"Anti-de Sitter Space"

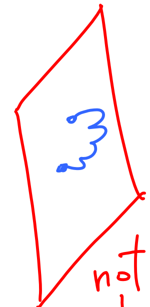
$$\text{String Theory} = \text{Particle Physics}$$



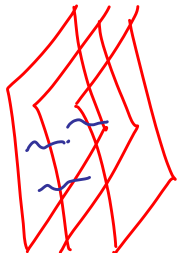
particles



strings



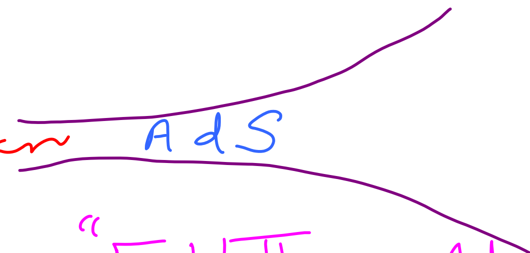
not just "strings, branes"



Conventional "field theory"

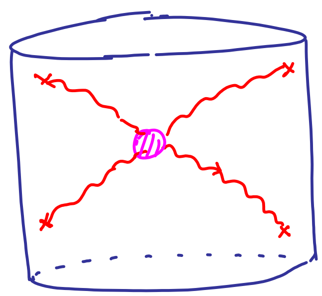
@ low energies

low-E

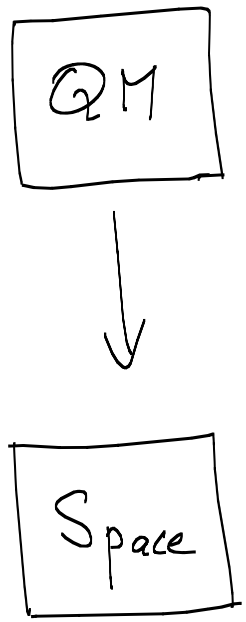


"Black Brane"

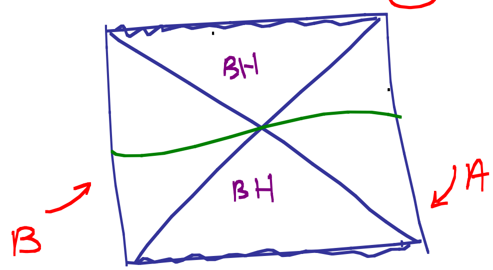
"Field Theory + AdS?"

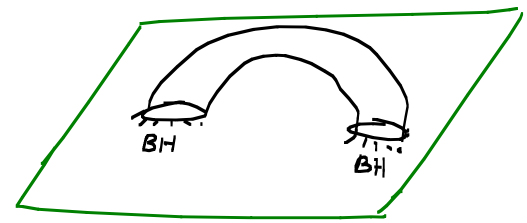


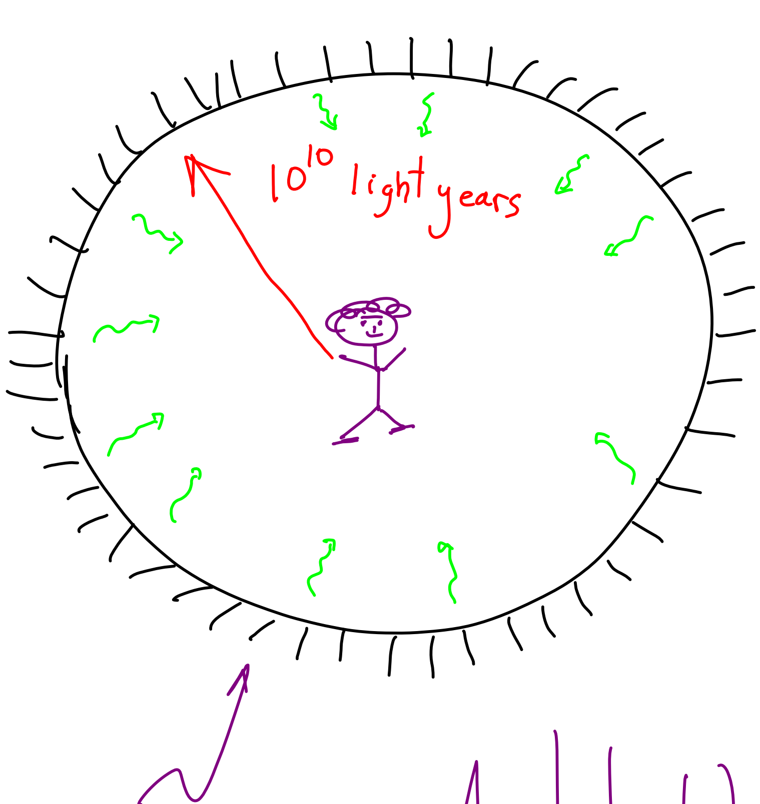
"No! Holography:
Field Theory = AdS!"



Holography already gives emergent space + gravity, with remarkable connections emerging between Quantum Entanglement and Geometry

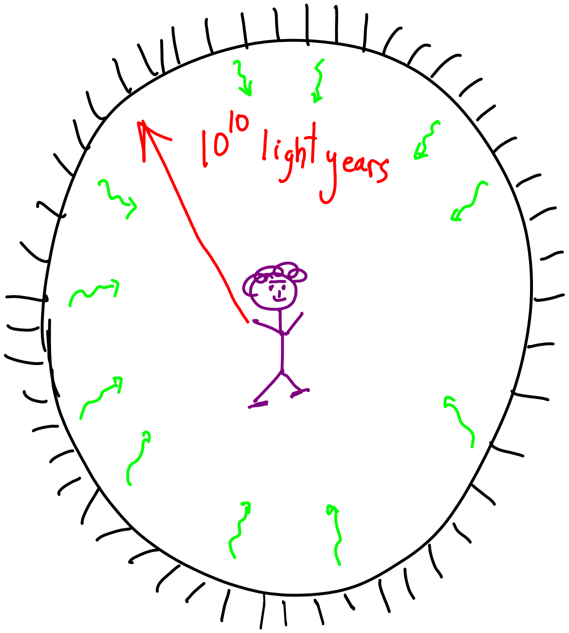






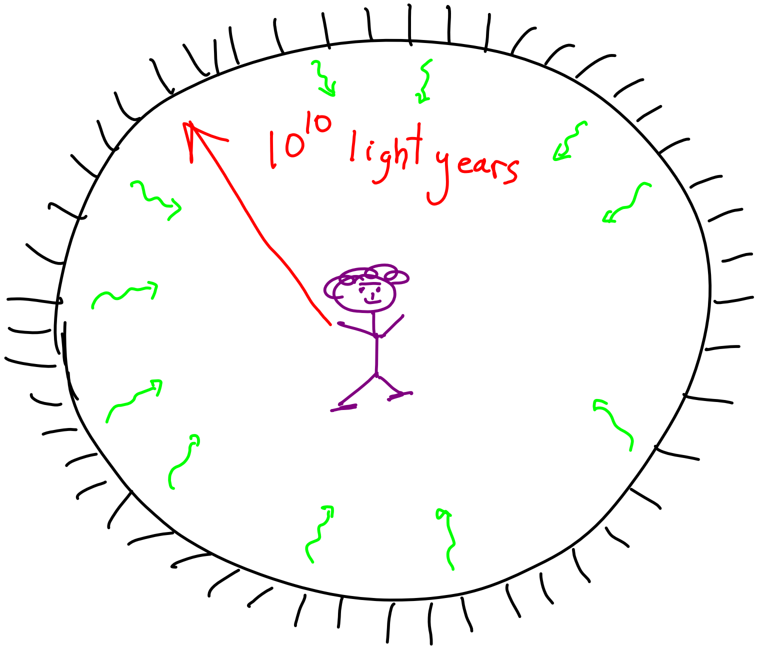
This is the first place the rules of QM cry uncle!

Horizon of Our Accelerating Universe



No precise Quantum Observables

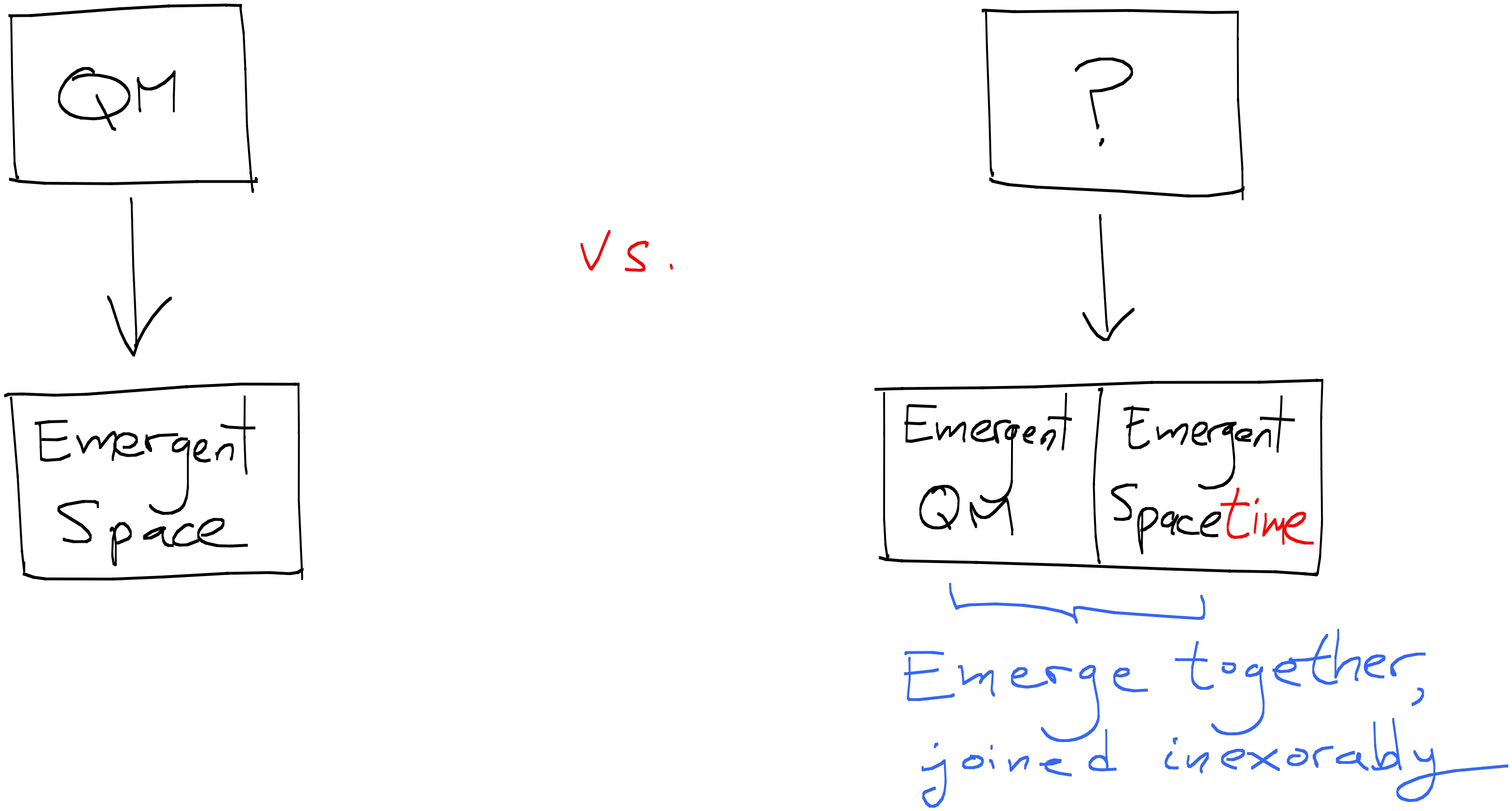
Late acceleration makes it <u>in principle</u> <u>impossible</u> to learn anything about initial singularity!



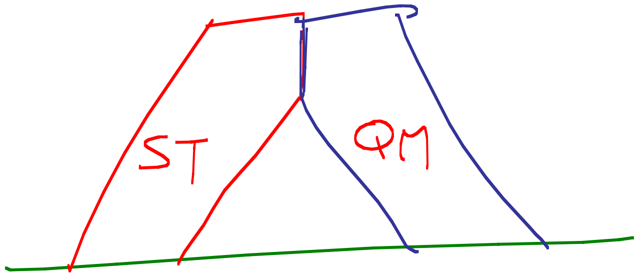
This is the first place the rules of QM cry uncle!

Emergent Extension of Space-Time

Quantum Mechanics?


QM
Emergent Space
vs.
?
Emergent QM
Emergent Spacetime
Emerge together, joined inexorably

Broad Clues



They Buttress Each Other, making each other more rigid + robust

QM makes "aether" impossible

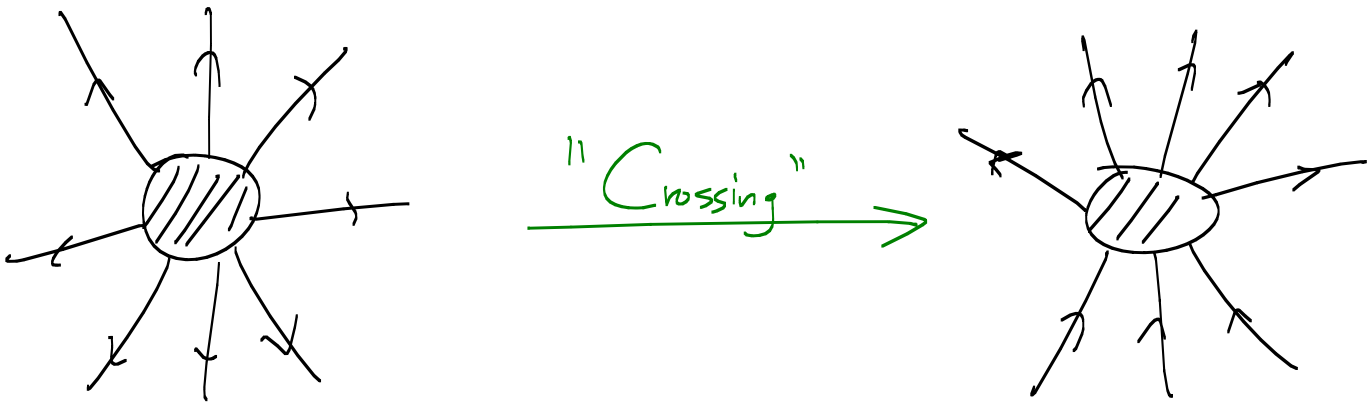
ST makes hidden var impossible

Broad Clues

Textbook formulation of QFT is Euclidean (x_1, x_2, x_3, x_4). We then analytically continue $x_4 \to it$.

to get Causal + Unitary physics hand-in-hand.

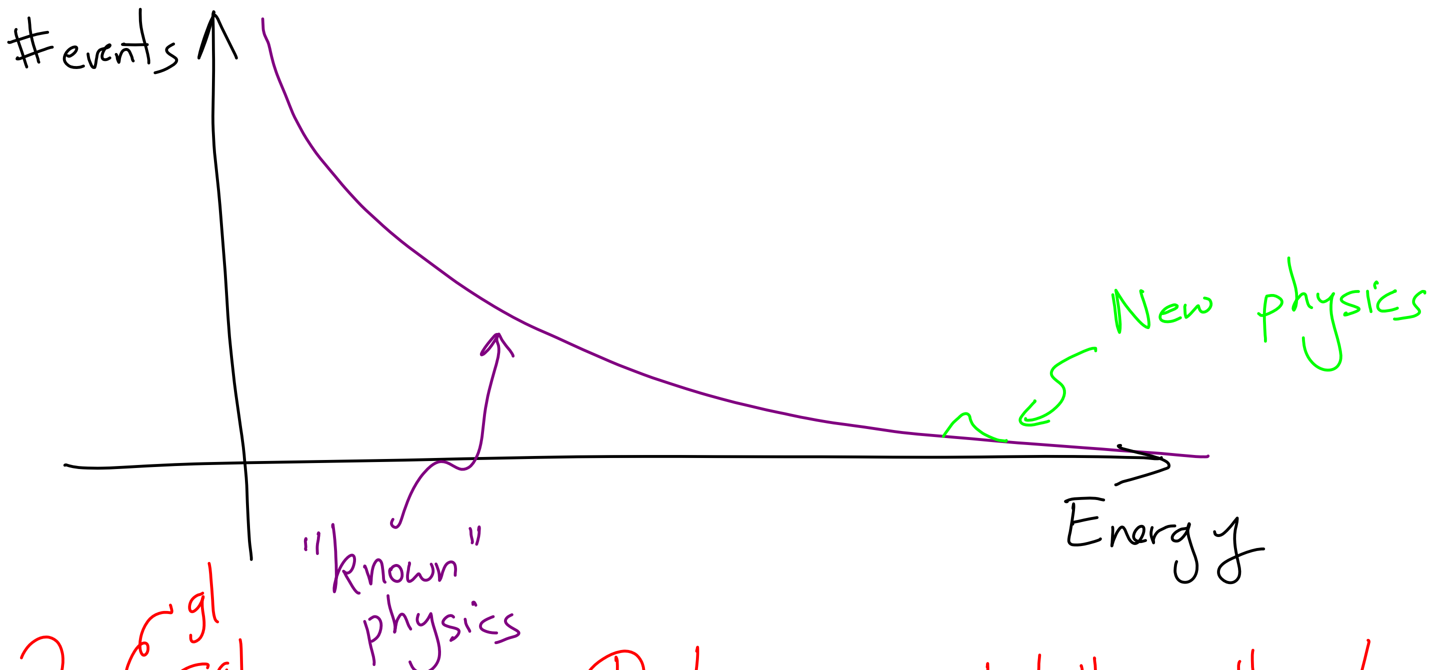
Clues in Scattering Amplitudes



Most natural object:
no "in", "out";
complex momenta

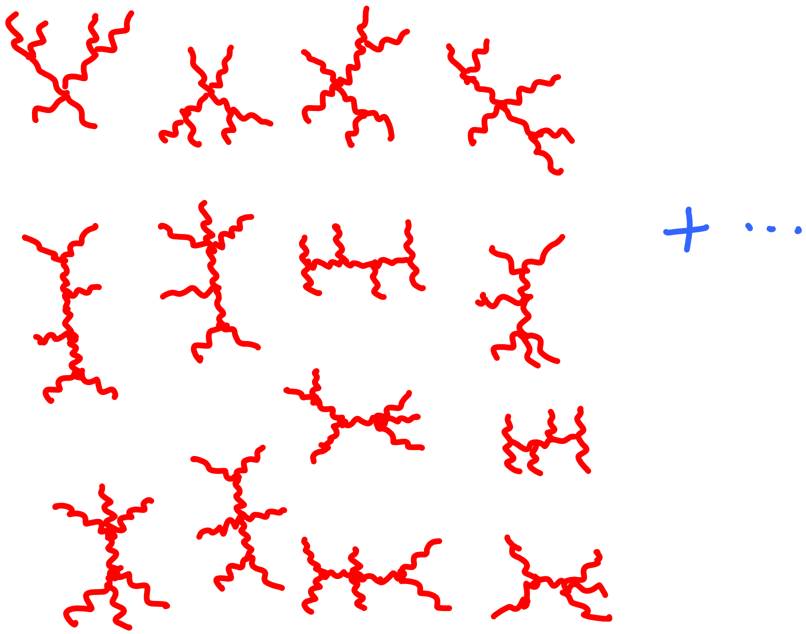
"in" $\longrightarrow$ "out"
+ Unitarity hand-in-hand

L. H. C.



Rates: ~ 1 billion collisions/sec
~ 10 top quarks/sec
~ 1 squark minute.

Feynman Tells Us What To Do



220 Diagrams

10's of thousands of terms ...

Result of a brute force calculation:

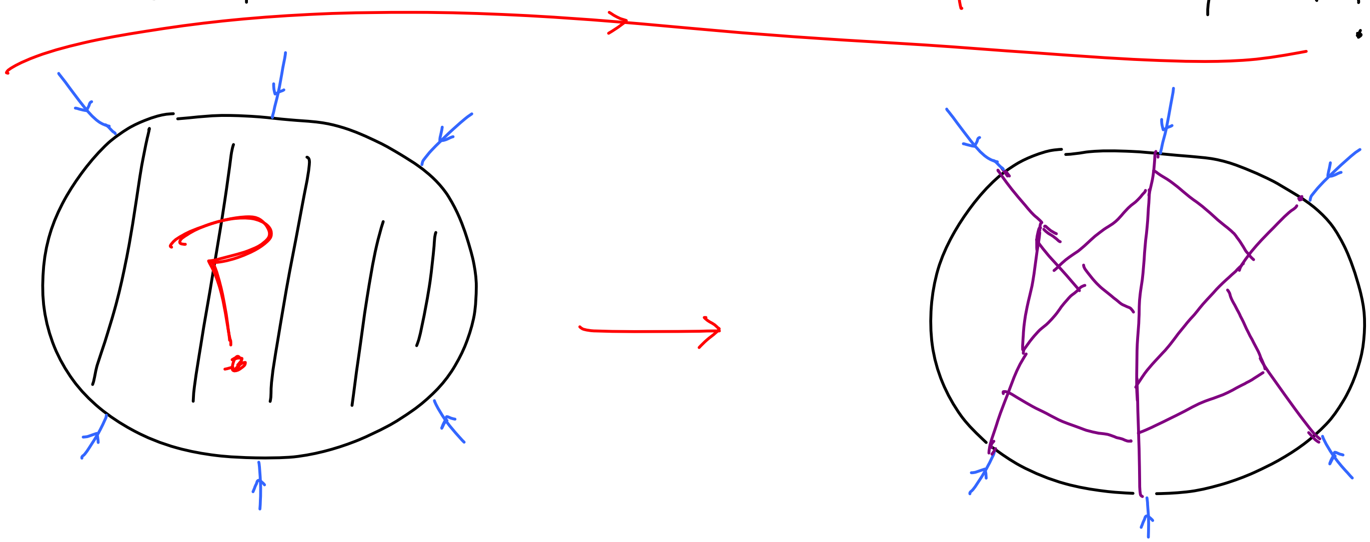
$k_1 \cdot k_4\, \varepsilon_2 \cdot k_1\, \varepsilon_1 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5$

+ 30 more pages

$$\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle} \quad (!)$$

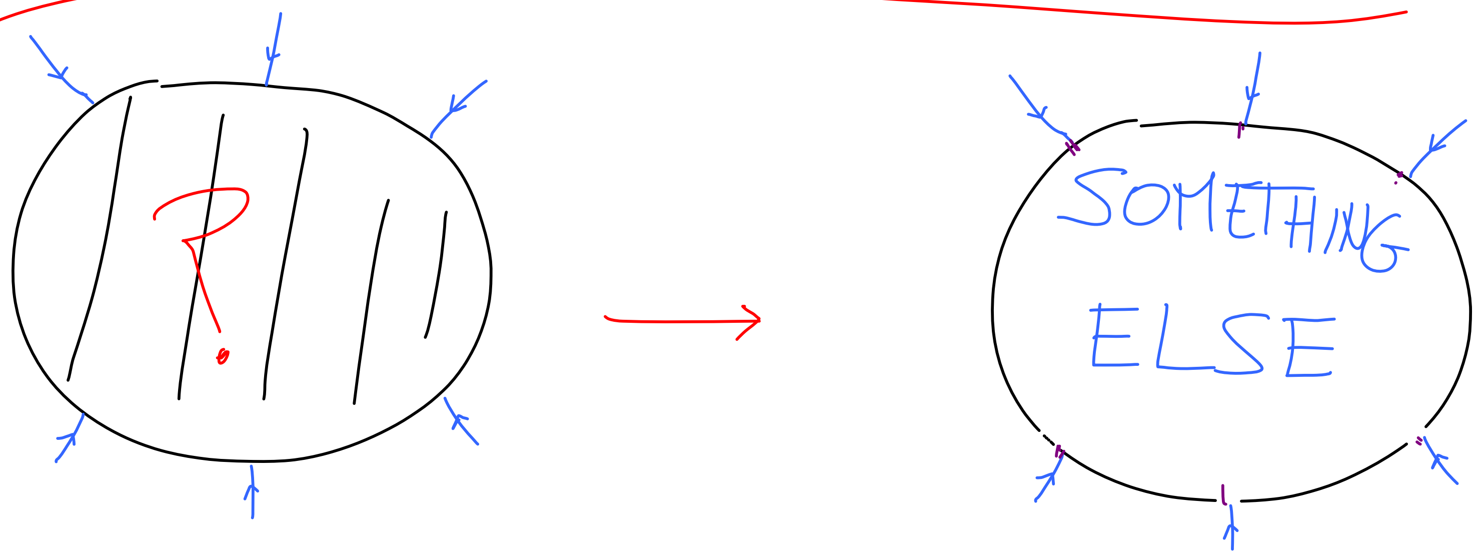
The standard way of doing physics makes usual rules of spacetime + QM manifest – but is obviously hiding some extraordinary new structures!

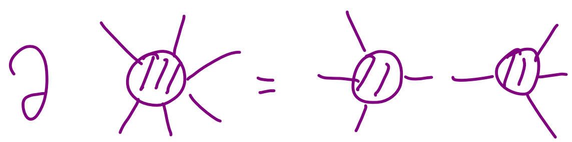
What is the Q to which A is the Answer?



"Quantum Collisions inside Spacetime"

What is the Q to which A is the Answer?





Deep Q left from 60's:

"How is Causality encoded in amplitudes measured at $\pm\infty$ time?"

We still don't know the A, not even completely in pert. th! Not a technicality: TIME + DYNAMICS

New Strategy: Look For

NEW PRINCIPLES, LAWS

from which CAUSAL, UNITARY

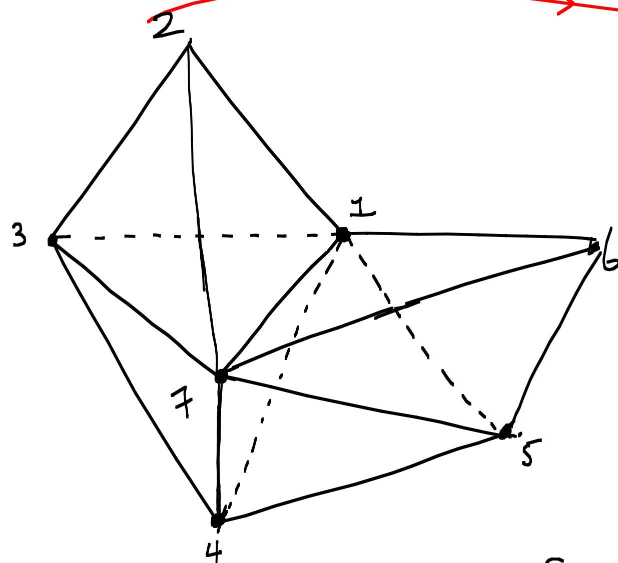
evolution — local Spacetime Physics + QM,

emerge together.

Shut Up + Calculate
(by hook or crook)

Look for hidden
mathematical
structures

Volume of This Shape



No spacetime,
No Lagrangian,
No Hilbert Space
⋮

Locality,
Unitarity
Together,
From Combinatorial
Geometry

Leading Amplitude for $[1^- 2^+ 3^+ 4^+ 5^- 6^+ 7^- 8^-]$ @LHC!

{Hundreds of Pages of Feynman Diagrams}

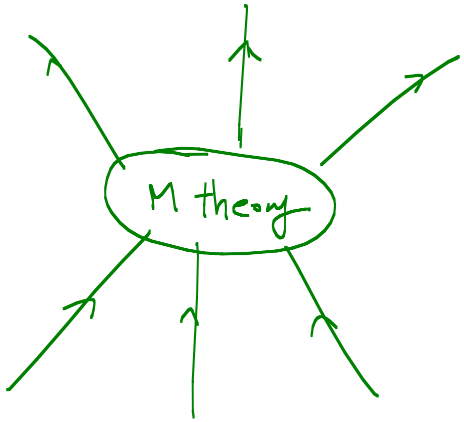
Contrasting Speculations

* What principle determines future Universe?
(At least for universes — not ours! — that aren't accelerating deep in future)

This problem has enough concrete avatars that it is possible to work on it today
(+ many of us do work on it)

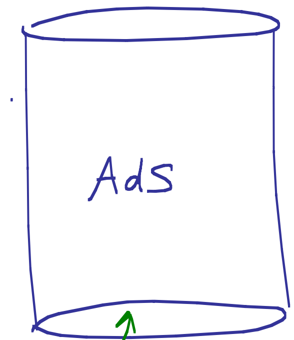
* Can we remove the system / ∞ observer dichotomy, with a unified theory of both? EXTEND [NOT MODIFY!] quantum mechanics to do so....

A Huge Tension



One Unified Theory!
Landscape of connected solutions

vs.



Any Quantum theory

Is a different theory in AdS!

Extension of QM is source of Unification?

Classical $\longrightarrow$ Quantum $\longrightarrow$ Quant. Grav in Flat/AdS $\longrightarrow$ Acc Univ

Fewer Observables

More Unified

All that's left
is system/observer divide

These "thoughts" are just too vague, far away to do anything with.

I don't know how to work on them!

I "shut up and calculate" with the hope that something lucky + magical might happen to bring them closer, so I <u>can</u> work on them someday; for now it's just fun daydreaming

Q: But don't you care about what reality "really is" — not just some abstract equations that describe it?

A: Whatever reality is, it has time and again proven to be vastly different than our preconceptions. I care too much too overly pollute things with my own prejudice-laden "words" and "thoughts" — I prefer instead to try and listen and follow what it's telling me to do next. We "listen" by "calculating".

SHUT UP + CALCULATE!