The Demand Bargaining Set: General Characterization and Application to Majority Games *

MASSIMO MORELLI* and MARIA MONTERO**

*Ohio State University and Institute for Advanced Study

**Dortmund University

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Abstract

The cooperative solution concept introduced here, the *demand bargaining set*, contains the core and is included in the Zhou bargaining set, eliminating the "dominated" coalition structures. The demand vectors belonging to the demand bargaining set are *self-stable.* In the class of constant-sum homogeneous weighted majority games the demand bargaining set is non-empty and predicts a *unique* demand vector, namely a *proportional* distribution within minimal winning coalitions. The noncooperative implementation of the demand bargaining set is obtained for all the games that satisfy the one-stage property.

Keywords: bargaining sets, stable demands, undominated coalition structures, weighted majority games, proportional payoffs.

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morelli@ias.edu, maria.montero@wiso.uni-dortmund.de

1 Introduction

The interest in the stability properties of coalitions and payoff allocations in coalition formation games can be traced back to the first half of the 20th century, and the discussion about which solution concept we should use to predict coalition structures and payoff allocations at the same time remains open. Value concepts yield an *ex ante* evaluation, hence they cannot offer predictions about the prevailing payoff distribution within the prevailing coalitions. On the other hand, solution concepts like the bargaining set, the stable sets, and the kernel avoid the emptiness problems of the core for a large class of games and do face the problem of predicting the possible *ex post* configurations of payoffs, but the set of solutions is often too large. Most solution concepts determine the distribution of gains *within* given coalitions or coalition structures, and hence are very helpful to model arbitration problems, where coalitions are formed before the bargaining process over the distribution of payoffs begins. In contrast, as pointed out in Bennett (1985), the aspirations approach seems well suited for situations where the formation of coalitions is endogenous.

The cooperative solution concept introduced in this paper, the *demand bargaining set*, shares the central role of payoff demands with the aspiration approach, but it does not limit attention to the domain of aspirations. While on the one hand the traditional imputation approach determines payoff distribution after having fixed a coalition structure, and while on the other hand the aspirations approach determines coalitional outcomes after having fixed a payoff distribution, we allow payoff distribution and coalition formation to be *simultaneously determined*. In spite of this larger strategic space, we will show that in constant-sum homogeneous weighted majority games our stable demands coincide with the balanced aspirations of the game.[1]

The idea behind the demand bargaining set can be given by means of a simple example. Consider three people who have to share a dollar. Let majority be enough for a decision, so that only two people will end up obtaining a positive share. Imagine that the three players first choose a *demand*, a share between 0 and 1. Then, given any triplet of demands, consider the majority coalitions with feasible demands. How do we establish that a pair, composed by the triplet of demands and a majority coalition compatible with it, constitutes a solution for this distributive problem?

[1]The concept of balanced aspirations is well explained in Bennett (1983).

We say that such a pair is a solution in the demand bargaining set if every objection to it can be countered *using the same demands* expressed in the proposed triplet. In this example the only triplet of demands that satisfies this self-stability criterion is $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$. To see this, call i and j the two players in a proposed majority coalition given these demands, and call k the excluded player. Any objection by – say – the coalition jk would have to give $\frac{1}{2} + \epsilon$ to j and $\frac{1}{2} - \epsilon$ to k, for some $\epsilon > 0$; But then this objection can be countered by the coalition ik, in which both players can be made better off with respect to the objection by reproposing the $\frac{1}{2}, \frac{1}{2}$ split. This triplet of demands is the only one such that it can be used to counter any objection. So equal demands are a sort of "social norm" in this example, and they are our prediction.

The demand bargaining set turns out to contain the core, and is a subset of the Zhou bargaining set. With respect to these and other solutions in the bargaining set tradition, there are two main innovations: (1) we make use of the fact that every allocation can be viewed as a *pair* consisting of a *demand vector* and a *coalition structure*; and (2) for every proposed pair the set of counterobjections (to objections to such a proposal) is restricted to include only those pairs that use the *same* demand vector as in the original proposal. The second feature is the one determining the selection of a subset of the Zhou bargaining set. In particular, the selection obtained through the demand bargaining set has a very general property: in grand-coalition superadditive games no "dominated" coalition structure can ever be part of a solution (Theorem 2).

In any constant-sum weighted majority game that admits a homogeneous representation the proportional payoff division is the *unique* stable outcome, and no previous bargaining set could make such a sharp prediction.[2] This result illustrates particularly well the role of the axiom requiring that counterobjections be limited to those that can use the original demand vector: the *social norm* of distributing payoffs proportionally to the relative contributions (within the minimal winning coalition) is the only one that can be used to counter all possible objections to a

[2]There is however an interesting connection with the von Neumann-Morgenstern main simple solution. Warwick and Druckman (2001) explain in detail the theoretical as well as empirical reasons to believe that the proportional payoff vector is the one that we should expect to arise in legislative bargaining contexts.

proposal where the norm itself is used. This type of *self-stability* is the key idea proposed by our concept.

In Section 4 the demand bargaining set is implemented for all superadditive games satisfying the one-stage property, following the methodology of Pérez-Castrillo and Wettstein (2000). The mechanism is simple and serves to illustrate further the forces behind our concept.

2 The Demand Bargaining Set

In this section we introduce the cooperative solution concept, defining it for any TU game and discussing some general properties.

2.1 Notation and Basic Definitions

Let N be the set of players, $N \equiv \{1, 2, \ldots, i, \ldots, n\}$. Let $S \subseteq N$ represent a generic coalition of players, $\mathcal{S}$ denote the set of possible coalitions, and $v : \mathcal{S} \to \mathcal{R}_+$ denote the characteristic function. If $v(i) = 0$ for all i in N, the pair (N, v) is called a zero-normalized TU game. Given any n-dimensional payoff vector x, $x(S)$ denotes the sum of the values corresponding to the components of x related to the members of S: $x(S) \equiv \sum_{i \in S} x_i$. We will denote by Σ the set of possible coalition structures (partitions of N), and σ will represent a generic element of that set.

The demand bargaining set (henceforth DBS) is defined on the space $\mathcal{X}$, where

$$\mathcal{X} = \{(\alpha, \sigma) \in (\mathcal{R}^n \times \Sigma) : \sum_{j \in S} \alpha_j \leq v(S) \text{ for all } S \in \sigma, |S| > 1\}.$$

Thus, a candidate element for the DBS is a pair (α, σ) where $\alpha \in \mathcal{R}^n$ is the demand vector specifying what each player should receive for his cooperation with other players in a coalition and σ is a *coalition structure* compatible with α in the sense that coalitions of more than one player must be able to afford the demands of their members. No such restriction is imposed on singletons.[3]

[3]The reason for not imposing any requirement on singletons is that α is interpreted as the demand players make for cooperating *with other players*.

For any given pair $(\alpha, \sigma) \in \mathcal{X}$, the corresponding *feasible allocation* α^σ is obtained using the following *payoff assignment rule*:

$$\alpha_i^\sigma = \begin{cases} \alpha_i & \text{if } i \in S \in \sigma : |S| > 1 \\ v(i) & \text{otherwise.} \end{cases} \tag{1}$$

In words, the demands are assigned as actual payoffs to the members of coalitions with more than one player; singletons receive $v(i)$ regardless of players' demands.

2.2 Solution Concept

Consider a proposed pair $(\alpha, \sigma) \in \mathcal{X}$.

Definition 1 *An objection by a coalition T against the proposal (α, σ) is an allocation vector y such that*

$$\begin{aligned} y_i &> \alpha_i^\sigma \text{ for all } i \in T \\ \text{and } \sum_{i \in T} y_i &\leq v(T) \text{ (i.e., } y \text{ is feasible for } T\text{).} \end{aligned}$$

Notice that objections are against actual payoffs, not against demands; thus, an objecting player may receive less than his demand in the objection.

Definition 2 *A coalition Z can counter (or, make a counterobjection to) the objection of T against the proposal (α, σ) iff*

(1) $Z \cap T \neq \emptyset$ and

(2) the original demand vector α is such that

$$\begin{aligned} \sum_{i \in Z} \alpha_i &\leq v(Z) \\ \alpha_i &\geq \alpha_i^\sigma \quad \forall i \in Z \\ \alpha_i &> y_i \text{ for all } i \in T \cap Z. \end{aligned}$$

Definition 3 *An objection to (α, σ) is* justified *iff it cannot be countered.*

Notice that with respect to the standard definition of a counterobjection (as used in most versions of the bargaining set) we restrict the set of possible counter-objections to include only those that can be derived using the same demand vector of the original proposal. We also require the inequality within $Z \cap T$ to be strict.[4] Because of the strict inequality, objections that use the original demand vector α are always justified.

Definition 4 *A pair* (α, σ) *in* $\mathcal{X}$ *belongs to the* demand bargaining set *iff there is no justified objection to it.*

Intuitively, the demand bargaining set tries to formalize the concept of a *stable social norm*. Suppose that at the beginning of the game players make payoff demands for cooperating with other players. After the demand vector α is known, a coalition structure σ compatible with it forms. When is α a stable social norm and σ a stable realization of it? We can distinguish two types of objections, depending on whether they conform with α.

1. Objections that respect the social norm (that is, objections in which players receive α or objections by singletons) are always justified.

2. Other objections (objections by some coalition T of more than one player in which players are not paid according to α) are a challenge to the established social norm. Players in T say: "α is unfair to us, we want to replace (our part of) α with y". This objection is not credible if some of the players in T can do better in some coalition Z *while still following the social norm* α.

2.3 Important Properties and Relation to Previous Concepts

Most solution concepts are made of two elements: a restriction to some "reasonable" space (aspirations or imputations) and a stability condition (e.g., objections can be countered). We have imposed no restrictions on the space $\mathcal{X}$ other than σ being compatible with α. We will see that some further restrictions arise *endogenously*, as

[4] The uniqueness of stable demands in Theorem 3 would hold even if we assumed a weak inequality (proof available upon request).

consequences of the stability condition: the predicted outcomes will be imputations and the predicted demand vectors will be (essentially) aspirations.

2.3.1 The Aspiration Approach

Like the DBS, the aspiration solution concepts also study the stability of demand vectors. However, as we will now show, there is an important difference with respect to the domain where the solution is defined.

Definition 5 *A demand vector $x \in \mathcal{R}^n$ is an* aspiration *iff it satisfies*

1. $x(S) \geq v(S)$ *for all* $S \subseteq N$ *(maximality);*

2. *For all* $i \in N$ *there exists a coalition* $S \ni i$ *such that* $x(S) \leq v(S)$ *(feasibility).*

Limiting attention to the space of aspirations, it is possible to define a variety of solution concepts.[5]

The DBS is defined in a larger space. Maximality and feasibility are *not* imposed on demand vectors. Moreover, if (α, σ) is in the DBS, α is not necessarily an aspiration.[6] This is due to the fact that a singleton i receives $v(i)$ regardless of her demand, so that her demand may be unfeasible or violate maximality. However, we can construct an aspiration α' such that (α', σ) is in the DBS and $\alpha_i'^{\sigma} = \alpha_i^{\sigma}$ for all i by setting $\alpha' = v(i)$ for each singleton whose demand is unfeasible or smaller than $v(i)$, and $\alpha_i' = \alpha_i$ for all other players (this change does not affect actual payoffs or objections, and since the players whose demands have changed could not be in any counterobjecting coalition previously, it makes counterobjections easier).

[5]Some important solution concepts are the *aspiration bargaining set*, obtained by adding the *partnership* condition (Albers (1974), Bennett (1983)) and the *aspiration core*, obtained by adding the *balancedness* condition (Cross (1967)). Denoting by $S_i(x)$ the set of coalitions containing i that are feasible given x, an aspiration vector x is partnered iff for any two players i and j either $S_i(x) = S_j(x)$ or $S_i(x) \backslash S_j(x)$ and $S_j(x) \backslash S_i(x)$ are both non-empty. An aspiration vector x is balanced iff it minimizes $\sum_{i \in N} x_i$ in the space of aspirations. Other solution concepts are the set of *equal gains* aspirations (Bennett (1983)) and the *aspiration kernel* (Bennett (1985)).

[6]Consider the following game: $N = \{1, 2, 3\}$, $v(1, 2) = 6$, $v(3) = 1$ and $v(S) = 0$ for all other S. The demand vector $(3, 3, x)$ is in the DBS for any x, though only vectors with $x \geq 1$ are maximal and only vectors with $x \leq 1$ are feasible.

Remark 1 *If (α, σ) is in the DBS, there exists an aspiration α' such that (α', σ) is in the DBS and $\alpha_i'^\sigma = \alpha_i^\sigma$ for all i.*

Thus, we could have imposed the requirements of feasibility and maximality without any essential change in the DBS.

2.3.2 Other Bargaining Sets and the Core

Something similar can be said about the traditional bargaining sets, defined on the space of imputations.[7]

Definition 6 *An imputation for a coalition structure σ is a payoff vector x satisfying*

1. $x(S) = v(S)$ *for all* $S \in \sigma$.
2. $x_i \geq v(i)$ *for all* $i \in N$. *(individual rationality)*

We do not require α^σ to be an imputation for σ, but this property arises endogenously.

Remark 2 *If (α, σ) is in the DBS, then α^σ is an imputation for σ.*

Proof. First, $\alpha^\sigma(S) = v(S)$ for all $S \in \sigma$. If S is a singleton, this is true by construction. If S is not a singleton, each player in S is currently receiving α. Then $\alpha^\sigma(S) < v(S)$ would imply that S has a justified objection in which each player receives $\alpha_i + \epsilon$.
Second, $x_i \geq v(i)$. If $\{i\} \in \sigma$, this is true by construction. Other players are receiving α_i, and $\alpha_i < v(i)$ would imply that $\{i\}$ has a justified objection. **QED.**

Since the stable outcomes in the sense of the DBS lie in the space of imputations, it is interesting to relate the DBS to other solution concepts that lie in this standard domain, like other bargaining sets and the core.[8]

[7] We use the terminology in Maschler (1992). An alternative term is individually rational payoff configurations.

[8] We refer to the extension of the core to an arbitrary coalition structure (see Aumann and Drèze (1974)).

Since the core is the set of imputations to which there are no objections, the DBS contains the core.

Remark 3 *If a payoff vector x is in the core for a coalition structure σ, then the pair (x, σ) is in the DBS.*

Remark 4 *If (α, σ) is in the DBS and the realized allocation is such that all players receive their demands, then α is in the core for σ. Thus, the DBS* coincides *with the core for the grand coalition. If the core of v is empty for all coalition structures, there is no allocation in the DBS such that all players receive their demands.*

After the seminal paper by Aumann and Maschler (1964) many other bargaining sets have been introduced, differring in the exact definitions of objection and counterobjection. Those by Mas-Colell (1989) and Zhou (1994) share with the DBS the feature that objections are not against a particular player or set. When compared with the Mas-Colell bargaining set, the DBS makes both objections and counterobjections more difficult, and none of the sets is contained in the other. A clear inclusion result can instead be obtained with respect to the Zhou bargaining set.

The DBS and the Zhou bargaining set share the same definition of objection. As for the definition of counterobjection, they cannot be directly ranked. The DBS makes justified objections easier since it requires counterobjections to use the original demand vector and some of the counterobjecting players to be strictly better off, whereas the Zhou bargaining set requires only that the counterobjecting players are weakly better off. On the other hand, counterobjecting could seem easier in our framework, since the DBS does not impose the requirements $Z \backslash T \neq \emptyset$ and $T \backslash Z \neq \emptyset$. However we are able to show (by contradiction) that the balance of these conflicting forces implies that the DBS is included in the Zhou bargaining set.

Theorem 1 *If (α, σ) is in the DBS, then $(\alpha^{\sigma}, \sigma)$ is in the Zhou bargaining set.*

Proof. Suppose (α, σ) is in the DBS but $(\alpha^{\sigma}, \sigma)$ is not in the Zhou bargaining set. Then, there must be a coalition T that has an objection y such that any counterobjecting coalition Z satisfies either $Z \subset T$ or $T \subset Z$.

Consider first the case in which T has an objection and $Z \subset T$ has a counterobjection. Then it must be the case that $\alpha_i > y_i > \alpha_i^\sigma$ for all $i \in Z$, thus Z itself has a justified objection and (α, σ) cannot be in the DBS.

Consider now the case in which T has an objection that can *only* be countered by supersets of T. Let Z be one of those supersets. Since Z is a superset of T, it follows that $\alpha_i > y_i$ for all i in T. Z itself has an objection in which one of the players in T (say, j) receives less than his demand and each other player in Z receives more than his demand. In order for (α, σ) to be in the DBS, there must be a counterobjection to this objection. Let Z' be one of the coalitions that have a counterobjection to this objection. Z' must contain player j and cannot contain any other player in T. But then Z' itself has a counterobjection to the original objection by coalition T, contradicting the assumption that this objection can only be countered by supersets of T.[9] **QED.**

Corollary 1 *The requirements $T \backslash Z \neq \emptyset$ and/or $Z \backslash T \neq \emptyset$ could have been imposed in definition 2 without changing the DBS.*

2.3.3 Elimination of Dominated Coalition Structures

An important property of the DBS is that it eliminates the "dominated" coalition structures for grand-coalition superadditive games,[10] and therefore constitutes a meaningful selection of the Zhou bargaining set. In order to show why such selection is meaningful, let us explain what a dominated coalition structure is, and then give an example.

Definition 7 *A coalition structure σ is* dominated *given α if either*

$$\sum_{i \in T} \alpha_i^\sigma < v(T), \text{ for some } T \subset S \text{ for some } S \in \sigma \tag{2}$$

or

$$\exists T : v(T) > \sum_{i \in T} \alpha_i^\sigma \text{ and } T \text{ is union of elements of } \sigma. \tag{3}$$

[9] Z' can be a superset of T only if T is a singleton. Notice that since the DBS only contains imputations, any objecting set T must have at least two players.

[10] A game is grand-coalition superadditive iff for any $\sigma \in \Sigma$, $v(N) \geq \sum_{S \in \sigma} v(S)$.

Example 1 Consider the players' set $N = \{1, 2, 3, 4\}$ with

$$v(T) = 4 \qquad \textit{if } |T| = 2$$
$$v(S) = |S| \quad \textit{otherwise.}$$

In this example (example 2.6 in Zhou (1994)) both the grand coalition and the all-singletons coalition structure are dominated. However, the grand coalition with equal payoff division is included in the Zhou bargaining set and in the Mas-Colell bargaining set. The same happens with the "all-singletons" coalition structure.[11] *Neither* of these unintuitive coalition structures is stable in the sense of the DBS. The general ability of the DBS to eliminate unreasonable coalition structures from being stable[12] is established in the following theorem.

Theorem 2 *If (N, v) is grand-coalition superadditive and (α, σ) is in the DBS, then σ cannot be dominated given α.*

Proof. If there was a coalition T satisfying condition (2), T itself would have a justified objection. As for condition (3), suppose $\exists T : v(T) > \sum_{i \in T} \alpha_i^\sigma$ and T is the union of elements of σ. If (α, σ) is such that all players in T are receiving their demands, T itself has a justified objection. If (α, σ) is such that not all players in T are receiving their demands, call Z the set of players who are receiving their demands. Condition (3) can be written as $v(T) > \sum_{i \in T \cap Z} \alpha_i + \sum_{i \in T \setminus Z} v(i)$. Since the game satisfies grand coalition superadditivity and T is the union of elements of σ, $v(N) > \sum_{i \in Z} \alpha_i + \sum_{i \in N \setminus Z} v(i)$. Consider an objection by the grand coalition that gives all players in Z more than their demands. A counterobjecting coalition C cannot include any players from Z. Thus, if a counterobjecting coalition exists, this counterobjecting coalition itself has a justified objection. **QED.**

Corollary 2 *If $v(N) > \sum_{S \in \sigma} v(S)$ for all $\sigma \in \Sigma$, only the grand coalition can be stable in the sense of the DBS. If moreover the core for the grand coalition is empty, then the DBS is empty.*

[11] The traditional bargaining set (Davis and Maschler (1967)) is always nonempty for *all* coalition structures (see Peleg (1967)).

[12] Notice that the elimination of dominated coalition structures may be at the cost of emptiness of the solution concept, since there are games that have only dominated coalition structures.

Remark 5 *Unlike the Zhou bargaining set, the DBS has the dummy player property.*[13]

A criticism often made to bargaining sets is that they are "too large". Theorems 1 and 2 hint that the DBS is less exposed to that problem than most other solution concepts of the same family. In particular, we will show below that the DBS predicts a unique payoff distribution for all constant-sum weighted majority games admitting a homogeneous representation, while other bargaining sets typically admit a continuum of payoff distributions.

3 Characterization for Weighted Majority Games

3.1 Definitions, Assumptions, and Auxiliary Results

A coalitional game (N, v) is a *simple game* iff $v(\emptyset) = 0,\ v(N) = 1,\ v(S) = 0$ or 1 and $v(S) = 1$ whenever $v(T) = 1$ for some $T \subset S$.

Denote by $\Omega \equiv \{S : v(S) = 1\}$ the set of *winning coalitions* (WC) and by $\Omega^m \equiv \{S : v(S) = 1,\ v(T) = 0 \text{ for all } T \subset S\}$ the set of *minimal winning coalitions* (MWC). A player will be called a *veto player* iff he belongs to all winning coalitions.

A simple game is called *proper* iff for all $S \in \Omega,\quad N \setminus S \notin \Omega$. It is called *strong* iff for all $S \notin \Omega,\quad N \setminus S \in \Omega$. Proper and strong simple games are called *constant-sum.*

A simple game (N, v) is a *weighted majority game* iff there exists a vector of non-negative weights w and a number q $(0 < q \leq \sum_{i=1}^n w_i)$ such that

$$S \in \Omega \iff \sum_{i \in S} w_i \geq q.$$

The pair $(q; w)$ is called a *representation* of the game. A weighted majority game admits a *homogeneous representation* iff there exists a representation $(q; w)$ such that

$$\sum_{i \in S} w_i = q \quad \text{for all } S \in \Omega^m. \tag{4}$$

A weighted majority game admitting a homogeneous representation is called a *homogeneous game.*

[13] Individual rationality guarantees that a dummy player i receives at least $v(i)$. He could in principle receive more than $v(i)$ in a coalition S with other players, but then $S \backslash \{i\}$ would have a justified objection.

Example 2 Let (N, v) be a four player game represented by (4;3,2,1,1). This representation is not homogeneous: there is one MWC with 5 votes, and three MWCs with 4 votes. An equivalent homogeneous representation is (3;2,1,1,1).

Denote by W^i the set of winning coalitions containing player i for a given game (N, v). Notice that $W^i \subseteq \Omega$, and $W^i = \Omega$ only if i is a veto player. Denote by μ^i the number of WCs in W^i. Similarly, denote by M^i the set of MWCs containing player i and m^i is the number of MWCs in M^i.

Lemma 1 *Let (N, v) be a constant-sum simple game. For every player i, either $m^i = 0$ or $m^i \geq 2$.*

Proof. Suppose that for some i there exists a MWC $S \in M^i$. Since the game is strong, the coalition $T \equiv \{N \setminus S\} \cup \{i\}$ must be a winning coalition. Either T itself is a MWC, or there must exist $Z \subset T$ such that $Z \in \Omega^m$, and such a coalition Z must contain i, otherwise the game would not be constant-sum. In either case, $m^i \geq 2$. **QED.**

Corollary 3 *If (N, v) is a constant-sum simple game, for every MWC S containing player i, we can find another MWC S' such that $S \cap S' = \{i\}$.*

Definition 8 *Player i and player j are of the same type iff the characteristic function is unchanged when permuting them.*

Remark 6 *If (N, v) is a weighted majority game and $\mu^i = \mu^j$, then i and j are of the same type.*

Proof. Suppose not. Then, without loss of generality, $w_i > w_j$. Consider the winning coalitions that contain either i or j, $\{W^i \cup W^j\} \backslash \{W^i \cap W^j\}$. If we take the coalitions in $W^j \backslash W^i$ and replace j by i, the resulting coalitions are all winning. If we take a coalition in $W^i \backslash W^j$ and replace i by j the resulting coalitions cannot be all winning, or i and j would be the same type. But then $\mu^i > \mu^j$, a contradiction. **QED.**

In general, if two players, i, j, have different numbers of votes ($w_i \neq w_j$), it does not follow that they must be of a different type. For example, consider the game represented by $(7; 5, 4, 3)$. Every player has the same number of WCs, and permuting them does not change the characteristic function, even though every player has a different weight. There is no reason why the player with 5 votes should have more bargaining power than the other two. In fact, an *equivalent* homogeneous representation of this game is $(2; 1, 1, 1)$. While it is always true that if $w_i = w_j$ then $\mu^i = \mu^j$, the converse is true (as established below) for the homogeneous representation of a constant-sum weighted majority game.

Remark 7 *If a constant-sum weighted majority game (N, v) admits a homogeneous representation $(q; w)$, then $\mu^i = \mu^j \longrightarrow w_i = w_j$, $\forall\ i : M^i \neq \emptyset$, $\forall\ j : M^j \neq \emptyset$.*

Proof. Suppose that $\mu^i = \mu^j$ and $w_i > w_j$. Consider all the WCs that contain either i or j: $\{W^i \cup W^j\} \backslash \{W^i \cap W^j\}$. The sets $W^i \backslash \{W^i \cap W^j\}$ and $W^j \backslash \{W^i \cap W^j\}$ are nonempty because of Corollary 3. If we take every single coalition in $W^j \backslash \{W^i \cap W^j\}$ and substitute j with i, we always obtain a WC with i, while when substituting i with j in every coalition in $W^i \backslash \{W^i \cap W^j\}$ this is not the case: homogeneity implies that there exists at least one MWC containing i but not j with exactly q votes, and this in turn implies that after the substitution the coalition would not be winning anymore. Hence $\mu^i > \mu^j$. Contradiction. **QED.**

Lemma 2 *Let (N, v) be a constant-sum weighted majority game. Let i and j be two players such that each of them belongs to at least one MWC. Then there is a MWC containing both i and j.*

Proof. Let S be a MWC containing i. If it also contains j, we are done. If it does not, consider $N \backslash S \cup \{i\}$. Because the game is strong, this coalition is winning. It includes a MWC $T \ni i$. If $T \ni j$, we are done. If not, this implies $w_j < w_i$. Now we take a coalition $S' \ni j$ and repeat the same process. This time we cannot throw away i, because it would imply that $w_i < w_j$, a contradiction. **QED.**

We will assume that $w_i < q$ for all $i \in N$.[14] In strong games, this assumption excludes the presence of veto players.

[14]Otherwise the coalitional game would be irrelevant. In such games the coalition structure of all

3.2 The Proportionality Result

Theorem 2 has the following implication for the selection of coalition structures in weighted majority games:

Lemma 3 *The only "candidate pairs" (α, σ) for the DBS of a constant-sum homogeneous game without veto players are those where σ includes a WC S with $\sum_{i \in S} \alpha_i = v(S)$.*

Theorem 3 *Let (N, v) be a constant-sum homogeneous game without veto players, and let $(q; w)$ be a homogeneous representation of (N, v). If there are no dummy players, the DBS of any such game is* non-empty *and consists of all pairs (α^*, σ) where the* unique *stable demand vector has*

$$\alpha_i^* = \frac{w_i}{q} \quad \textit{for all } i$$

and σ contains a MWC S and the players in $N \backslash S$ as singletons.

If there are dummy players, the DBS is non-empty and consists of pairs (α, σ) where $\alpha_i = \alpha_i^*$ for all non-dummy players and σ contains any winning coalition S feasible for α.

Proof. We first provide the proof for the case in which there are no dummy players.

Consider a pair where σ consists of a MWC S and the players in $N \backslash S$ as singletons and the demand vector is α^*. We first show that (α^*, σ) is in the DBS of the game.

Since (N, v) is proper, any objecting coalition T must contain some agents in common with S (i.e., $T \cap S \neq \emptyset$). T can make an objection to (α^*, σ) only if there exists a payoff vector y feasible for T such that $y_i > \frac{w_i}{q}$ for all $i \in T \cap S$. Define $Z \equiv \{i \in T : y_i \geq \frac{w_i}{q}\}$. Because $Z \supseteq \{T \cap S\}$ and all agents in $T \cap S$ must receive strictly more than $\frac{w_i}{q}$, it follows that Z is a losing coalition (Z winning would be

singletons is stable with any demand vector, which is to be expected since cooperation with other players is superfluous.

unfeasible). Thus, there must be a WC $C \subseteq N\backslash Z$, and a MWC $C' \subseteq C$. Since the game is homogeneous, α^* is feasible for any MWC, thus coalition C' has a counterobjection: since the game is proper, $C' \cap T \neq \emptyset$. Moreover, the fact that C' is included in $N\backslash Z$ ensures that all players in $C' \cap T$ are strictly better-off.

We have shown that no objection to (α^*, σ) can be justified, because every objection would lead to a counterobjection using α^* once again. In order to complete the proof, we now have to show that any pair (α, σ) with $\alpha \neq \alpha^*$ is vulnerable to justified objections.

Suppose (α, σ) is in the DBS but $\alpha \neq \alpha^*$. We define the sets

$U \equiv \{i \in N : \alpha_i < \alpha_i^*\}$, the set of "underdemanding" players.

$F \equiv \{i \in N : \alpha_i = \alpha_i^*\}$, the set of players demanding exactly α^*.

$O \equiv \{i \in N : \alpha_i > \alpha_i^*\}$, the set of "overdemanding" players.

There are two possibilities:

Case 1) $U = \emptyset$. Since $U = \emptyset$ and $\alpha \neq \alpha^*$, it must be the case that $\alpha_i \geq \alpha_i^*$ for all i and $\alpha_j > \alpha_j^*$ for some j. We know from Lemma 3 that σ contains a winning coalition S such that $\sum_{i \in S} \alpha_i = v(S)$. Since $U = \emptyset$, coalitions containing overdemanding players are unfeasible, thus S must be such that $\alpha_i = \alpha_i^*$ for all $i \in S$, therefore $j \notin S$. Any MWC $T \ni j$ has a justified objection. In this objection, any player $i \in \{T \cap F\}$ receives $\alpha_i^* + \epsilon$ and any player in $T \cap O$ receives some positive payoff. Notice that $j \in T$ and $T \in \Omega^m$ ensures that players in $T \cap F$ have less than q votes, so that this payoff distribution is feasible. Since $S \cap O$ is empty, any positive payoff makes the players in $T \cap O$ better-off, hance we indeed have an objection. There is no possible counterobjection, since it would have to include some players in $T \cap O$, and any coalition containing overdemanding players is unfeasible.

Case 2): $U \neq \emptyset$. This implies $O \neq \emptyset$ (otherwise any MWC containing a player from U would have a justified objection). Again, we know from Lemma 3 that σ contains a winning coalition S such that $\sum_{i \in S} \alpha_i = v(S)$. We will distinguish two cases:

Case 2a) $S \subseteq F$. Take any player $j \in U$, and any MWC $T \ni j$. Denote $T \cup U$ by Z. Coalition Z has a justified objection in which every member of $Z\backslash O$ gets $\alpha_i^* + \epsilon$ and every member of $Z \cap O$ gets a positive payoff. To see this, notice that this payoff division is feasible for Z if ϵ is small enough.[15] $S \subseteq F$ implies that all players in Z are made better-off by the objection. Finally, since a counterobjection must include at least one player from $Z \cap O$, and it can include no players from U, no counterobjecting coalition is feasible given α.

Case 2b) S is not a subset of F. Lemma 3 implies $S \cap O \neq \emptyset$ and $S \cap U \neq \emptyset$. Notice that S need not be a MWC. There is a MWC $S' \subseteq S$ such that $S' \cap O \neq \emptyset$ and $S' \cap U \neq \emptyset$, and $S \cap O \subseteq S'$. Let $Z := (N \backslash S') \cup U$. As in case 2a, Z has a justified objection in which every member of $Z \backslash O$ gets $\alpha_i^* + \epsilon$ and every member of $Z \cap O$ gets a positive payoff. Note that the difference with case 2a is that, in the construction of Z, we had to make sure that $Z \cap (S \cap O) = \emptyset$.

Given that α^* is the only stable demand vector and that dominated coalition structures are excluded from the DBS, σ must contain a minimal winning coalition.

If there are dummy players, their actual payoff is always zero but their demands are not constrained. A dummy player can be part of a winning coalition in σ if his demand is zero. **QED.**

Remark 8 *Notice that the DBS depends only on the characteristic function, and hence it is invariant to the particular representation chosen.*

Remark 9 *Note that what matters is the homogeneous representation. For example, in every 3-player constant-sum game without veto players any homogeneous representation gives equal weights to the three players, and hence the unique stable demand vector is $\alpha^* = (1/2, 1/2, 1/2)$.*

Example 3 *(apex game)* Consider the game $(4; 3, 1, 1, 1, 1)$. The DBS predicts either a coalition of the four small players with payoff division $(1/4, 1/4, 1/4, 1/4)$ or a coalition of the large player and a small player with payoff division $(3/4, 1/4)$.

For the sake of illustration, we show that coalition structure $\{\{1,2\},\{3\},\{4\},\{5\}\}$ together with the demand vector $(3/4, 1/4, 1/4, 1/4, 1/4)$ is in the DBS. An objection by, say, $\{1,3\}$ must give player 3 less than $1/4$, and therefore can be countered

[15] This follows because $Z \backslash O$ is a losing coalition. If it were winning, one could find a subset of $Z \backslash O$ having exactly q votes, and containing at least one player from U (because of the way Z has been constructed, $Z \backslash U$ is losing), and this set would have a justified objection.

by $\{2,3,4,5\}$; an objection by $\{2,3,4,5\}$ must give one of the players (say, player 5) less than 1/4, and thus can be countered by $\{1,5\}$. As an example of other demand vectors not being stable, consider the demand vector $(1/2, 1/2, 1/2, 1/2, 1/2)$ together with the same coalition structure. An objection by $\{1,3\}$ with $(3/4, 1/4)$ cannot be countered.

Remark 10 *If (N, v) is a constant-sum homogeneous game without veto players or dummies, then the associated stable demand vector α^* is the only solution of the following program:*

$$\begin{array}{rl} & \min \sum_{i\in N} \alpha_i \\ s.t.\ \sum_{i\in S} \alpha_i & \geq 1 \text{ for all } S \in \Omega^m. \end{array} \tag{5}$$

This implies that α^* is a balanced aspiration.

Proof. Suppose α is a solution to the minimization problem (that is, a balanced aspiration) but $\alpha \neq \alpha^*$. This implies $U \neq \emptyset$. Take $i \in U$ such that $i \in \arg\min_{j\in U} w_j$ and a MWC S containing i. Because of maximality of balanced aspirations, the sum of the demands in this MWC is at least 1. In order for α to have a smaller sum than α^*, there must be at least another player from U in $N\backslash S$. Consider the set $N\backslash S \cup \{i\}$. Because the game is constant-sum, this is a WC. If it is also a MWC, it contradicts maximality. Otherwise, there is a MWC included in it. In order for maximality to be satisfied, there must have been some player $j \in U$ that has been thrown away. Because we could take j away and still have a WC, $w_j < w_i$, a contradiction. **QED.**

Recall the following result shown by von Neumann and Morgenstern (1944): If for a constant-sum simple game (N, v) there is a vector $x = (x_1, ..., x_n)$ with each $x_i \geq 0$ such that $x(S) = 1$ whenever S is a MWC, the set of imputation vectors

$$\{z^S : z_i^S = x_i\ \forall i \in S,\ z_i = 0\ \forall i \in N\backslash S,\ S \in \Omega^m\}$$

is a stable set, and was called the *Main Simple Solution* by von Neumann and Morgenstern. The weighted majority games we have studied in this section are a subset of this set of games (with $x = \alpha^*$), hence the set of imputations $\{\alpha^{*\sigma}\}$ derived from the set of pairs $\{(\alpha^*, \sigma) \in DBS\}$ coincides with the main simple solution for this class of games. This means that among the many stable sets of constant-sum homogeneous games, the DBS selects the one that was singled out by von Neumann and Morgenstern as the most meaningful one. Furthermore, the DBS makes the same predictions as the main simple solution for *all* constant-sum simple games admitting a main simple solution, even if they are not weighted majority games. More precisely,

Remark 11 *Let (N, v) be a constant-sum simple game admitting a main simple solution with associated vector x. If there are no veto or dummy players, the DBS consists of all pairs (α^*, σ) where $\alpha_i^* = x_i$ for all i and σ contains a MWC S and the players in $N \backslash S$ as singletons. If there are dummy players, the DBS consists of pairs (α, σ) where $\alpha_i = x_i$ for all nondummy players and σ contains any winning coalition feasible for α.*

The proof is identical to that of Theorem 3. It only requires that the game is constant-sum and that all minimal winning coalitions can afford α^*; it does not require the game to be a weighted majority game.

Despite this coincidence of predictions, the set of imputations predicted by the DBS need not be a von Neumann Morgenstern stable set for general games - not only external stability but also internal stability can fail (see Morelli and Montero (2001)).

3.3 Proportional Payoffs in Other Solution Concepts

In constant-sum homogeneous games without veto players, the prediction that any minimal winning coalition can form and that the payoff division will be proportional seems very compelling to us. Corollary 3 implies that α^* is partnered, equal gains and in the aspiration kernel. These aspiration solution concepts, however, may admit demand vectors other than α^* for this kind of games.[16]

No solution concept in the bargaining set family makes the proportionality prediction as the unique prediction, but proportional payoffs may be stable according

[16] Consider the game $(6; 4, 2, 2, 1, 1, 1)$. The demand vector $(\frac{14}{18}, \frac{7}{18}, \frac{7}{18}, \frac{2}{18}, \frac{2}{18}, \frac{2}{18})$ is partnered, equal gains and in the aspiration kernel, but it is not proportional to the homogeneous weights.

to some solution concepts: this is the case for the nucleolus for the grand coalition (though not for other coalitions) as proved by Peleg (1968). For minimal winning coalitions, the proportional payoffs are always in the classical bargaining set. Thus, for constant-sum homogeneous games the DBS selects a subset of the classical bargaining set.

Remark 12 *Let (N, v) be a constant-sum homogeneous game. The proportional payoff division is in the classical bargaining set for all coalition structures σ containing a minimal winning coalition S.*

Proof. Let i, $j \in S$. Suppose i has an objection against j via coalition T. If $w_i \geq w_j$, then j can counterobject by forming a MWC Z with players from $N \backslash S$ (always possible by corollary 3). Players in $Z \cap T$ can be paid the same as i offered, the rest can receive 0 and j keeps at least what i got in the objection; since $w_i \geq w_j$, this is at least j's original payoff. Now suppose $w_i < w_j$. Take a MWC Z formed by j and players from $N \backslash S$. Notice that, because of the way Z has been constructed, any player $k \notin S \cup Z$ has $w_k < w_j$ (that is, when constructing Z we cannot "throw away" a player larger than j). Does Z have a counterobjection? If so, we are done; if not, this means that the players in $Z \cap T$ are on the aggregate "overdemanding". Thus, $T \backslash Z$ are on the aggregate "underdemanding". Any underdemanding player $k \in T \backslash Z$ is outside S and therefore has $w_k < w_j$. Now consider a coalition T' containing k such that $T' \cap T = \{k\}$. This coalition has a counterobjection: it includes j, excludes i and can afford α^*. **QED.**

It may be interesting to note that the kernel (Davis and Maschler (1967)) predicts an equal (rather than proportional) division for all minimal winning coalitions, regardless of the weights of the players. The same can be said of some of its supersets (Granot and Maschler (1997), Potters and Sudhölter (2001)).

4 Implementation of the DBS

In this section, we provide an implementation of the DBS via a simple mechanism in which an auxiliary set of individuals compete over the n players in the cooperative game, following Pérez-Castrillo and Wettstein (2000). We will assume that the cooperative game (N, v) is zero-normalized, superadditive and has the following property: $v(S) > \sum_{i \in S} v(i)$ implies $v(N \backslash S) = \sum_{i \in N \backslash S} v(i)$. This property is called the *one-stage property* (Selten, 1981) and implies that no more than one profitable coalition can be formed at a time.

The mechanism is played by four auxiliary individuals called principals. The principals have lexicographic preferences: in the first place they want to maximize profits but, other things equal, they prefer to hire as many agents as possible.[17]

The *mechanism M* is played as follows:

Stage 1. Principal 1 (P1) chooses (α, S), with $\alpha \in \mathcal{R}^n$ and $S \subseteq N$. The bid of P1 for agent i, x_i, is computed in the following way: $x_i = \alpha_i$ for $i \in S$ and 0 otherwise.

Stage 2. P2 and P3 simultaneously choose y^2 and $y^3 \in \mathcal{R}^n$. We will denote $\max(y_i^2, y_i^3)$ by y_i. Given the bid vectors x, y^2 and y^3, each agent is provisionally assigned to the principal that offers the highest price (or wage). Ties are broken in favor of the principal with the lowest index. Let T^j be the set of players provisionally assigned to principal $j = 1, 2, 3$. If one principal gets all the agents, the game ends and the principal that got the agents receives $v(N)$ less the wages of the n agents. If no principal has got all the agents, the game moves to the next stage.

Stage 3. P4 may hire any set Z of agents under the following conditions:

1) He has to offer them $z_i = \alpha_i$;

2) Z must contain elements of both T^1 and $T^2 \cup T^3$;

3) z must make all the players in Z weakly better-off, and players in $Z \cap (T^2 \cup T^3)$ strictly better-off.

Finally, principal j $(j = 1, 2, 3)$ hires the agents in $T^j \backslash Z$ and pays them the wage offered; P4 hires the agents in Z and pays them z.

Theorem 4 *The mechanism M implements the DBS in SPE.*

Proof. We first construct an equilibrium of the mechanism M for any (α, σ) in the DBS.

[17] This type of preferences is also assumed by Pérez-Castrillo and Wettstein in one of their mechanisms.

Consider the following strategies: P1 submits (α, S), where S is the set of players receiving their demands given (α, σ), that is, $S = \{i \in N : \alpha_i^\sigma = \alpha_i\}$. After observing this choice, P2 and P3 bid $y^2 = y^3 = x = \alpha^\sigma$ (and play an equilibrium in all other subgames). If the game reaches stage 3, P4 plays a best response.

Given these strategies, P1 gets all the agents and all principals make zero profits. To show that this is an equilibrium, we prove that there is no profitable deviation at stage 1 or 2 (since at stage 3 this is the case by construction).

Let us first rule out that P1 has a profitable deviation. This cannot be the case because P2 would not be playing a best response.[18]

Any profitable deviation by P2 or P3 corresponds to an objection to (α, σ). The third stage would be reached after the deviation (an objection that takes all the players is excluded by theorem 2). Since (α, σ) is in the DBS, there is a counterobjection to this objection. P4 hiring some agents corresponds to a counterobjection and, since P4 prefers to hire more agents rather than less, he will always counterobject. The one-stage property ensures that the deviator (P2 or P3) would be left with a negative profit.

Now we prove that the outcome of any SPE of the mechanism M is in the DBS (provided nonempty).

First, all principals make nonnegative profits in equilibrium. Second, they all make zero profits (if some principal is making positive profits, P2 or P3 could bid ϵ more for all agents the other principals would be getting, get all the agents and end the game). Third, P1 must have all the agents. Because of the one-stage property, P1 can secure all the agents by choosing (α, S) such that the corresponding (α, σ) is in the DBS, so any strategy combination in which P1 does not get all the agents cannot be an equilibrium. Fourth, (α, σ) is in the DBS. Suppose not. Then P2 is not playing a best response, since he can make an objection, secure some agents, and P4 cannot do anything since there is no counterobjection.[19] **QED.**

[18]Since P2, P3 and P4 play best responses in all subgames, after the deviation by P1 all principals make nonnegative profits. Moreover, P3 and P4 pay for the agents at least what P1 offered them. It follows that P2 cannot be playing a best response: slightly overbidding P1 (and possibly P3) and thus getting all the agents and finishing the game would be profitable.

[19]In the mechanisms of Pérez-Castrillo and Wettstein (2000) the designer does not need to know the characteristic function; in our mechanism, the game is zero-normalized, so the designer knows

5 Concluding Remarks

Payoff distribution and coalition formation should be studied simultaneously. Since value concepts give only an *ex ante* evaluation of the prospects of different players, they cannot be used to predict the *ex post* payoff distribution in an "equilibrium" coalition structure. Solution concepts that keep the spirit of core-like competition, respecting individual rationality as well as group rationality, seem more appropriate for this task. The demand bargaining set is, we believe, a meaningful addition to the set of concepts of this family. In fact, it is a selection of the Zhou bargaining set that manages to eliminate from the set of solutions all the counterintuitive solutions that are dominated, and gives a precise prediction for the important class of constant-sum homogeneous games. The selection of undominated coalition structures makes the DBS, in our view, a very useful concept, with a much sharper predictive power than previous concepts in the family of bargaining sets. It is not always nonempty, but the proportional prediction for constant-sum homogeneous games suggests that it can be used in important allocation and distribution problems where the core is empty.

However, predictive power was not the only motivation of the paper. The main conceptual contribution of the demand bargaining set is the idea itself of self-stability of demands. A set of "claims" by the players is considered stable if the same claims can be used by some subset of the players to counter any possible objection to the assignment of those claims. Intuitively, this is an important requirement in distributive politics or in any distributive problem in legislatures: the advocates of any given distributive "norm" (like "to everyone according to its contribution") should be able, if they want that norm to prevail, to counter any objection by making reference to the norm itself. Countering an objection with a completely unrelated counterobjection would not be convincing as much as an argument that uses the same norm originally proposed. Asking that counterobjections have to

$v(i)$ for all i in N. The extension to a completely unknown characteristic functions is straightforward: let P1 complete the vector x himself and add a final stage in which the agents accept or reject the wage offer (this would have been a problem for the original mechanisms of Pérez-Castrillo and Wettstein since they implement bargaining sets that are not individually rational). In equilibrium P1 will complete the vector x with $v(i)$ for all $i \notin S$.

use exactly the initial demand vector is an extreme form of consistency with the initial proposed norm, and in future research it could be interesting to study weaker forms of consistency, but we believe that this concept will serve at least as an useful benchmark for this type of investigation of the way social norms are agreed upon.

Morelli (1999) studies demand bargaining games that produce similar predictions to the DBS for homogeneous weighted majority games. However, the connection between those noncooperative games and the DBS is not one-to-one. Selten (1981) implements the set of partnered aspirations, which is a superset of the DBS for constant-sum homogeneous games. Here, instead, we managed to obtain a tight implementation result.

References

[1] ALBERS, W. (1974). Zwei Lösungkonzepte für Kooperative Mehrpersonenspiele, die auf Anspruchsniveaus der Spieler Basieren. *OR-Verfahren* (Methods of Operations Research) 21, 1-13.

[2] AUMANN, R.J., and J.H. DRÈZE (1974), "Cooperative Games with Coalition Structures", *International Journal of Game Theory* **3**, 217-137.

[3] AUMANN, R. and M. MASCHLER (1964), "The Bargaining Set for Cooperative Games", in: M. Dresher, L.S. Shapley and A.W. Tucker, eds., *Advances in Game Theory.* Princeton: Princeton University Press, 443-476.

[4] BENNETT, E. (1983), "The Aspirations Approach to Predicting Coalition Formation and Payoff Distribution in Side Payment Games," *International Journal of Game Theory* **12**, 1-28.

[5] BENNETT, E. (1985), "Exogenous vs. Endogenous Coalition Formation", *Economie Appliquée* **37**, 611-635.

[6] CROSS, J. (1967), "Some Theoretical Characteristics of Economic and Political Coalitions", *Journal of Conflict Resolution* **11**, 184-195.

[7] DAVIS, M. and M. MASCHLER (1965), "The Kernel of a Cooperative Game," *Naval Research Logistics Quarterly* **12**, 223-259.

[8] DAVIS, M., and M. MASCHLER (1967), "Existence of Stable Payoff Configurations for Cooperative Games", in M. Shubik (ed.), *Essays in Mathematical Economics in Honor of Oskar Morgenstern*, Princeton University Press, 39-52.

[9] GRANOT, D. and M. MASCHLER (1997), "The Reactive Bargaining Set: Structure, Dynamics and Extension to NTU Games", *International Journal of Game Theory* **26**, 75-95.

[10] MASCHLER, M. (1992), "The Bargaining Set, Kernel, and Nucleolus," in R. Aumann and S. Hart (eds.), *Handbook of Game Theory with Applications*, 591-667.

[11] MAS-COLELL, A. (1989), “An Equivalence Result for a Bargaining Set,” *Journal of Mathematical Economics*, **18**, 129-138.

[12] MORELLI, M. (1999), “Demand Competition and Policy Compromise in Legislative Bargaining,” *American Political Science Review* **93**, 809-20.

[13] MORELLI, M. and M. MONTERO (2001), ”The Stable Demand Set: General Characterization and Application to Weighted Majority Games”, Ohio State University Working Paper 0103.

[14] PELEG, B. (1967), ”Existence Theorem for the Bargaining Set $\mathcal{M}_1^{(i)}$”, in M. Shubik (ed.), *Essays in Mathematical Economics in Honor of Oskar Morgenstern*, Princeton University Press, 53-56.

[15] PELEG, B. (1968), “On Weights of Constant Sum Majority Games,” *SIAM Journal of Applied Mathematics* **16**, 527-532.

[16] PÉREZ-CASTRILLO, D. and D. WETTSTEIN (2000), ”Implementation of Bargaining Sets via Simple Mechanisms”, *Games and Economic Behavior* **31**, 106-120.

[17] SUDHÖLTER, P. and J.A.M. POTTERS (2001), “The Semireactive Bargaining Set of a Cooperative Game,” *International Journal of Game Theory*, **30**, 117-39.

[18] SELTEN, R. (1981), “A Non-Cooperative Model of Characteristic Function Bargaining,” in V. Böhm and H. Nachtkamp (eds.), *Essays in Game Theory and Mathematical Economics in Honor of Oscar Morgenstern*, 131-151. Mannheim: Bibliographisches Institut.

[19] VON-NEUMANN, J. and O. MORGENSTERN (1944), *Theory of Games and Economic Behavior,* Princeton N.J., Princeton University Press.

[20] WARWICK, P.V. and J.N. DRUCKMAN (2001), “Portfolio Salience and the Proportionality of Payoffs in Coalition Governments”, *British Journal of Political Science*, **31**, 627-49.

[21] ZHOU, L. (1994) "A New Bargaining Set of an N-Person Game and Endogenous Coalition Formation", *Games and Economic Behavior* **6**, 512-526.