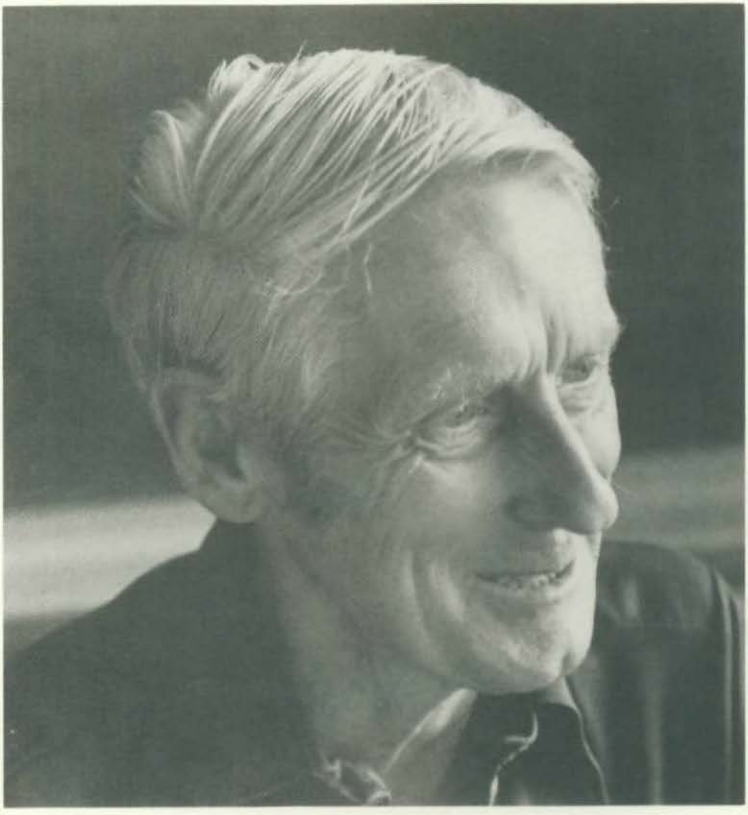
DEANE MONTGOMERY

1909–1992

INSTITUTE FOR ADVANCED STUDY



RESOLUTION

By the Board of Trustees

"THE BOARD notes with great sorrow the death of DEANE MONTGOMERY, Professor Emeritus in the School of Mathematics, on March 15, 1992.

DEANE MONTGOMERY was born in Minnesota in 1909 and studied at the University of Iowa where he received his Ph.D. in 1933. His first contact with mathematics at the Institute was in Princeton University's Fine Hall in 1934, at that time the only meeting place for all Princeton mathematicians since Fuld Hall was not yet built. Professor Montgomery taught at Smith College and Yale University and was a Member in the Institute's School of Mathematics in 1945–46, returning in 1948 as a permanent Member until 1951 when he became a member of the Faculty. For nearly forty years, as Professor until 1980 and then as Professor Emeritus, Deane Montgomery was a familiar and revered figure in our midst. Concerned always for the training and encouragement of younger mathematicians, he made his Topology Seminar into a forcing ground for important results in that field. His quiet dedication to mathematics, his firm support for the highest standards of scholarship, and his care for the Institute are a legacy which has enriched us all.

We extend to Deane Montgomery's widow and family our deepest sympathies and express herein our admiration of him and our appreciation for the distinction he brought both to his profession and the Institute for Advanced Study."

MEMORIAL COMMENTS

The following recollections on Deane Montgomery are reproduced in the order they were given. A. Selberg, G. D. Mostow, C. T. Yang and R. Fintushel delivered them in person at the Memorial Service. L. Zippin, K. Chandrasekharan and R. Bott could not attend; their contributions were read by A. Borel, the first one at the Memorial Service, and the last two after a dinner honoring the speakers. Also included is an obituary written by A. Borel for the Notices of the American Mathematical Society.

DEANE MONTGOMERY: IN MEMORIAM

I would like to welcome you all here today to this gathering in honor of Deane Montgomery. Deane was a central figure in the development of mathematics during the middle part of this century. He filled this role as a scholar, as an educator and as a leader of the mathematics community. Deane Montgomery was associated with the Institute for Advanced Study for approximately half a century. During this period he helped very significantly to shape the School of Mathematics as well as the Institute. He came to an institution that was in its infancy, yet in some ways already venerable. In his service as a Faculty member he was instrumental in enabling the Institute to meet its responsibility to the subject of mathematics and to the mathematical community and therefore to form as an institution. Today we will hear remembrances from a number of Deane's colleagues and friends.

November 13, 1992
Princeton,
New Jersey

PHILLIP A. GRIFFITHS
Director
Institute for Advanced Study

Atle Selberg
Institute for Advanced Study
Princeton, New Jersey

My first association with the Institute was as a temporary member here for the year 1947–48. I then went to Syracuse University for a year. In the fall of 1948, I received an offer to come back to the Institute as a permanent member, a status that — I am happy to say — is no longer in use here; it did have some drawbacks! At that time I also learned that Deane Montgomery had come from Yale to join the Institute as a permanent member, and when I myself arrived here in the fall of 1949 we finally met.

My first impression was that he was a remarkably handsome man. He was just 40 then, and hearing that he had spent about ten years teaching at Smith College, I couldn't help thinking that mathematics probably gained considerably in popularity there while he was holding forth on the subject. As I learned to know him more, it was his character, his honesty and integrity that impressed me.

That we both were permanent members meant that we had no formal duties except being in residence during the academic year. We were not really on the faculty and received an annual "grant-in-aid" rather than a salary. As the tax laws were then interpreted, this grant was considered tax free.

Shortly after my arrival Oswald Veblen asked if we would sit in on the School meetings in order to assist in the consideration of the applications for memberships and grants. So Deane and I, in spite of having no duties, dutifully sat down to study the applications and were present at these School meetings, offering our comments on the merits of the applicants. However, we were not allowed to vote, and if anything outside of the applications came up, we could not even comment, though we at times felt sorely tempted to do so.

Our common, somewhat awkward status — not on the faculty, but still in the faculty in some sense — was undoubtedly what brought us close together quite early. It certainly was not common interests in mathematics; we hardly ever discussed mathematics except in connection with some application for membership, and

in later years, in connection with the consideration of a permanent appointment in the School of Mathematics.

Veblen did take quite an interest in Deane and myself. The three of us often went out to lunch together, always to the so called "stag room" of the Nassau Tavern — now the Nassau Inn. Veblen impressed me by always being able to get lamb chops regardless of what was on the menu! During these lunches as well as at times when we dropped in on him at his office, Veblen would talk about the Institute and give us his reminiscences from the early years of its history and development. I believe that Veblen more than anyone else shaped the Institute in the form it came to take. He always maintained that the temporary members constituted the most important part of the Institute; even to the extent that he thought it could have functioned successfully without a faculty, if there would have been a competent outside panel which would select a suitable mix of young and more senior members for each year.

In retrospect, I think Veblen was concerned about the future of the Institute, and in particular about the stability of the School of Mathematics, and therefore wanted to transmit his ideas concerning the Institute to us as the youngest permanent members of the School to ensure continuity. Deane probably took Veblen's teaching even more to heart than I, for in later years it was he more than anyone who, so to say, took over Veblen's mantle and carried on his legacy in the somewhat critical period that followed. It was very appropriate and fitting that on Veblen's death in 1960, Deane inherited not only his office in Fuld Hall but also his archives of minutes from Faculty and School meetings from the earliest years.

The mathematics faculty became rather depleted in the years immediately following Veblen's retirement in 1950. Herman Weyl retired the next year. Actually the difference in their age was about six years, but Einstein and Veblen had special terms in their appointments and could stay on till the age of 70, while the retirement age for others at that time was 65. In 1951 Carl Ludwig Siegel decided to return to Göttingen in Germany, and James Alexander decided to withdraw from the Institute as a professor at about the same time. On the faculty only Marston Morse and von Neumann were left of the mathematicians, and it must be added

that von Neumann was not very involved with mathematics any more, but was pursuing other interests. A few years later he accepted an appointment to the Atomic Energy Commission and left for Washington.

This state of affairs may have been the reason that Deane and I both were appointed professors in 1951. A contributing reason could also have been that the Internal Revenue Service had changed the rules so that only a small fraction of our grant-in-aid was now considered tax free.

Our promotion meant that counting von Neumann there were only four professors left in Mathematics. As this seemed rather inadequate, it was clear that we had to exert ourselves to restore the mathematics faculty to a reasonable strength with new appointments, particularly since Marston Morse was scheduled to retire in 1957. In fact, he did stay on till 1962 since the retirement age was raised to 70 before his retirement.

Our united efforts led to some excellent appointments in the years following: Hassler Whitney 1952, Kurt Gödel 1953, Arne Beurling 1954, Armand Borel 1957 and André Weil 1958. With this added strength we felt that the School was again on a secure basis at least for the near future.

Throughout this time and later, Deane, who was a very outgoing person, was extremely active in the mathematical life at the Institute. First of all, he ran a seminar in topology every year, where the most recent work in the field was presented and discussed. He also had more contact with the temporary members outside his own fields of interest than most of us. This extended to the social life also; Deane and Kay invited every year all of the members of the School with spouses to their home. This became more and more difficult with the years as the number of members increased from about 25 in the early years to over 60. Besides Kay and Deane, only Louise and Marston Morse managed to carry on this scale of hospitality to the members in later years. Deane knew not only the members of the School but nearly everyone in the Institute. His interest in and concern for the Institute was not only with the academic side, but encompassed buildings and grounds as well as finances. He was the only faculty member I knew of

who asked to have the comprehensive statement of Institute finances sent to him each year. Most of us were satisfied if we just had the School budget in hand.

In 1962, the efforts of the School to make the best new appointment in anticipation of Marston Morse's retirement that year brought on a prolonged period of conflict. I shall not go into the substance of this conflict here, but its impact on Deane was particularly hard. More than anyone he carried on the concern for and dedication to the School and to the Institute as a whole that Veblen had exemplified, particularly a concern for the very highest academic standard. But, unlike Veblen, Deane was not really cut out for conflict. Veblen had been politically very astute and skillful, and he could engage in conflict and then shut it out once he went into his office or to his home. Deane was not talented in this way. He was too straight-forward for that, and when he cared deeply and felt strongly about a cause, he carried his concerns with him 24 hours a day. When he suffered a heart attack in the 60s, I thought it was brought on by the high degree of stress that these years exposed him to. Fortunately he recovered, but he had to keep a rather strict regimen of diet and exercise after this. It did not interfere with his mathematical work however.

Deane was a man who liked to work with others. Often, when I knocked on his office door to talk about some School or Institute matter or simply to have a chat, he would have one or more persons in there discussing some mathematical problem. Over the years, he particularly had two long-term collaborators that I remember. They did not reside in Princeton but came regularly once a week to see him. In the early years it was Leo Zippin, and in the later years C. T. Yang. I must confess that looking at this memorial program I realize for the first time what C. T. stands for. Deane always used only the initials.

Deane's interest in and concern for the School and the Institute continued also after his retirement. As he was always very gregarious and friendly and talked with everyone, or at least nearly everyone, he knew much more about what was going on than I did. To a large extent he was my main source of information, and after he moved to North Carolina, I felt often a bit "out of the loop," in addition to my general sense of personal loss.

It was undoubtedly a heartrending decision for him to make that move and so to separate himself from the Institute which had been such a large part of his life. However, increasing age and decreasing health made it necessary for Kay and him to move to some type of retirement home, and they felt that what was available closer to the Institute was not the appropriate choice in their case.

It also felt hard for us, his colleagues; he had been a large part of our lives too, perhaps particularly so of mine. We had shared a long time, some of it rather difficult, here.

Last spring, I had returned from a six-month stay abroad and I was thinking of getting in touch with Deane again. The news of his death came as a great shock to me. But he will surely long live in memory here and also in other places where former members of our School reside. Deane did play an important role in a number of organizations. He was active in the American Mathematical Society and held several offices there, including that of president. He was active as well in the International Mathematical Union and served as its president for a four-year term. Deane was universally highly respected by the whole mathematical community; and in spite of the sometimes bitter conflicts in the past, I believe he came ultimately to be regarded with respect and affection by the whole Institute community.

G. Daniel Mostow
Yale University
New Haven, Connecticut

1. Deane Montgomery is one of the very few people I have encountered who has remained a hero to me as I have passed from youth to middle age to the age that is now mine.

Heroes are very rare, in modern times especially. How can one pay homage to a hero? I think that I might do that best by relating my personal interactions with him and surmising what I can about the inner Deane.

I first met Deane in September of 1948 when he joined the faculty of the Institute as a Permanent Member and I was starting my second year of membership. We had lots to talk about mathematically because at that time, before Chevalley's book "Theory of Lie Groups" was published, we were two of the very few American mathematicians familiar with Lie groups generally and who worked on transformation groups in particular. He had heard about the results in my Ph.D. thesis from Leo Zippin whom I had met a year earlier, and I felt flattered by Deane's interest.

What took me entirely by surprise one morning was his asking me, "How is the baby getting along?" Throughout my graduate years at Harvard, no faculty member had ever expressed any interest in my personal life, and I accepted the generation gap between faculty and students as unbridgeable.

With Deane there was no generation gap. He was my first genuine friend of an older generation, i.e. fourteen years my elder.

In 1948, the Institute was smaller than it is today, Fuld Hall and the newly completed building C were its only buildings for mathematics. The von Neumann Computer Center housed only computer staff. In those days, before National Science Foundation grants made foreign travel commonplace, most members from abroad were visiting our country for the first time. It was usual to sit around the common room during and after tea, conversing about topics of common interest. The Americans, especially the bachelors, would often talk about the new cars which were just beginning to become more easily available in that post World War II period. The foreign members would often give voice to their

impressions about life in the United States, and we would try to learn from them about life in their countries. Sometimes the conversation would lapse into gossip.

In such sessions, the opinion was volunteered more than once by one or another of the foreign mathematicians that "Deane Montgomery exemplifies the best of America" or "Deane is the best of Americans." This was hardly news to the Americans.

Deane's disposition was remarkably friendly, at all times and to all people.

I still remember how astounded I was when, in comparing our families, it turned out that he was an only child. How could an only child be so unspoiled? So generous with his time to young mathematicians, listening patiently to the enthusiastic discoveries of the night before? Listening to the unburdening of personal problems and offering advice — but only when asked. He was the one I consulted at career junctures; his advice was invariably distilled wisdom.

2. Deane himself spent most of his research life tackling difficult problems and knew the loneliness of being stuck. His encouragement helped many a young mathematician who was stuck get over the obstacle. That is why so many of us here today feel so indebted to him.

He used to say that if he got absolutely nowhere with a problem after a month's effort, he would put the problem aside, at least for a while, and take up another until he would get a new idea.

I did have the privilege of seeing him excitedly coming back to Hilbert's fifth problem after having set it aside for the n^{th} time. He had just received word from Andy Gleason that Andy had succeeded in proving the existence of a connected arc in any finite dimensional locally compact group. This theorem may sound innocuous, but Deane realized its import and immediately embarked on the final push, together with his long-time collaborator Leo Zippin, to solve Hilbert's fifth. The final solution is presented in two papers that appeared back-to-back in the Annals of Mathematics v. 56, 1952. Gleason's: *Groups without small subgroups,* pp. 193–212, received by the editors June 13, 1952 and Montgomery-Zippin's: *Small subgroups of finite dimensional groups,* pp. 213–241, received by the editors March 28, 1952.

The second paper, uses the result of Gleason's paper, which Andy had communicated to them. This paper marked the climax of a long strenuous effort that had lasted more than a generation. Deane was very pleased with this success, in his quiet way. But immediately thereafter he resumed his program of research on transformation groups.

(In this audience I cannot refrain from the parenthetical historical comment: Gleason's arc resulted from his remarkable idea of constructing a semi-group of subsets; according to Gleason, that idea came to him while reading Hille's book "Semi-group of operators on Hilbert space" — a wonderful instance of unpredictable pregnancies in mathematics.)

3. Deane had a very strong sense of family and his devotion to daughter Mary and son Dick was matched by their love for him. Those who knew him then realize how deeply he and Kay suffered when their son Dick returned home to Princeton with cancer, to spend the terminal 10 months of his life at home. Deane slept in a sleeping bag in the hall outside Dick's doorway, so that he might tend to Dick, if needed. The morning after Dick died, Deane gazed out the window into his garden where he had planted flowers at the base of a tree. Dick had been fond of looking at the flowers, reporting to Deane how they were doing. Gazing at the flowers that next morning, Deane said: "I wish I had planted 10 times as many." That was as much as he could voice in their sorrow.

It remains to offer some remarks about Deane's role as a member of the Institute's faculty. This role poses a paradox.

I have already described how the members of the School of Mathematics perceived Deane. However, the Deane perceived by some of the Institute's administration did not match the members' description at all. The key to this contrasting behavior lies in Deane's background.

Fortunately he loved to relate stories about the region of his birth to his children, and thanks to his daughter Mary, we can learn a bit of his family history.

Deane was born September 2, 1909, in Weaver, Minnesota, a town of about 100 inhabitants. Both his grandfathers had been

pioneers. In their time, the land in Minnesota was just beginning to be opened up for farming. His paternal grandfather John Montgomery, was of Scottish-Irish origin. In the 1840's, during the famine in Ireland, John Montgomery, together with his two brothers Richard and George, emigrated to the United States. Probably because they could not write, the brothers lost track of each other. In Deane's words to his children Mary and Dick, "John Montgomery came to Minnesota, walked into the woods with his axe, chose a plot along the river for his farm, built his log cabin, and started farming." There were Indians in the region and the Grandfather Montgomery got along well with them. The Indians would fish and hunt near the farm, and after some time would give John part of their catch in return for storage services. The Indians were friendly, but had the disconcerting practice of walking into the cabin unceremoniously. That was one of the reasons his wife, Grandmother Montgomery, prevailed upon him to move into Weaver.

Deane's maternal Grandfather Hitchcock had emigrated from England and was also a Minnesota pioneer. Both of Deane's parents were born in log cabins.

Deane started school in Weaver's one-room school house. He couldn't avoid overhearing and absorbing the lessons of the older pupils, and as a result, he skipped several grades.

His 8th grade class was the largest in the school's history — 3 pupils.

Deane's father died when Deane was 11.

At age 14, he started high school at Wabasha — the first in his family to go to high school. His first year he lived at home, commuting on an early morning train. The second year, his mother allowed him to board away from home in Wabasha. The high school was not big. As Deane told Mary and Dick, "it had 12 boys — one of whom was unfortunately crippled, so I had to play on the football team. The team made only one touchdown all season. I was on the bottom of a pileup when it happened, so I didn't even see the touchdown."

His father had left enough money so that Deane could plan to attend college. The University of Minnesota was ruled out however, because Deane's mother thought it was not safe for Methodists. That meant enrolling at Hamline College.

Summertime, Deane had to work to supplement the money left by his father. Work meant farm work, and Deane was proud of having been "champion cabbage planter."

From his widowed mother Deane inherited a rigorous egalitarianism. She believed that all people are equal under God and that everyone was as good as anyone else — an attitude Deane kept throughout his career. He also inherited from her the habit of understatement.

Deane graduated from Hamline at age 20, went on to the University of Iowa where he got his Ph.D. in 1933. He visited Harvard then Princeton, as a National Research Council Fellow. From the savings he accumulated by frugal living on his Fellowship, he and Kay were able to join Ted Martin and his wife on a summer tour of Europe.

They enjoyed that trip, but that seems to have started his appetite for foreign travel. For the rest of his years, tourism for Deane was travel with his family, enjoying the drive through the countryside. Cities were avoided. Rather he enjoyed pointing out the different stands of crops, or calling out "Look, there's a Northern Shrike." At home he kept a rain gauge so that he could know just how much rain had fallen.

I go into Deane's roots in order to recapture his perspective.

In his view, mathematics is a subject which the public cannot experience — in fact, only mathematicians really care about what is going on in mathematics. If a mathematician's work is not appreciated by mathematicians, then all the effort directed into that mathematician's training and all the hard work is in vain — a sad end to a noble beginning.

Given Deane's egalitarian inclinations, he felt deeply that no mathematical achievement should be belittled. I recall his remark in 1948 about the bruised ego of a well-known Dutch mathematician whose theorem was not getting the recognition he expected. "If we don't show appreciation, then who will?."

He went out of his way to encourage young mathematicians. He treated them, and that included me, as equals. In their younger days Deane and Kay were wonderful hosts to the young members and their wives.

The keen awareness of being not far removed from the back woods of Minnesota, reinforced by his provincial education, carried over to Deane's view of American mathematics. To Deane, American mathematics was young in comparison with the mathematics tradition in Europe and doing mathematics was pioneering. As a consequence he felt a special obligation to protect and nurture mathematics in the United States. He was keenly aware of how singular the Bamberger gift was that supported mathematics at the Institute for Advanced Study. He knew then, as we still know, how difficult it is to raise endowment funds for mathematics. Over his years on the IAS faculty, any proposal to expand the Institute's activities before the acquisition of new endowment funds adequate to pay for the expansion was regarded by Deane as the threat of a dangerous animal slinking out of the woods.

And so it occurred that the mild-mannered Deane, of whom the members in the School of Mathematics were so fond, fought with iron-will against some of the Institute's Directors when he sensed peril.

Fortunately, he lived to see American mathematics grow from youth to healthy maturity. And if one reads the list of talks presented year after year in his seminars, one senses that he could take much satisfaction from his contribution to the subject he loved so deeply and to which his papers have made such profound contributions.

Deane Montgomery is a hero of American mathematics. For us who knew him, remembering Deane is a blessing.

Chung-Tao Yang
University of Pennsylvania
Philadelphia, Pennsylvania

In my mind, Professor Deane Montgomery was much more than a friend. In fact, I always regarded him as a respected teacher as well as a beloved uncle. For this reason, I named my son after him. For the same reason, I never called him by his first name. This is a Chinese tradition I have followed all the time.

For many years, Professor Deane Montgomery reserved one day a week for me to visit him. Most of the time, we discussed mathematics and suggested to each other ideas on how to attack certain mathematical problems. Occasionally we also chatted about other matters. It is impossible for me to express how much I learned from him. When we worked on the paper "The existence of a slice," one day he casually commented that a certain fact was obvious. It took me two whole weeks to work out what he meant. The depth of his thought was no doubt beyond me. The paper on smooth pseudo-free actions on homotopy 7-spheres was our answer to a question he had asked from time to time a few years earlier.

To me, Professor Deane Montgomery was not only a most outstanding mathematician, but also the greatest human being I have ever met. During my long association with him, I could observe that he took a very keen interest in helping young mathematicians who had received their degrees from less prestigious universities. His capacity seemed unlimited and his virtue was greatly appreciated and admired. I am sure that besides all of us here, many more people will miss him badly.

Ronald Fintushel
Michigan State University
East Lansing, Michigan

One afternoon during my third year as a graduate student at S.U.N.Y. Binghamton, my adviser, Louis McAuley, popped into my office and asked if I would like to talk with Deane Montgomery about my thesis. "Certainly," I answered, tongue-in-cheek, thinking that he must be joking. "How could Deane Montgomery, one of the founders of the theory of transformation groups, possibly be interested in hearing about the work of a student at S.U.N.Y. Binghamton?" Half an hour later my adviser appeared once more, announcing that, sure enough, we were driving to Princeton early the next morning.

That next day was a turning point in my career in mathematics. I appeared at Deane Montgomery's office with more than a little trepidation, but Deane's warmth and sincerity put me at ease. His office was expansive with a beautiful view of the Institute grounds. It had a large library and was arranged with a dual purpose. The rear was furnished comfortably in order to facilitate conversation. (Later I would learn that the Institute's Faculty of Mathematics would often hold their meetings there.) We talked for a while about the graduate program at Binghamton, and then moved to the blackboard at the front of the office. I explained my thesis work and Deane encouraged me and offered some ideas for future research. I returned to Binghamton invigorated and excited about mathematics. Within a year, due in a large part to Deane's help, I had a position at Tulane University.

There are dozens of mathematicians who have told me similar stories about their relationships with Deane Montgomery. He was justly famous for his efforts in helping young topologists and more generally, making all visitors feel welcome at the Institute. He was especially ardent at searching for students like myself from smaller, less prestigious graduate programs, and encouraging their careers.

In 1979–1981, I was fortunate to serve as Deane Montgomery's assistant. Throughout his career at the Institute, Deane had 21 assistants. Although most of them were mainly interested in some

aspect of transformation groups, this was not always the case. For example, M. Kuranishi, E. Moise, and C. D. Papakyriakopoulos were his assistants. And again, many were chosen from outside "prestige" departments. In order to occupy that wonderful office in the corner of Fuld Hall adjacent to Deane Montgomery's, his assistants assumed two duties — to meet with him once a week in a private seminar to study a topic of the assistant's choice, and to allow Deane to buy him a cup of coffee afterward.

During our weekly sessions I learned much about this remarkable man. Deane often spoke of his career and of the history of the Institute. I have excoriated myself several times since for not keeping a journal, for most of the details have drifted away with time. However there are certain basic aspects of these conversations which I will never forget — Deane's love of mathematics and his joy at the success of others, his gentleness and personal humility, his abhorrence of pretense in any form, his pride in the Institute and conviction to uphold its standards. Most of all, he absolutely never gave false praise. His midwestern upbringing and mathematical training at the University of Iowa gave him a point of view that often served as a refreshing foil to the intense sophistication all around him. I remember one Thursday morning topology seminar whose topic was not *au courant* and whose presentation was, to put it kindly, rough at the edges. Afterwards, when asked for my opinion, I mumbled noncommitedly. Deane's response was quite different. "I thoroughly enjoyed that," he said, and called the speaker "salt of the earth," one of his highest forms of praise.

Deane had the knack of making people feel comfortable and important. He knew that I was an avid runner; so we often talked about our exercise schedules. Deane was himself an early morning exerciser who enjoyed going for a long walk before coming to his office. Now, I am a person who likes to work in the early morning, but in my two years at the Institute, I never arrived at Fuld Hall before Deane. Usually his door was closed and I could hear the muffled sound of conversation with an early guest. Many mathematicians who came to the Institute began their visits by calling on Deane.

Because of his humility and personal distance for self-promotion, mathematicians whose work does not involve the study of transformation groups are often unaware of his many contributions to topology. Others have spoken and written about his solution of Hilbert's fifth problem, but perhaps not enough is said about his later work, especially his joint work with C. T. Yang. In a long series of papers written in the late 1960's and early 70's, they used the study of group actions on homotopy 7-spheres to showcase and test the growing new techniques of differential topology, especially index theory and surgery theory. At a time when much work in topology consisted in building these machines, their papers demonstrated the beauty of applying this theory to unfurl complexities of symmetry and structure.

As a part of this series, Montgomery and Yang studied pseudo-free circle actions, those that have no points fixed by the entire circle group, but which have isolated circles which are pointwise fixed by finite cyclic subgroups. Since a linear action of this sort on a $2n - 1$ sphere can have at most n such exceptional orbits, it was natural to ask whether such a restriction existed for smooth actions. In their papers they found a beautiful structure theory for such actions on homotopy 7-spheres and they showed that one can find examples with arbitrarily many exceptional orbits.

When I first came to the Institute, I was interested in the same question for pseudo-free circle actions on the 5-sphere, and my discussions with Deane encouraged me further. It was around this problem that Ron Stern and I first began our collaboration, and although the problem itself remains unsolved, it has been a major motivation for most of our work since then. It has served as a testing ground for our knowledge of Kirby calculus, of the theory of singular spaces, and finally of gauge theory, without ever revealing all its secrets. Yet in turn it has taught us much about 4-dimensional topology. In recent years there has been a resurgence of excitement among young researchers in calculating gauge-theoretic invariants of Seifert fiber spaces, and many facets of their interest can be traced back to the papers of Montgomery and Yang via this route.

The admiration of the mathematical community for Deane was universal. There was a large conference held in honor of his 75th

birthday at the University of Colorado and also a conference at the University of North Carolina in honor of Deane's 80th birthday. At each of these conferences many of the mathematicians spoke extemporaneously about the ways that their lives and careers had been touched by their friendship and mathematical association with Deane Montgomery. I found the story of one of the participants to be particularly moving. He recounted how, early in his career at an east coast university, his desire to be a mathematician was nearly overwhelmed by anti-semitism. It was Deane Montgomery who helped him gain the resolve to fight the bigotry and to persevere in his work.

After I left the Institute, Deane and I kept up a steady correspondence. He had developed an interest in gauge theory, and this was often a topic of discussion. I could always count on his letters for support and advice. In many ways I felt that Deane Montgomery was my mathematical father. In this sense, I have many siblings. His mathematics and his unwavering character have inspired all of us. By his actions, he has shown us how to conduct our relationships with our own colleagues and students. Although we all miss him terribly, if we follow his example, his spirit will never die.

Leo Zippin

City University of New York
Queens College, Flushing, New York

As the friends of Deane assembled here know, the names Montgomery and Zippin were paired over a period of many years.

It once happened at a mathematical meeting, a secretary who was filling out name labels for participants, made mine out as "Montgomery Zippin." When this was related to Deane, I was told that he replied, "We are both of us, flattered." Travelling in Europe, wherever mathematics was practiced, I found that the combination, as a unit, was very well known.

I never considered myself Deane's equal, except in the field of geometry where I had a keen spatial intuition. But our individual strengths combined to make for a very rewarding mathematical partnership. Deane had no shortcomings. Whatever he was supposed to do, as part of his job, he did and did superbly.

Through the years we were in constant touch. We met about once a month, either in Princeton or New York. When one or the other was travelling about the country, there were frequent telephone conversations, and many letters were exchanged.

When we abandoned research we remained in touch. We wrote to each other when he moved to Chapel Hill. I regretted that I was not able to attend the gala affair that the department held for him. I did send a telegram expressing my great happiness on the occasion.

Once again, because of advanced emphysema and vascular problems, I can not be with you.

I had two wonderful conversations with Deane just before he died. His daughter, Mary, had said that I might call him at the hospital. He sounded quite himself, cheerful, bantering, expecting to be discharged within a day or two. He joked about the fact that when he got home, the "girls" Kay and Mary would have to get used to doing more of the marketing and cooking. Then he said, "I am a little tired and I had better lie down to rest." Those were his last words to me.

K. Chandrasekharan
Federal School of Technology
Zürich, Switzerland

I salute the memory of Deane, a mathematician of exceptional originality, and a human being of exceptional integrity and strength of character, and a dear friend, who was upright, steadfast, strong, generous, and truly modest. He was held in high esteem throughout the mathematical world, as was signified by his unopposed election as President of The International Mathematical Union. He delighted in helping young mathematicians, not just from North America and Europe, but from Asia, Australia, Africa and Latin America. The School of Mathematics at Princeton owes more to him than to any other after Oswald Veblen. And the greatest tribute to his memory is to keep that School flourishing in its pristine splendor.

Raoul Bott

Harvard University
Cambridge, Massachusetts

It grieves me that I can not join you in person for this celebration of Deane Montgomery, but I will be with you in spirit.

Just today I received from Michael Atiyah a copy of an old photograph taken by Air India on the occasion of the conference at Tata in 1964. And there in our midst stands Deane with that wonderful slightly shy "American" grin, that we must all remember with pleasure, but now tinged with regret. The picture took me right back to his modest office in the math building.

Deane's office always seemed to me an oasis of calm and good will. Already the first time I entered it as a young man in '49, I was conscious of this quality. And Deane was always *there* — he kept regular hours — and he *listened* when one talked! He treated us youngsters as equals and if one had a problem, he was the first one we would turn to.

The Montgomerys were also wonderful hosts. At their parties liquor flowed freely and we all felt easy. I remember misbehaving at many of these and will never forget one memorable such occasion, which ended with a bunch of us playing marbles with Johnny von Neumann on the living room floor; Deane presiding over it all with great humor.

All of us — and especially those he helped nurture early in their careers — will always remember him not only with respect for his mathematical achievements, but also with great affection.

Armand Borel
Institute for Advanced Study
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DEANE MONTGOMERY (1909–1992)*

Deane Montgomery was born on September 2, 1909, in Weaver, Minnesota. He received a B.A. from Hamline University in 1929, a M.S. in 1930 and a Ph.D. in 1933 from the University of Iowa.

After having held various fellowships at Harvard University, Princeton University, and the Institute for Advanced Study, he went to Smith College, where he was successively assistant professor (1935–38), associate professor (1938–1941) and professor (1941–1946). During that period, he was also a Guggenheim fellow at the Institute and a visiting associate professor at Princeton University. After two years at Yale University as an associate professor (1946–1948), he came to the Institute, where he was a permanent member from 1948 to 1951 and a professor from 1951 to 1980, at which time he became emeritus.

His thesis adviser had been E. W. Chittenden, and he had a solid background in real analysis and point set topology. His initial research interests focused on the latter, to which he devoted his first four papers. In the tradition of L. E. J. Brouwer and "Polish topology," they already show considerable technical strength and expertise. As soon as he came to Harvard and Princeton, he broadened his interests, first to algebraic or geometric topology (initially on his own and in a private study group including N. Steenrod and Garrett Birkhoff), and then gradually to transformation groups, which became his major interest for the rest of his career.

His first papers in that area, many written in collaboration with Leo Zippin, were in part in the spirit of earlier work of Brouwer and Kerejarkto, aiming at characterizing groups of

* Reprinted from "Deane Montgomery 1909–1992," by Armand Borel, *Notices of the American Mathematical Society,* Volume 39, Number 7, September 1992, by permission of the American Mathematical Society.

familiar euclidean motions such as translations or rotations by topological conditions. They were motivated by questions on the foundations of geometry and, foremost, by Hilbert's fifth problem. In the broad sense, the latter asks, given a locally euclidean topological group acting effectively (i.e., no element $\neq 1$ acts trivially) on an analytic manifold, whether coordinates can be introduced to make the group and the operation analytic (the answer is no). In its narrow sense, it asks whether a locally euclidean topological group is, after a suitable change of coordinates, a (n analytic) Lie group. Variants of the first problem and the second one became points of major interest in the next fifteen years or so, but not of sole interest, though.

Among Deane's contributions to the first question, let me mention the following results pertaining to a (separable metric) locally compact group G acting effectively on a manifold M: (*i*) If G is compact, M analytic, and each transformation is analytic, then G is a Lie group (1945); (*ii*) (with S. Bochner, 1946). If M is C^2 and every transformation is C^2, and no element $\neq 1$ leaves pointwise fixed a nonempty open subset, then G is a Lie group; (*iii*) (with S. Bochner, 1947). If M is a compact complex analytic manifold and G the group of automorphisms of M, then G is a complex Lie group acting holomorphically. On the fifth problem proper, after a series of papers with L. Zippin, Deane gave a positive solution in dimension three (1948). Then came shortly afterwards the decisive results proved jointly with L. Zippin: The existence of a closed subgroup isomorphic to $\mathbf{R}$ in a locally compact, noncompact, connected, separable metric group of strictly positive finite dimension (1951) (also established by A. Gleason) and then the reduction to groups without small subgroups (1952). Since A. Gleason had just proved that such a group is a Lie group, that gave a positive answer to Hilbert's fifth problem. In fact, the whole investigation had been carried out for separable metric finite-dimensional locally compact groups and it was shown more generally that such a group is a "generalized Lie group," i.e., possesses an open subgroup that is a projective limit of Lie groups, hence is a Lie group if it is locally connected. The

assumption of finite dimensionality was soon removed by H. Yamabe, who was Deane's assistant at the time.

This was the climax of a major effort and, as I remember it, some people were mildly curious to see where Deane would turn, now that this big problem had been solved. But he did not have to look around at all. Apart from writing with L. Zippin a systematic exposition of the work on the fifth problem (1955), he just went back full time to what was really his main interest (and is already the subject matter of the last chapter of that book): Lie groups (especially compact Lie groups) of transformations on manifolds, so that, in the context of his whole work, the contributions to the fifth problem appear almost as a digression, albeit a most important one.

Even during that hot pursuit, Lie transformation groups were very much on his mind, and he brought a number of interesting contributions, in particular in joint works with L. Zippin and with H. Samelson. In fact, two papers with H. Samelson on compact Lie groups transitive on spheres or tori (1943) have a special place in my memory: When I was an assistant in Zürich, H. Hopf once gave me copies of them, and I could generalize and sharpen some of their results. This led to my first single author paper, which I submitted for publication in the *Proceedings of the AMS* to Deane, then an editor; my first contact with him.

The general problem in transformation groups is, roughly, to relate the structures of the group G, the manifold M operated upon, the orbits, fixed points, and the quotient space. At the time, there was one body of special, but deep, work, that of P. A. Smith on homeomorphisms of prime power order of homology spheres or acyclic spaces. Very little was known otherwise and Deane was a prime mover in the development of a general theory, which he pushed in many directions. He and various collaborators proved a number of foundational results, as well as more special ones, which often opened up fruitful directions for others. A survey of these contributions and of work they led to is given by F. Raymond and R.

Schultz in the Proceedings of a Conference honoring Deane on his 75th anniversary (*Contemporary Mathematics,* vol. 36 (1983)) and I shall not try to duplicate it. It ranges from basic results such as the existence of a slice (with C. T. Yang, 1957), a powerful tool to study a group action near an orbit, the existence of a principal type of orbits (with C. T. Yang, 1958), to more special ones, such as actions on euclidean space or spheres with orbits of small codimension or the existence of smooth actions of SO_3 on euclidean space without fixed points (with P. E. Conner, 1962). In a first phase, the emphasis was on continuity, i.e., on topological properties, but Deane kept up with the great advances of differential topology and soon veered more and more to differentiable actions, adapting techniques and points of view of differential topology. This led to his last major effort, a long series of joint papers with C. T. Yang on free or semi-free (i.e., free outside the fixed point set) actions of the circle group on homotopy 7-spheres, which produced notably many interesting examples of homotopy complex projective 3-spaces (1966–1973).

During his tenure as a professor at the Institute, Deane was at the center of activity in topology (algebraic, geometric, differential), one of the highlights in the life of the School, first by his seminar, a perennial feature and a meeting ground for topologists in the Princeton community, but also in more informal ways. He frequently organized seminars in his office, usually with some younger members with whom he would go through some recent developments. He was always seeking out and encouraging young mathematicians. He and his wife Kay would regularly and very warmly receive the visiting members at their home. Maybe remembering his own beginnings in an out of the way place, he had a special interest, and talent, in finding out people with considerable potential among some applicants from rather isolated places about whom not much information was available.

His concern for the Institute went far beyond his immediate scientific interests and was all encompassing. He had a very high view of the role the Institute should play and served this

ideal with unwavering and thoroughly unselfish loyalty. In day to day contacts, he was very kind, informal, full of understanding, always ready to help, and struck one as a very mild person, but deceptively so for anyone who, in his eyes, would threaten the Institute's standards, and who would then soon see rising an iron-willed and formidable opponent. His care for the highest standards at the Institute, later gratefully acknowledged in citations by the Trustees, was not always universally understood or shared at the time, so that he and like-minded colleagues had to weather some rather stormy moments, during which he was totally unshakable.

His abiding interest in the welfare of mathematics also led him to accept a number of official positions. In particular, he was Vice President (1952–1953), elected Trustee (1955–1961) and President (1960–1963, includes terms as President-Elect and Ex-President) of the AMS, where he also served on a number of committees, and President of the International Mathematical Union (1974–1978).

Honors, too, came his way: Honorary Doctor of Science from Hamline University (1954), Yeshiva University (1961), the University of Illinois (1977), and the University of Michigan (1986), as well a Doctor of Laws degree from Tulane University (1967), election to the National Academy of Sciences in 1955, to the American Academy of Arts and Sciences, and the American Philosophical Society in 1958; and receipt of the Steele Prize of the AMS in 1988.

Deane was an early riser and it was a rare event for anyone to be at the Institute before him. Being very gregarious, he talked to practically everybody working in any capacity at the Institute, which won him the respect and affection of members and staff alike and gave him an exhaustive knowledge of the Institute. Through O. Veblen, to whom he had been very close during the latter's late years, it reached to the very beginnings of the Institute so that he was a walking encyclopedia about all aspects of the Institute's history and operations.

In 1988 he and Kay moved to Chapel Hill, N.C. to be close to their daughter and granddaughters. That prospect did not

fully compensate for the severance of the ties with an institution which had meant and still meant so much to him, and it was altogether a rather sad occasion, the sadness of which was hardly mitigated by promises to keep in touch. Being myself a fairly early riser, I often started my day by knocking at his door, sure to find him, to have a chat, mostly about mathematics, mathematicians and Institute affairs. That I have not been able to do so after his departure has left for me a void which could not be filled.

Deane died in his sleep in Chapel Hill, on March 15, 1992. He is survived by his wife, his daughter Mary and two granddaughters.

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